THE PROBLEM OF LOGIC

BY THE SAME AUTHOR

THE PROBLEM OF FREEDOM IN ITS RELATION TO PSYCHOLOGY

("IN PERSONAL IDEALISM." EDITED BY HENRY STURT)

LONDON: MACMILLAN AND CO., LTD. 1902

A PHILOSOPHICAL INTRODUCTION TO ETHICS

LONDON: SWAN SONNENSCHEIN AND CO., LTD. 1904

RUDOLF EUCKEN'S PHILOSOPHY OF LIFE

Second Edition

With Frontispiece Portrait of Rudolf Eucken

Crown 8vo., cloth, price 3s. 6d. net

LONDON: ADAM AND CHARLES BLACK, SOHO SQUARE. 1907

AGENTS

AMERICA . .	THE MACMILLAN COMPANY 64 & 66 FIFTH AVENUE, NEW YORK
CANADA	THE MACMILLAN COMPANY OF CANADA, LTD. 27 RICHMOND STREET WEST, TORONTO
INDIA	MACMILLAN & COMPANY, LTD. MACMILLAN BUILDING, BOMBAY 309 BOW BAZAAR STREET, CALCUTTA
AUSTRALASIA.	OXFORD UNIVERSITY PRESS, MELBOURNE

THE
PROBLEM OF LOGIC

BY

W. R. BOYCE GIBSON, M.A. (OXON.)

LECTURER IN PHILOSOPHY AT THE UNIVERSITY OF LONDON

WITH THE CO-OPERATION OF

AUGUSTA KLEIN

LONDON

ADAM AND CHARLES BLACK

1908

TO

MY WIFE

PREFACE.

THE present volume has grown up and taken shape under the chastening influences of College teaching. No teacher of Logic would wish to underestimate the value of the education he receives from his students; and since my education has for nine years been advancing along these lines, my claim to have learnt Logic through teaching it may be accepted in sincerest good faith. A first and most grateful acknowledgment is due to my many fellow-workers at Hampstead (at the New College Centre, and at Westfield College) who by their doubts, difficulties, criticisms, and suggestions have had so much to do with the shaping of this book.

But there is a still more intimate sense in which the book is the work of many rather than of one. From the time when it was first decided to reconstruct the College lectures with a view to publication, I was privileged to enjoy the invaluable sympathy and assistance of Professor G. F. Stout. Professor Stout most kindly consented to read through these lectures, and returned them to me shortly afterwards accompanied by a small volume of criticisms. It would be hard to exaggerate the value of these criticisms. On such fundamental heads as the Laws of Thought, the interrelation of Categorical, Disjunctive, and Hypothetical Propositions, and the essential meaning of the Disjunctive and Hypothetical Judgments, the substance of Professor Stout's contentions was adopted, and will be easily recognized by all who are familiar with the Professor's logical views. Many extracts from these criticisms will be found in the present volume. Professor Stout has also allowed me to look through a large part of his own Class lectures in Logic, and has helped me in many other ways, not least through certain conversations which we have had together over fundamental logical principles.

Miss Klein's collaboration dates from the first revision of the work—from the spring of 1905. Since that date, every change in the treatment—and the reconstructions have been drastic—has been subjected to the friendliest but most unsparing criticism. No point of divergence between us but has been thoroughly discussed, and transmuted into a point of common agreement.

If, in addition to the reading of the proofs, the verifying of the quotations and the elaboration of the scientific illustrations and allusions, I may single out two respects in which Miss Klein's co-operation has been particularly valuable, I would mention her revision of the work in the interest of consistency, and her revision of it in the interest of clear expression. To show the importance of these revisions, it would be necessary to publish the original draft side by side with the final product; but as this course is not practicable, I can only assure the reader that, however he may suffer from the defects of the present treatise, his sufferings, but for these revisions, would have been incalculably worse. It has, indeed, become increasingly evident to me, as the work proceeded, that it could no longer be honestly regarded as one man's work. The original draft was the work of one; the reconstruction is the work of three.

With regard to the help derived from published treatises on Logic, my heaviest obligation has been to the works of Mill and of Sigwart, to Professor Bosanquet's 'Logic,' and to Mr. Joseph's 'Introduction to Logic.' My indebtedness to Mr. Joseph is indirect rather than direct, our points of view being quite different. But though I have been unable to assimilate either the Aristotelian or the Baconian elements which figure so prominently in Mr. Joseph's treatment, I have every reason to be grateful that his work appeared early enough to allow of my making full use of it in revising my own. Among other works which have been particularly useful to me, I would specially mention Professor Minto's treatise, 'Logic, Inductive and Deductive' (notably the Introduction to Book II., dealing with the Logic of Science), and Mr. Alfred Sidgwick's books, notably 'The Use of Words in Reasoning'; but I have also profited much by the treatises of Dr. Keynes, Dr. Mellone, Professor Carveth Read, Mr. St. George Stock, Dr. Venn, and Professor Welton. I would, in addition, gratefully acknowledge the help given me by Miss Strudwick, of the Goldsmiths' College, New Cross, in connexion with the scientific illustrations on pp. 59-62. In conclusion, I would add that if I have appeared to ignore the work of such writers as Professor Dewey or Dr. Schiller, it is not through any lack of sympathy or appreciation. I am, indeed, persuaded that the drift of the present work is convergent with that of the Pragmatic Reformation, and that the stress laid on relevancy is a vital bond of union between ourselves and the Pragmatists. But the central contentions of Pragmatism concern the Logic of Experience, and cannot, therefore, be appropriately or adequately treated in the pages that follow. We hope to consider them in a later work.

The present volume aspires to be the first part of a 'complete' treatise on Logic, of which the second will deal, or attempt to deal, with the Logical Problem in its more philosophical aspect. Some

brief indication as to this programme will be found in the Introduction. Here it may be enough to state that the Religious Idealism in which the author's own conviction culminates seems to him to call imperatively for a frank and fruitful co-operation between the Idealism of the Hegelian School on the one hand, and the Psychologism of the Pragmatic and Genetic movements on the other. In attempting this reconciliation, so far as it is relevant to the requirements of a logical treatise, the author ventures to hope that he may be found working in the service of that liberating movement in Philosophy which, in his own mind, is centrally associated with the work and personality of Professor Eucken.

The promise of a sequel is no doubt a convenient shield for sheltering an author—though, indeed, only temporarily—from any charge of incompleteness in his treatment. I would claim this shelter as regards the discussion of the principles of Mathematics in their logical bearing. I hope to deal with this important problem in the sequel.

I am much more doubtful with regard to the general problem of Symbolic Logic. Whether, in postponing the discussion of this department of Logic, I am or am not shelving its consideration altogether, I am not now prepared to say. In no case would I contest the interest and importance of Symbolic Logic; but whether the limitations of my programme—or of my own powers—may not render its discussion irrelevant—or impracticable—is, perhaps, a pardonable question.

The distinctive feature of the present volume will, I think, be found in the dominating position assigned to the idea of *relevancy*. The fundamental concepts of Truth and Reality have been defined in the light of this category, and the principle of *Fidelity to Relevant Fact* has been adopted as the master-key to all the main positions, including the central problem of a Formal treatment, and its relation to a material treatment of Logic. I would also draw attention to the distinction between the functions respectively assigned to the Laws of Non-Contradiction and Excluded Middle. This distinction will be found to be directly connected with that between a Formal and a material treatment of the logical problem.

In conclusion, I would gratefully acknowledge the work done by Miss Klein in the framing of the Index. The Index is her work, and she alone is responsible for it.

W. R. BOYCE GIBSON.

CHARDONNE SUR VEVEY,
May 10, 1908.

ERRATA.

Page 337, line 30, *instead of* 'Neptune's irregularities,' *read* 'The irregularities of Uranus.'

Page 404, line 27, *instead of* 'Agreement,' *read* 'Difference.'

CONTENTS.

THE PROBLEM OF LOGIC

I.

INTRODUCTION.

Logic is the mind's systematic attempt to understand the nature and the conditions of the search after Truth. To the question, 'What is Truth?' we would answer by suggesting the following provisional definition:

> Truth is the Unity of ideas as systematically organized through the control exercised by relevant fact.
>
> *Or—*
>
> Truth is the Unity of Thought as systematically organized through the control exercised by that aspect of Reality which is relevant to the purpose of the thinker.

With a view to bringing out the meaning of these definitions, we must state in the first place that we do not regard Truth as a datum, but as a problem. The truth we seek cannot be that from which we start, for were truth already attained at the outset, no sufficient reason could be assigned for proceeding any further with the quest. We might, of course, regard the Truth as given, and devote our energies to its systematic exposition and application. But, in that case, we should have radically to alter our definition of Logic. Logic would no longer deal with the Search after Truth, but would be busied solely with the question of its consistent presentation. Logic would just mean Consistency-Logic, and might be defined as the mind's systematic attempt to understand the nature and the conditions of a correct presentation of the Truth. But, valuable as such a Consistency-Logic would be, its logical value would lie, not in its relation to a system of given truth, but in its analysis and development of the laws of consistent thinking.

We would draw attention, in the second place, to the fact that Truth is defined as a Unity, and that to define Truth as a Unity is to ground logical inquiry on a monistic basis. We cannot, of course, justify monistic faith by merely asserting it, nor, by asserting it,

make our meaning clear. 'Monism' is a catchword as dear to the Rationalism of Hegel as it is to that of Haeckel, and we suspect that much that calls itself Pluralism is but Monism in the making. It is indeed a much-abused word, and we introduce it thus bluntly, at the outset of our inquiry, not as a dogma but as a problem. To justify our monistic faith we need here do no more than justify our right to accept the struggle for complete unity of thought as the fundamental mark of the truth-seeker, and the attempt to define the nature and conditions of such a struggle as the distinctive function of Logic.

We might justify this right by presenting it as a necessity of our logical reason, and contend that it is meaningless to suppose that Unity of Thought and Purpose can be ultimately satisfied by anything short of the Unity of the Universe. Or we might defend our monistic faith as a postulate limiting the scope of our inquiry, and proceed confidently with our venture, more than content with the perfect freedom conferred upon us by our own self-limitation. We would prefer, however, to point quite simply to a certain insatiability of logical appetite as the best justification for our Monism. For if we forgo or evade the struggle after Unity, we really do limit ourselves in quite a literal and painful sense. We renounce the hope of a logical conquest that shall leave us nothing foreign or unsubdued to mock us with its alien nature. We abdicate a fraction of our empire, and must live in perpetual dread of border troubles, of disturbances emanating from those shadowy entities—the dim hosts of the ununifiable. And can one imagine thought surveying such a chaos from the edge of its own self-limited domain and still deliberately disclaiming a redemptive mission? Is it not thought's nature rather to weep because, like Alexander, it sees no further worlds to conquer? Our sufficient apology, then, for regarding the Truth-problem as a search after Unity is that logical ambition can be satisfied with nothing less, and cannot endure the sight of chaos battening for lack of its two-edged sword.

We turn, in the third place, to our contention that *relevant fact* is the agency which controls the process through which our thinking becomes systematized. The precise function of the expression 'relevant fact' is to indicate that truth implies at once a reference to purpose and a reference to reality; and the second of the two definitions of Truth that we have given explicitly brings out this implication.

Thought submits itself to fact as the experimenter submits himself to the object experimented on. As the experimenter determines the conditions under which the experiment shall take place, so thought selects and determines the aspect under which the facts shall be thought. The purpose of the inquiry, be it that of the physicist, the biologist, the artist, or the mystic, determines the range of fact within which the student of Nature recognizes an

objective control. It is true that to conquer Nature we must obey her ; but we must know clearly what it is that we obey, and to this end must first select and mark out the domain that we have then to conquer through submission. The investigator of Nature is thus at once self-controlled by his own purpose, and outwardly controlled by the facts in so far as they are germane to that purpose. In a word, he is controlled throughout by 'relevant fact'—*i.e.*, by the objective nature of that aspect of the universe which is relevant to his subjective interest.

We thus reach the conclusion that the conception of Truth from which we set out itself determines the principle which must dominate and inform our whole attempt to realize it. If the Unity of our thought is to be shaped through the pressure of relevant fact, then fidelity to relevant fact must be the fundamental principle through which growth in Truth is determined, and it must also figure as the standard or criterion of any inquiry into the conditions of its attainment. So we take it as our guiding clue through the mazes of the logical problem.* We shall realize its determinative influence, not only in the problems of definition and division, where it operates in the interests of Order and non-ambiguity, but even in fixing what we mean by meaning—the 'meaning' which these processes serve to develop. Again, the reference which the principle implies to purpose, and through purpose to reality, will be found to enter into the very conception of a complete logical judgment ; whilst, in methodology and the problem of scientific explanation, this principle of fidelity to relevant fact will be explicitly sustained as the fundamental principle and standard of Induction, and rendered determinate in the light of the Inductive Postulate.

Let us now apply these general considerations to the special case of the present inquiry. The truth we have in view is Truth in so far as it can relevantly serve as an Ideal for a pre-philosophical Logic. When preparing for more difficult flights—*e.g.*, for a truth-journey down the abysmal depths of personality—Logic might reasonably desire to equip itself with a more penetrating conception of Truth than is required for its more preliminary labours. If Truth is, in all cases, the Ideal which we aim at progressively realizing through Knowledge, and is conceived as that which can adequately satisfy the thinker's will to know, then the Truth-Ideal will vary with the view we take of Knowledge, and also with the depth of this will to know. By Knowledge we may understand Self-Knowledge, and the depth of the truth-interest will then be measured by the depth of the

* The relation of this Principle to the Laws of Thought may be stated in the simplest way by saying that the former presupposes the latter. But it is only in the purely Formal treatment of the logical problem, in connexion with the problem of Inference, that the Laws of Thought, as we understand them, can be accepted as an adequate logical standard. Where the truth-interest is present, a concreter principle —operating, of course, in conformity with the Laws of Thought—is required to give positive direction to our thinking.

self that is seeking for truth-satisfaction ; we should then be concerned with the profoundest questions—with Freedom, Personality, Perfection, Immortality, God—questions which spring from the unrest and dissatisfaction of our deepest self. But if by Knowledge we understand, not Self-Knowledge, but Knowledge about Things, Knowledge of that which we apprehend through the senses, we may well be content with a less intimate specification of the meaning of Truth. We reach this more restricted conception of Truth through marking out the realm of fact which we take to be relevant to the limited requirements of a pre-philosophical treatment—in a word, by defining what we here mean by Reality.

Under ' Reality ' we shall include two main aspects of Fact :

1. The world as common sense understands it (or some conventionally restricted fragment of it).
2. Nature, understood as the subject-matter of Science.

In bringing the worlds of Science and Common Sense thus closely together, we are making an assumption which it is important to notice. We are assuming that the attitude of common sense to the more or less fragmentary world within which its interest is restricted is, on its own humbler level, similar to the attitude of Science towards Nature. It may, however, be objected with good reason that in thus characterizing the common-sense attitude towards reality as pre-scientific, we are doing injustice to the ordinary consciousness, which, over and above its interest in a world external to it, has interests of a personal and social kind. The objection in itself is perfectly legitimate. The ordinary consciousness is religious as well as practical, and has inward as well as outward looking views as to the nature of truth. If sense-experience rests its beliefs on an ' I have seen,' the intuitionism of the moral and religious consciousness rests its beliefs on an ' I have felt.' In the one case truth is taken to be the truth about an object, the truth about a fact ; in the other, it is taken as the truth for a subject, the truth for a person.

The implications of the more inward conception of the meaning of truth are of fundamental importance, and, at a more advanced stage of logical inquiry, their discussion becomes imperative. But for our present purpose—*i.e.*, for the purposes of a pre-philosophical Logic—we propose to ignore this personal, inward interpretation of the truth-problem, and the deeper view of Reality which would correspond to it. At the same time we must remember that the definition we have provisionally laid down does not do full justice to the truth as it is presented to common sense. It imposes a restriction which reduces common sense to an infra-scientific standpoint. Only when common sense is thus restricted can Science be regarded as its completion and rectification. Only when we have eliminated as irrelevant the relation of truth to personal

experience can we fairly describe Science as organized Common Sense.

The deliberate exclusion of the personal element from the definition of truth may appear to some to be unjustifiable even when the definition is given solely from the scientific and infra-scientific points of view. The objection may be raised that, since the reference to reality which is implied in all truth-seeking, whether scientific or infra-scientific, can be characterized and defined only through relation to logical purpose, we cannot study Reality at any stage without introducing the personal element. It is quite true that the truth-definition which we have adopted explicitly includes a reference to purpose. But this mere *reference* to purpose in no way commits us to a personalistic view of truth or of reality. On the contrary, it may so define the reference to reality as to render such a view irrelevant and impossible. How this reality-reference has been defined, in the interest of a pre-philosophical treatment, we have already seen. The limitation ensures that Truth shall be truth about fact, and not the truth of personal realization. To reach the philosophical conception of truth, we must study Fact in the light of a philosophical truth-interest, and adopt a correspondingly philosophical conception of reality.

It is true that reference to purpose *implies* reference to a deeper reality than that reality of nature the conception of which it serves to define, and that in this important sense the scientific point of view implies and presupposes the philosophic ; but the implication remains latent, and the scientific and pre-scientific conceptions of truth and reality correspondingly impersonal and objective.

There are, we may say, three main stages in the life of Logic. In its first, formal, or common-sense stage, Logic presents itself as a propædeutic, or preliminary discipline, and the truth-ideal which it then presents to thought is truth as involving the relation of thought not to the reality of the Natural Order, but to a reality of a more or less restricted and conventional kind. The point of view, in a word, is essentially formal in the sense of conventional. There is no reference to a permanent order like that of Nature as conceived by Science, but only to such conventionally restricted aspects of it as answer to the requirements of some particular purpose.

In the second, real, or scientific stage, the casual, disconnected grasp on Reality which these conventional restrictions involve is definitely abandoned. Thought ceases to play with Reality in the interests of discussion, or of other requirements of practical intercourse. Armed with the idea of natural law, it now disposes itself to face the full force of that great realm of fact which has no limit but that of the applicability of the idea itself.

And yet this second stage is not final. It presupposes a relation of externality between fact and idea, and is broken through when this externality is done away with, and Truth shows itself as the

intimate oneness of idea with fact. The complete setting forth of this unity is the function of a philosophical Logic. Briefly, it amounts to the idealizing of fact and the realizing of ideas within a conception of experienced fact larger than is possible to Science or appropriate to its restricted point of view. In this third stage, Thought, as Hegel would say, finds itself at home with itself, freed from all fettering abstraction, and at the very heart of the reality it is its mission to understand. What remains is then just the systematic articulation of the structure of this experienced fact, at once most real and most ideal—the Logic of spiritual experience.

This Personalistic Logic, as already stated, lies beyond the scope of the present treatise; the following course covers only the first two stages. The earlier stages, however, are essential to the proper grasp of the third and last. For the lessons of each earlier stage are taken up into the succeeding one in a form determined by the richer, concreter conditions of the latter. Thus, what is gained at the one level is not lost at the next, but transcended and redeemed. The 'Reason' of Philosophy must have assimilated the 'Understanding' of Science, the passion for distinctness and precision, which is characteristic of the scientific attitude, and its loyalty to relevant fact. Loyalty to ideals can bestead Philosophy but little if it does not, in its own appropriate way, include reverence for fact as an integral requisite of all true spiritual experience.

In the foregoing attempt to define the point of view adopted in the following treatise, the meaning of the word 'formal' deserves particular consideration. For it is more customary to identify the term 'Formal Logic' with a Logic of Validity than with a treatment in which is implied a merely 'formal' reference to reality. In particular, the word 'formal' is associated with the so-called Forms or Formal Laws of Thought as the principles upon which consistent thinking ultimately depends. Thus, in using this ambiguous term, it is essential that we should not confuse the two meanings. We propose, therefore, in the interests of clearness, to adopt the following device. When 'Formal' is being used in its fundamental sense of 'abstractly valid,' we shall employ a capital F; when it is being used in the sense of 'conventional,' we shall write the word with a small 'f.' Should the word open the sentence, and the capital letter be indispensable, we shall leave it to the context to decide in which of its two senses the word is being used.*

The distinction between a formal and a real logical treatment is a distinction within a unity. Both methods equally imply a funda-

* Perhaps the strongest reason for retaining two such closely similar words to designate meanings apparently so different is that the meanings are not so unrelated as they appear to be. A 'Formal' treatment of Logic might be considered as a 'limiting case' of a 'formal' treatment of the subject—the case, namely, where the conventional restriction put upon the meaning of Reality is such as to reduce it to an essentially hypothetical status (*vide* p. 145).

mental respect for consistency, and they both involve a reference to reality, though the reference is occasional in the one case and systematic in the other. We do not, then, propose to keep the two methods separate. We propose, on the contrary, to discuss the real in close connexion with the formal aspect, and thereby to secure a unity of treatment which would be forfeited by the attempt to deal with the two aspects successively and in isolation from each other. When we are interested in emphasizing what is common to these two types of logical treatment, we propose to use the word 'material' to cover both. Thus, a material logical treatment may be either formal or real.

In contrast with a material treatment of Logic, we have what is customarily known as a purely Formal treatment. We shall find that at a certain stage in the development of our subject it becomes essential to abstract entirely from the reference of thought to reality as we have defined it (*vide* p. 4),* and to concentrate our whole attention on the logical conditions of valid thinking. When our logical interest is thus rigidly restricted, and reduced to an interest in validity, the treatment ceases to be material, and becomes Formal.

The chapters on the Laws of Thought and their application to the problems of Opposition, Eduction, and Syllogism are the chapters essential to a strictly Formal treatment. The ideal of (material) truth, which alone gives meaning to the distinction between 'formal' and 'real,' here gives place to the ideal of validity. The reference to reality implied in all reasoning whatsoever is tacitly ignored as 'accidental,' and the primary logical requisite, the requisite of validity, monopolizes the attention. Whatever reference to truth or falsity there is in Formal Logic is wholly hypothetical. If the statements 'All donkeys are daffodils' and 'All dragons are donkeys' are both accepted, accepted as though they were true (whether, as a matter of fact, they *are* true is here a completely irrelevant question), then Formal Logic insists that the statement 'All dragons are daffodils' must also be accepted, accepted as though it were true.

The Validity-Ideal, which is regulative of a Formal logical treatment, implies the twofold requisite of *Self-consistency* and of *Inter-consistency*. A statement or an argument is self-consistent when it so hangs together that thought may pass through it, as it were, from beginning to end without falling into contradiction with itself by the way. The statement, 'Square tables are round' violates this fundamental requirement. So does the following argument:

> 'All men are rational animals.
> Nebuchadnezzar was a man.
> Therefore, Nebuchadnezzar was not a rational animal.'

* *Vide* note, p. 9.

We cannot maintain, without illegitimate variation in our use of words, that all men are rational, and that one is not so.

The Interconsistency of our statements is as important as their Self-Consistency. The diligent reader may discover on different pages of a connected treatise statements which no charity can construe as interconsistent. The statements may be separated by more than a hundred pages, but the requisite of Interconsistency will still compel a logical readjustment of the passages such as will make them maintainable together by one and the same thinker in one and the same discourse. The coherency of our thinking is essentially dependent upon a faithful observance of the requisite of interconsistency.

Logical Consistency should be carefully distinguished from Material Compatibility. Whether the assertion that my friend takes no regular exercise is compatible with the statement that he continues to enjoy robust health, and is in that sense 'consistent' with it, is a question that concerns material truth. A treatment which ignores all considerations of truth and falsity* cannot possibly say anything relevant upon the matter.

Logical Consistency should be distinguished from logical Validity. The meaning of the former is at once wider and more negative than that of the latter. Consistency implies mere freedom from self-contradiction ; Validity, a connexion so close that the severing of it would involve a contradiction. If we say 'Some people are reasonable,' it is quite consistent to add 'Some people are not reasonable'; but, as we shall see (*vide* p. 174), we could not validly infer that some people are not reasonable from the statement that some people are. An argument is said to be valid when the conclusion drawn from the premisses is such that we must accept it, once the premisses have been accepted. A conclusion drawn in this way from its premisses is said to be drawn from them with logical necessity, and is known as a valid conclusion. So, again, the proposition 'If all men are mortals, some mortals are men' is a valid proposition, since the acceptance of the 'if' clause necessitates our accepting its consequent. The statement 'If all men are mortals, all mortals are men,' is invalid if taken as asserting a logical connexion, though it is not inconsistent.

We should also note the distinctively negative character of Logical Consistency. Logical Consistency does not amount to systematic coherency. The coherency of a scientific system means much more than mere freedom from self-contradiction.

We conclude this Introductory Chapter with the following brief résumé of its main points :

Logic is the Science of Right Thinking.
To think rightly we must think both consistently and truly.

* *Vide* note, p. 9.

To think consistently is to avoid all self-contradiction. If we think as logical necessity requires, our thought is said to be valid.

Consistent = not involving contradiction = not inconsistent.

Valid = involving logical necessity.

Inconsistent = involving self-contradiction.

Invalid = not involving logical necessity.

To think truly is to think under the control exercised by that aspect of Reality which is relevant to the purpose of our thinking.

Under Reality, as relevant to the truth-interest of a pre-philosophical discipline, we include the world of Common Sense —the world in relation to our various practical interests— and Nature as understood by Science.

In either case, this reality is conceived as having a nature sufficiently stable to control our tentative thought about it.

When the reality we have in view is limited by some practical interest, the logical ideal is satisfied in proportion as our ideas adjust themselves to the control exercised by this conventionally limited reality. Ideas so adjusted may be said to be *formally* or conventionally true, true in relation to our restricted practical purpose.

When the reality is Nature as conceived by Science, the controlling of our ideas through reality is said to give us real or scientific truth.

Finally, when our sole interest is in the validity of our thinking, the question whether the reference of our thought to reality is *formal* or *real* ceases to be relevant ; for we are here no longer concerned that our thought shall be true, but only that it shall be valid.

The treatment of right thinking which is thus exclusively regulated by the Ideal of Validity is known as Formal Logic. Whatever reference there is to truth or falsity in Formal Logic is wholly hypothetical.

The *Formal* treatment of right thinking should be carefully distinguished from a *formal* reference to reality, a Formal treatment being a treatment in accordance with the Formal Laws of Thought, the laws of logical Validity.

By 'Formal' we mean dominated by the Ideal of Validity.

By 'formal' we mean 'conventional.'

NOTE.—There is a certain misconception with regard to our use of the term 'Formal,' which our very definition of a Formal treatment may have served to foster. We have stated that a logical treatment can be called Formal only in so far as we abstract from all reference to truth or reality ; and if the definitions which, in the interest of a pre-philosophical treatment, we have given of these

same terms are not borne carefully in mind, the reader may be left with a very poor opinion as to the status of Formal Logic. Formal Logic will seem to be concerned essentially with some abstract department of Non-Being.

If we turn, however, to the definitions of truth and reality, as given on pp. 1, 4, or in the résumé, p. 9, we shall readily see that no such disparagement of a Formal treatment is intended or implied. In abstracting from all reference to reality *as we have defined it*, we do not abstract from all reference to all reality. It is only when the pre-philosophical definition of reality which we have adopted is mistaken for the ultimate meaning of reality that a Formal treatment of Thought appears unreal, and, in its detailed application, tends to degenerate into mere mechanical drudgery, on the one hand, or, on the other, into irresponsible explorations within a purely artificial world.

The abstraction from all reference to *material* reality and truth still leaves us with the reference of thought to itself ; and when this self-reference of thought, together with the problem of Validity which it involves, is studied under the redeeming conditions of philosophical insight, Formal Thinking gains a vital, a spiritually vital significance. Assuming a *philosophical* definition of Truth—as we understand the term 'philosophical'—the interest in Validity is itself an interest in Truth.

II.

LOGIC IN ITS RELATION TO LANGUAGE.

CHAPTER I.

II. (i.) WORDS, THEIR FUNCTION AND RIGHT USE.

The Function of Words.

PROPOSING as we do to start in the humblest and most methodical way in our investigation of the nature and conditions of Truth, we look first to the tool or instrument we shall be dependent on all through—namely, Language.

Logic, like every other science, depends on language, written or spoken, as its only suitable instrument. In Grammar, which considers words in themselves and in relation to each other, Language is the subject-matter treated of as well as the instrument; but it is not so in Logic. Logic is concerned with language only as an instrument of thought, and its aim is so to handle the instrument as to make it a help and not a hindrance to correct thinking. Since thought can be handled only in verbal form, the regulative function of Logic, directed primarily upon thought itself, is inevitably pressed upon language as well. Language must reveal thought and not falsify it.

Rhetoric, too, is concerned with language and the right use of words. But whereas Logic aims at the right use of words with a view to correct thinking, Rhetoric aims at the right use of words with a view to persuasion. The purpose of Rhetoric is to prove practically effective, and its appeal is therefore made to the whole man, to his emotions and humours as well as to his reason. As a science, at any rate, Logic is concerned with theoretical soundness rather than with practical efficiency. As an art it may be said to aim at practical efficiency, but its appeal is still made exclusively to the reason.

Over this instrument, Speech, Logic proposes to exercise appropriate supervision. But supervision, to be logical, must be in accordance with the nature of what is supervised. Before we consider the right use of words, we must learn something of their natural function in relation to thought.

The main function of words is to fix meanings or ideas both in our own minds and in those of our fellows. If I wish to see an object clearly, I bring it into the focus of vision. This I do instinctively through the help of a number of delicate eye move-

ments. There are movements of convergence of the two eyes, of accommodation to near or far vision, and focusing movements. These contractions of the eye-muscles enable us to fixate the objects we look at. Similarly, we fixate smells by setting our nasal muscles in action, and so inhaling or sniffing upwards. We fixate a taste by setting the muscles of the palate in action, and pressing the food on to the palate. So with the ear-muscles in hearing. A horse will 'prick its ears' to fix a sound. It is in a perfectly analogous sense that we utilize the muscles of lips, tongue, larynx, for toning and articulating our breath into sounds that bring our meaning fixedly before us. Thus, we control the utterance of our thought by means of a certain special set of muscles, the muscles involved in controlling the breath so as to produce articulate sounds.

The function of words is to fix ideas, and this in a twofold sense. For not only do they serve to *impress* meanings on ourselves who think; they also serve to *express* our meanings to others, and are then known as expressive signs.

These should be distinguished from substitute signs. An expressive sign is meant to express meaning, whereas a substitute sign is a counter which can be manipulated without our knowing what idea it stands for (*cf.* Stout, 'Analytic Psychology,' p. 193). Thus, algebraical symbols are used as substitute signs. I may start by positing that x shall stand for the number of cows a certain farmer bought; but I may go on to solve the equation $x^2+3x+2=20$ without thinking any more about the cows. I am concerned solely with the algebraical laws according to which I may profitably operate upon the sign. It is only when the value of x is found that I think about the cows again.

Such substitute signs are not words. If I say 'S is P,' or 'All S is P,' S and P are not words. They would be 'words' only if they were intended to fixate attention on the letters of the alphabet indicated. They are mere symbols, and do not call attention to their meaning. 'A word,' it has been well said, 'is an instrument for thinking about the meaning which it expresses; a substitute sign is a means of not thinking about the meaning which it symbolizes' (*ibid.*, p. 194).

But to return to the natural function of expressive signs, which is to fix meanings with a view to rendering them unambiguous and stable. Meanings are naturally volatile; in Hegel's expressive phrase, they have hands and feet. It is indeed no easy task for words to keep even pace with the march of thought. While the meaning runs through a succession of changes, the word has a way of remaining unchanged.

The change in the meaning of a word tends to take place in one of two opposite directions: it may become more generalized, or it may become more specialized.

Instances of Generalization:

(*a*) 'Journey' and 'journal.' 'Journey' (Fr. *journée*) was originally one day's march. 'Journal,' originally a daily paper, has been generalized to include 'weekly' as well.

(*b*) 'Charm' and 'enchant.' From Lat. *carmen*, 'song or incantation,' and 'incantare.' In Elizabethan English both words involved the notion of 'spell, magical power.' Portia says to Brutus: 'I charm you. . . . That you unfold to me why you are heavy' ('Julius Cæsar,' II. i. 271). Here 'charm' means 'lay a spell upon,' and so 'adjure.' *Cf.* Milton's 'Samson Agonistes,' 934: 'Thy fair enchanted cup and warbling charms.' As the belief in magic declined, the meanings of both words widened, so as to include influences other than magical. *Cf.* also 'villain' and 'clerk.'

Instances of Specialization:

(*a*) 'Success.' In Elizabethan English its usual sense is 'result,' 'fortune,' whether good or bad. *Cf.* 'Troilus and Cressida,' II. ii. 117: 'Nor fear of bad success in a bad cause.'

(*b*) 'Stare,' 'to stiffen, stand on end,' is used in Shakespeare of hair as well as eyes. Brutus says to Cæsar's ghost: 'Thou mak'st my hair to stare' ('Julius Cæsar,' IV. iii. 280).

(*c*) 'Knave' was originally 'boy' (German '*Knabe*'). The word seems to have been specialized in so far as it now implies dishonesty, and at the same time generalized to include man as well as boy.

But the intrinsic vitality of thought presses in a still more fundamental way against the pretensions of language to fix it. The meaning of words is always tending to vary with the context. Adopting Professor Stout's terminology, we may conveniently refer to meaning fixed by context as 'occasional' meaning, and oppose it to meaning fixed by usage, to what we may call the 'dictionary' meaning of a word. We may look upon the usual interpretation as a sort of fictitious mean position about which the meaning of the term oscillates, and the occasional meanings as the slightly divergent positions where the balance has oscillated somewhat from the mean position. Thus, if we compare together the following expressions: 'the Queen of Sheba,' 'the Queen of the May,' 'the Queen of the hive,' 'the Queen of Hearts,' 'the Queen of puddings,' we shall notice that the word 'Queen' rings differently in the different phrases. Its living meaning varies from phrase to phrase: a queen in Solomon's palace is not a queen in the same sense as in a pack of cards or even in a hive.

As an illustration of the influence which context exercises over meaning, Professor Bosanquet's analogy ('Essentials of Logic,' p. 55) may be appropriately cited. He is speaking of a very fine Turner landscape which in 1892 was in the 'Old Masters' Exhibition' at Burlington House—the picture of the two bridges at Walton-

on-Thames. The picture is full of detail—figures, animals, trees, and a curving river-bed. Experts tell us that the organic unity of the parts of that picture is such that, if we were to cut out the smallest appreciable fragment of all this detail, the whole effect of the picture would be destroyed. Now consider this patch of colour which we will suppose has been cut out. If seen on a piece of paper by itself, it might be devoid of all significance ; but put it back into its proper place, and it shares at once in the whole beauty and meaning of the picture, takes its part in the picture's life. So a word (colourless enough when seen by itself in its usual meaning as conventionalized by definition), when placed in an appropriate setting, takes on at once the glow of the context.

The Right Use of Words (*Logical Aspect*).

The essential function of words being to fix meanings, the supervision which Logic exercises over them must consist in guiding and rectifying this intrinsic tendency of language so as to make it the best possible medium for expressing the truth. The essential fact we have to reckon with in this regulation of the function of language as the expression of ideas is that ideas show an intrinsic plasticity and indefiniteness, that meanings grow and vary with the context. Hence, any policy which tends ruthlessly to stereotype the meaning of words would obviously run counter to the proper fulfilling of the essential function of language, which is to express thought. If such definite fixity is imposed upon the use of a word, it will be for special purposes, as when, in the case of the elaborate technology of Science, every other requisite of expression is subordinated to the paramount desideratum of precision.

This natural tendency of words to fix the meanings they express receives its true logical guidance from the Principle of Non-Ambiguity. This is not the same as the Principle of Identity to be discussed further on, and if we venture to call it the first law of correct and consistent thinking, it is first not for thought itself, but for us who are making our way gradually towards the more inward principles that express most truly the nature of our thinking. It is essentially a limiting or negative principle. It insists, in the interests of right thinking, that the natural indefiniteness and fluency of our meaning shall never reach the point of ambiguity. But it has no quarrel with an appropriate indefiniteness in the use of words, provided this indefiniteness is definite enough for the purpose—*i.e.*, does not amount to ambiguity. In this sense we see the truth of the saying that Logic is the medicine of the mind. It is only when ambiguity is felt that Logic presses upon us its remedy of definition.

In interpreting and regulating the tendency in language to render our thinking determinate, Logic has not infrequently to unfix in order to fix better. It unfixes the casual non-purposive associations

that have grown up at random, undisciplined by reference to any self-consciously held ideal, practical or theoretical. Language, if unthinkingly used, plays the tyrant over our thinking. We may easily become the slaves of words. We may allow a word to gather about it a cluster of subjective associations with which we insist on investing it whenever it is used, never troubling to inquire whether the word in the new context, or as used by the author we are studying, does not mean something quite different from such meaning as we have come to attach to it. In the interests of right thinking, words should stand loose from such associations, so as to take on any desired meaning, the logical ideal requiring only that the meaning shall not involve any ambiguity or unreasoned inconsistency.

CHAPTER II.

II. (ii.) DEFINITION AND THE PREDICABLES.

Definition per Genus et Differentiam.

In ordinary talk we are not over-careful of the right use of words, provided we can make ourselves sufficiently intelligible for practical purposes. If a friend happens to use a word with which we are not familiar, we ask him what he means by it ; but we are, as a rule, quite satisfied with his answer if it be sufficiently definite to show us what he is referring to. We are satisfied if he *describes* to us the meaning of the unfamiliar word. Mr. Alfred Sidgwick has given a name to this kind of information. He calls it ‘ translation.’ ‘ Description ’ seems, however, a simpler and more satisfactory term. Description in this sense consists in giving a general account of a word’s meaning. It gives us the rough meaning of the word. Mr. Sidgwick is anxious, and rightly so, that we should not confuse description (or unelaborated definition) with definition proper. Etymologically, definition means marking out the limits or boundaries of the use of words, and this, as a rule, we never trouble to do in ordinary discourse. We are content to speak with a certain amount of useful vagueness. The words we use are clear enough at their centre, but they have misty edges. Indeed, apart from a certain inherent indefiniteness of contour, they would cease to be really useful ; for it is the very indefiniteness of words which permits of their taking on different shades of meaning according to context. But, as Mr. Sidgwick points out, indefiniteness does not mean ambiguity, though it is a precondition of it. If a word were definite through and through, with clear-cut edges in addition to a well-marked centre, it could never be ambiguous. Words become

ambiguous when their inherent indefiniteness has become such that it perplexes the meaning of what we say. Take the word 'Liberal.' 'The indefiniteness,' says Mr. Sidgwick,* 'which was latent in the name up to the beginning of April, 1886, became a few months afterwards so patent as to cause ambiguity; within what used to be called the Liberal party there had come to light two sub-classes, each of which denied to the other the right to the name.' The single meaning had split in two; the word had no longer one well-marked centre, but two; and so long as we were not told, on being spoken to about Liberals, whether C_1 or C was being referred to, ambiguity would arise.

We conclude, then, that if we would use our words rightly, we must be able—(1) to recognize the point at which definition becomes necessary; and (2) to know how to set about discovering the definition when required.

To sum up as regards (1), we have to recognize that, even when there is doubt as to the meaning of a term in an assertion, a definition is not necessarily called for. To define a word formally is to mark off its edges from the encroachments of other words, and there is no point in being precise about the edges if there is uncertainty about the centre. A definition, in fact, is rarely wanted unless the rough meaning of a word is already known. If the difficulty in grasping the meaning of a sentence arises from unfamiliarity with any word, description is called for, not definition; but if an actual difficulty is felt in applying a familiar word correctly in a given case—that is, whenever the latent indefiniteness natural to the word is actually causing ambiguity—then definition is called for.†

If it is called for, how are we to set about the work of defining? The natural answer is: Through a process of Comparison. Words at their outer edges are in contact with other words, and the respective sphere of influence of each can be marked out only by comparing and adjusting the meanings. To define a word, we must compare it with such words as are most closely related to it in meaning. This gives us the Genus and Differentia. The genus includes the marks which the word has in common with the rest; the differentia those which distinguish it from them.

We may express this result in a slightly different form. Definition, we may say, is the process whereby we assign to a word—(1) its class-designation, and (2) the specific difference which serves to distinguish it from all other words that share the same class-designation.

Experience shows that, though nothing is in all respects like any other thing, yet things can be separated out into groups, each group comprising all those different objects which resemble each other in

* A. Sidgwick, 'The Use of Words in Reasoning,' p. 196.
† *Ibid.*, p. 49.

certain points—P_1, P_2, P_3, P_4. The objects are then said to be *classed*, and the class-name defined, by these common marks—P_1, P_2, P_3, P_4. Anything that possesses these common marks is then designated by the class-name, also called the general name. Further, the class-name, as such, cannot possibly specify distinctions between the included sub-classes. The name 'horse' cannot inform me whether a cart-horse or a race-horse is in question. If I wish, therefore, to specify a particular section of a class, or, in other words, to differentiate a species from the genus, I must add a qualifying mark, or differentia. Thus, if I wish to define the kind or species of vehicle known as 'omnibus,' I ask myself: What is the *genus* or class under which this species falls, and what is the differentia, or specific mark, whereby it is distinguishable from whatever other species fall under the same genus? Now, practically, as we have seen, we answer this question by bringing together as many words with closely related meanings as possible, and comparing them. Let us compare 'omnibus,' for example, with 'tram.' The terms agree in designating four-wheeled public vehicles; they differ essentially in this: that, whereas the one designates such vehicles of this kind as are confined to rails, the other designates such as are not confined to rails.

Genus: Four-wheeled public vehicle.

Species:	Omnibus.	Tram.
Differentia:	Not confined to rails.	Confined to rails.

If we had compared the two terms 'omnibus' and 'cab,' we should have had some such result as this:

Genus: Four-wheeled public vehicle, not confined to rails.

Species:	Omnibus.	Cab.
Differentia:	Keeping to well-defined routes.	Not keeping to well-defined routes.

If we had compared 'omnibus,' 'cab,' 'tram' together, we should have had some such result as this:

Genus: Four-wheeled public vehicle.

Species:	Omnibus.	Tram.	Cab.
Differentia:	Keeping to well-defined routes, and not confined to rails.	Keeping to well-defined routes, and confined to rails.	Not keeping to well-defined routes, and not confined to rails.

This defining by direct comparison, and by assigning genus and differentia, is by far the most convenient for practical purposes; for it is of the essence of practical requirement that it should adapt itself to the exigencies of the specific occasion. The definition found

by consulting a dictionary is likely to have this defect: that it will not precisely suit the occasion. The only way in which to make definition relevant is to select for ourselves the kindred terms with which the term in question is in risk of being confused, and then to note, from the point of view that happens to be interesting us, the differentia which distinguishes its use from that of all these kindred terms.

The Relation of Genus to Differentia.

Taken together, Genus and Differentia state the marks essential *to the definition.* They include just those features which are logically *indispensable* for the unambiguous statement of our meaning.

The relation, however, between the two types of definition-mark —the generic and the specific—cannot be adequately represented by placing them side by side as though they were of co-ordinate significance. The differentia, as the specific mark, specifies, and therefore logically presupposes, the generic mark or genus: it is a specification of the genus. And, though the process of comparison through which our occasional definitions are framed does not explicitly bring out this connexion, the connexion is none the less definitely implied. It is concealed only by the logical incompleteness of the comparison process as we conduct it. Were this process thorough-going, the marks of agreement between two terms would include not only determinate, but also indeterminate marks, so far as these latter were relevant to our purpose in defining; and the differentia would then reveal itself quite naturally as a specification of one or other of these indeterminate marks of agreement. To define 'tram,' we compare it with 'omnibus,' from the point of view, say, of public transit. The two terms agree *determinately* in signifying four-wheeled public vehicles, but they also agree *indeterminately* in requiring *some* distinctive method of proceeding from starting-point to destination. The differentia 'confined to rails' just specifies what this distinctive method must be in the case of a tram. It is thus only in relation to the *indeterminate* elements of the genus that we could endorse Mr. Joseph's contention that 'the genus is the general type or plan, the differentia the "specific" mode in which that is realized or developed.'*

Let us take an illustration suggested by Mr. Joseph himself. The genus of Δ and N might be taken as 'plane rectilinear three-sided construction, possessing *some* specifiable arrangement of the three sides.' The differentia of the term 'triangle'—namely, 'enclosing a space'—would then be a specification of the above indeterminate mark; in the case of a triangular construction, the sides are so arranged as to enclose a space.†

* Joseph, 'An Introduction to Logic,' p. 68. *Cf.* p. 70.

† Mr. Joseph points out that the conception of 'species' as the specification of the 'genus' forbids our describing a genus as a larger class *including* the smaller

If further justification be required for the admission of the indeterminate mark into the structure of a definition, we may find it in the fact that it is necessitated by the very nature of the generalization process through which our definition is reached. The process of Generalization—or of its main feature, Abstraction—may be so understood as to stultify the attempt to connect genus and species vitally together. We may understand by it a process whereby differences are ruthlessly eliminated, and points of agreement reduced to mere identities—identities disengaged from all relation to difference. But if the abstraction of genus from species implies this logical isolation of the marks of agreement from the marks of difference, it is manifestly impossible to consider the species as specifications of the genus. If in mounting, through generalization, from species to genus, we sever the vital bond between the lower and the higher class, we cannot, when descending, through differentiation, from genus to species, behave as though this bond were still unsevered.

But it is surely gratuitous to suppose that generalization (or abstraction) is a devitalizing process of this kind. It is, of course, possible to conceive it after this fashion, and the Formal Logician has almost invariably done so. But just in so far as we embrace a true conception of identity, and abandon the old static view of it as typified in the formula 'A is A,' we are compelled to entertain new ideas about Abstraction. To abstract agreement from difference, we find, is not to isolate them one from another, but to connect them in a new way. It is through the Abstraction process itself that the difference becomes a specification of the agreement—the agreement a generalization of the difference. Abstraction does not take us from differences that have no identical element to identities that are out of all relation to differences : it takes us from the determinate to the relatively indeterminate. But the indeterminate so reached still points back to the specifications from which it has been abstracted. 'Colour' does not mean that which is neither violet, nor red, nor blue, nor any other colour; it means 'colour of some kind,' and, when its meaning is pressed a little further, it is seen to signify 'violet, *or* red, *or* blue, *or* some other colour.' As abstracted from these differences, it still stands to them in what

---

classes or species within it, and consequently renders the attempt to represent the relation by means of two circles, one within the other, entirely misleading. 'The word "class,"' he says (*ibid.*, p. 69), 'suggests a collection, whereas the genus of anything is not a collection to which it belongs, but a scheme which it realizes.' Now, in so far as we are reading the class in intension or conno-denotation (*vide* p. 72), it is undoubtedly necessary, in the sense above described, to consider it 'as something realized in its various members in a particular way' (*ibid.*, p. 71) ; but from the point of view of extension (*vide* p. 158) it is at least reasonable, and may be purposive, to depict the objects indicated by the class-term as included within the larger number of objects indicated by a second class-term. But to admit this is to admit that the one class (extensively defined) can be included within the other class (also extensively defined).

is at least a pre-disjunctive relation. The genus, as abstracted from the species, still points back to the species from which it has been abstracted. A man is a rational (animal of *some* kind); an animal is a sentient (organism of *some* kind). We conclude, then, that Generalization (or Abstraction), when properly interpreted, works in the service of the logical evolution of meaning. The genus, *qua* abstract vestige, is potentially the rudiment or germ of which the species are the specifications. It requires but the interest in the logical development of meaning to transform it actually from the one to the other.*

The Predicables.

The theory of practical definition, as outlined in the foregoing discussion, is closely connected with the Aristotelian doctrine of the Predicables. The Predicables, for Aristotle, were the various kinds of attribute which might be predicated of a subject. If I make the statement 'S is P' (where S is a class-concept), P may stand to S in any one of five possible relations. It may be its *definition—i.e.*, it may give the genus and differentia of S. Or it may be the *genus* alone or the *differentia* alone. Finally, it may give a property or *proprium* of S, or else an *accident*. These 'heads of predicables,' as they are sometimes called, 'have passed,' to quote Mr. Joseph again, 'into the language of science and of ordinary conversation. We ask how to define virtue, momentum, air, or a triangle; we say that the pansy is a species of Viola, limited monarchy a species of constitution; that one genus contains more species than another; that the crab and the lobster are generically different; that man is differentiated from the lower animals by the possession of reason; that quinine is a medicine with many valuable properties; that the jury brought in a verdict of accidental death; and so forth' (*ibid.*, p. 54).

There is a later scheme of Predicables connected with the name of Porphyry, a logician who wrote some six hundred years after Aristotle. Superficially, the sole difference between Porphyry's scheme of Predicables, as given in his Εἰσαγωγή, and the older scheme of Aristotle himself, appears to be the substitution of the predicable of 'species' for the predicable of 'definition.' The predicables, for Porphyry, are genus, species, differentia, proprium, and accidens. But the substitution in question conceals a more fundamental disagreement between the two schemes. In the case of Aristotle the subject-term meant a common nature, a kind, species, or universal, and not the individual object as such. The predicables were, therefore, one and all, predicated about a species, and

* *Cf.* with the above the discussion in Chapter VIII., p. 88. The distinction between the abstraction implied in generalization and the abstraction which results in 'abstract terms' should be noted.

it would have been obviously tautological to predicate the species of itself, and therefore illogical to include the 'species' among the predicables. With Porphyry the subject about which the predicable was predicated might not only be a species, but an individual object. In this latter case it was reasonable to predicate the species under which it stood, and so 'species' found its place among the predicables.

In adopting the Aristotelian scheme of predicables, we at the same time reinterpret it; for the point of view from which we regard the whole problem of the predicables is essentially different from Aristotle's. Aristotle's outlook was objective. He considered the content of the object as such, and not in its relation to the intent—*i.e.*, the intention—of the subject. To define a thing was to state that which made it what it was, and was therefore essential to its existence. But if we admit that 'essential' necessarily means 'essential from a certain point of view,' and thus admit the principle that definition is strictly relative to purpose, we have qualified the Aristotelian standpoint in a way so vital as to preclude any appeal to the authority of Aristotle.

With a view to bringing out the positive significance of the position which we have adopted in regard to the problem of definition, we turn now to the vexed question of the OBJECT of Definition. Meaning, we would say, is the direct object of definition. What, then, do we understand by Meaning?

Meaning, as we conceive it, is, in the first place, a product of thought in its relation to reality, or of reality in relation to thought. Meaning, in other words, is the meaning of an object for a subject; or, more specifically, it tells us what an object *is* in relation to a specified interest or purpose.

Meaning is thus a product of objective Nature and subjective interest, or, if we prefer it, of objective content and subjective intent. It must not only be the meaning of what *is*, and so objective in regard to content; it must be *our* meaning, and so subjective in regard to the definer's intention or intent.

Again, in defining meaning we may have in view either some restricted practical purpose or the broader interests of Science. This distinction we may appropriately equate to the familiar distinction between *formal* and *real* definition. The *formal* definition is a conventional definition framed to fit a specific interest that involves no more than a merely fragmentary hold on objective reality. The framing of *real* definitions, on the other hand, is ultimately controlled by one and the same unvarying ideal—namely, that of bringing the greatest possible simplicity and order into our grasp of Nature.

Meanings, again, are fixed and made definite through the use of words. Hence, to define the meaning of an object is at the same time to define the meaning of the word which symbolizes it. We

do not, of course, define words apart from their meaning. What is defined by the term 'rational animal' is not the mere sound-sign 'man,' but its meaning. If by words we mean the mere sound-signs in themselves, we cannot be said to define words, nor even to 'describe' them, but only to utilize them as sensory supports for meanings which can be defined.

The question 'What is it that we define, things, meanings, or words?' has been the theme of immemorial controversy. There have been three rival parties. The *realists* have maintained that it is *things* that we define; the conceptualists, that we define *meanings*; the nominalists, that we define *words* and *names*. The controversy hinged on the meaning of the 'universal.' The realists held that things had, in all those relations in which they resembled each other, a common or universal nature, and that, in defining this *common* nature, we were defining what was at least as genuine and indispensable a constituent of reality as was the individual nature of objects. The conceptualists held that the universal element existed, not in the objects themselves, but only in the thought which conceived them; the true universal was the concept. Finally, the nominalists held that things called by the same name had nothing in common but the name. The universal was thus a mere convenience of language. The only true existent, whether in reality or in thought, was the individual, and the individual was conceived by the nominalist in a sense which excluded the presence within it of any universal nature.

The conflict between these rival views was a conflict between abstractions which, far from being intrinsically hostile to each other, were, in reality, mutually complementary and indispensable.

We have already suggested that the definition of *meaning* is always at the same time the definition of an object, and to this extent the definition is realistic: definition is always definition of objective content. On the other hand, such objective content, we hold, is definable only in relation to subjective intent, so that, in defining the object, we are defining it as conceived in the light of this or that specific interest. To this extent our point of view might be characterized as conceptualistic. Still, it is not abstract, but, shall we say, concrete conceptualism. The conceptualism we have adopted is simply realism tempered by the requisite of reference to purpose.

According to the interest or purpose engaged, this plastic conceptualism may bear any shade of meaning, from the limiting case of a *mere* conceptualism to an idealism in which the realistic element is completely transfigured. If what is essential to me in defining a term is primarily and predominantly this, that *my* meaning shall be clearly and unambiguously understood, the nature of the object counts for little in the definition, and my meaning has but a vanishing reference to objective reality. This is logical conceptualism in a

strict but still intelligible sense. It is governed by an interest in the logical purity of meanings as such. If, on the other hand, my interest in the meaning of an object—the interest that it has for *me*, the subject—lies primarily in discovering what that object means, or tends to mean, within the spiritual unity of the universe, the conceptualism is transformed into idealism, and my definition will answer to the logical requirements of idealistic conviction.

We have finally to add that the true logical nominalism, in its relation to the problem of definition, is indistinguishable from conceptualism. To define a word is to define its meaning : we do not define as a mere sound-complex the aggregate of vowels and consonants which make up a word. When we say 'Man is a rational animal,' we are not defining the mere verbal label or sign represented by the three letters m, a, n, arranged in a certain order. All definition of meaning is at the same time verbal definition, and *vice versa*. The distinction between nominalism and conceptualism, in definition, is a distinction without a difference.

The statement that we do not define mere sound-complexes as such may easily be misunderstood. It may be taken to mean that we do not even define the *meanings* of symbols *qua* symbols. But this is by no means implied in the statement. Any and every meaning, as we hope eventually to show, is definable in some true sense of the word. The *meanings* of symbols as such are indeed definable. I define the conventional symbol 'man' when I say : '"Man" is a conventional verbal symbol representing the concept "rational animal."' Every symbol has, in fact, a twofold meaning : the meaning of the symbol *qua* symbol, and the meaning of the idea which is symbolized by the symbol. The meaning, in a word, may be the meaning either of the sign or of the signification. When I say 'man' *means* a rational animal, I am defining the meaning of the sign ; when I say 'man' *is* a rational animal, I am defining the meaning of the significate.*

We now proceed to apply the logical doctrine of meaning and of definition, as we have just been formulating it, to the non-defining predicables, *property* and *accident*.

A *property* or *proprium* is an attribute which, though not necessary to the definition itself, is still relevant to the defining interest. It is thus already present by implication in the meaning which an object has for us in the light of a specified interest.

Thus, in the geometrical proposition 'The equilateral triangle is equiangular,' the predicate states a proprium of the subject. 'Triangle' is the genus, 'equal-sided' the differentia, 'equiangular' the proprium. The equiangularity of an equilateral triangle is implied in the system of spatial relations, apart from which an equilateral triangle has no geometrical meaning, and our geometrical interest no real object. The geometrical interest in an equilateral

* For a further development of this point, *cf.* pp. 115, 121.

triangle presupposes this reference to the nature of Space, and the equilateral triangle is conceived as constructed in Space as Geometry treats of it. The very construction furnishes the definition. We trace out a plane rectilinear figure with three equal sides, enclosing a space—*i.e.*, a triangle with three equal sides. But when we come to examine the 'properties' of the triangle as thus constructed, we discover that one of these is 'equiangularity.' As a further property of an equilateral triangle, *qua* triangle, we have the fact that the three internal angles are collectively equal to two right angles.

Let us look a little more closely at the relation between proprium and definition. A definition, as we have seen, is the definition of an objective nature *qua* related to some definite interest or point of view. It would, however, be irrelevant to include within the definition whatever was relevant to the interest; for the function of Definition does not extend beyond the removal of ambiguity, and there may be much that is perfectly relevant to the interest, but which, so far as mere non-ambiguity is concerned, need not be explicitly stated. The propria, therefore, develop, from the point at which Definition stops, the meaning of the objective nature that is being defined. What the definition states is only that fraction of the essence which its own logical principle—the principle of non-ambiguity—requires it to state. The residue is developed in the form of propria.

We must distinguish between two types of propria—two at least, for we may eventually find it convenient to add a third. Properties may be either 'implied' or 'characteristic.' They are 'implied' when they are deducible with logical necessity from the nature we are interpreting, as fixed by the definition in strict relevance to the defining interest. Thus 'equiangular' is an 'implied' property of 'equilateral triangle,' for it can be deduced with logical necessity from the geometrical space-construction defined by 'three-sided plane rectilinear figure enclosing a space' and by the differentia of 'equal-sidedness.'

A property is 'characteristic' when it predicates of the nature we are interpreting an attribute which, without being 'implied,' can be shown by observation or experience to be both typical of that nature and relevant to our interpreting interest. Thus, from the point of view of biological science, such attributes as 'contractile,' 'irritable,' 'assimilating food,' 'reproducing itself after its kind,' would be characteristic properties of an 'organism.'

The Meaning of 'Essence.'

By 'essence' or 'essential meaning' we aim at expressing the contact between an objective nature and a subjective interest. What is indispensable to the conception of 'essence' is this interplay

between content and intent. It will thus be seen that, from the logical point of view, the point of view of right-thinking, 'essence' and 'meaning' are synonymous terms. All meaning is essential meaning, though some types of meaning are more intimately essential than others. From the point of view we have adopted, the non-essential or *accidental*—that which implies no interplay between content and intent—is logically *meaningless*. It is meaningless for the interest in question, and therefore meaningless for right-thinking, which is so constructed as to be unable to assimilate the irrelevant as such.

Some types of meaning, we have just said, are more intimately essential than others. In so far as the intent is an interest in defining the content up to the point required for satisfying the principle of non-ambiguity, the essence of our meaning is given by genus and differentia. In so far as the intent takes us beyond genus and differentia to other marks which are still relevant to it and characteristic of the content, the essence of our meaning is given more inclusively by propria as well, by 'implied' or 'characteristic' properties. But there is yet a third form of interplay between content and intent. The essence of our meaning becomes still more inclusive if we reckon among the marks which are relevant to our intent, and in this sense essential to it, features which, though relevant, are problematic. Thus a building *may be* a palace, a palace *may be* the palace of a king. From the point of view of a general interest in buildings as edifices for social uses, the possibility of being a palace is a perfectly relevant mark of a building, and the possibility of a palace being a royal palace a perfectly relevant mark of a palace. Such 'problematic' properties, as we may call them, need not be actually realized in any concrete instances of the meaning or nature in question. Any type of building which the architect could imagine, plan, and realize if need be, would be a problematic property of 'building.' It might be convenient to give a special name to such problematic properties as were not only capable of realization, but actually realized in at least one concrete instance or occasion. We might refer to these as 'occasional' properties. Thus, from the architect's point of view, it would be an occasional property of a building to be a palace or a country-house. Problematic properties which were not occasional in this sense might be referred to as 'purely problematic.' It might be possible to build a house which should have the precise shape of an elephant or of an icosahedron; but, until such houses are actually built, the device in question remains a purely problematic property.

Problematic properties should not be identified with *accidents* or accidental marks, as we have defined them above. The genuine *accidents*, from the general point of view we have adopted, must be marks which are irrelevant to our intent, and so entirely outside the interplay of intent and content. Thus, in a flower, the

colour, which to the artist is essential, is to the botanist relatively accidental, whilst the microscopic characters so important to the botanist are, from the artist's point of view, entirely negligible. Again, if my interest lies in the assuaging of my thirst, tumbler, mug, and other appropriate vessels are all alike to me: the handle of the mug and its absence in the tumbler are mere accidents, for they do not in any way affect the fulfilling of my interest. So, again, despite the fact that the burning of wood and the rusting of iron are both processes of oxidation, and so chemically akin, they are still essentially different for the person who is seeking warmth. To such an one the resemblances which interest the chemist are purely irrelevant, and in this sense accidental.

It may be objected that accidents as pure 'irrelevants' are not predicables at all, for no one can logically predicate of a subject what is irrelevant to it. Subject and predicate are united in the interest which prompts the making of the statement, and, as so united, are relevant to each other. This may very well be granted, in which case the 'accidents' of Aristotle's scheme become identical with the 'problematic properties' of the scheme that we have adopted, and the accident, in the guise of a realizable possibility, enters, in an intelligible way, into the essence of our meaning. The predicables are then reducible to *four*—definition, genus, differentia, and property; a property being either 'implied,' 'characteristic,' or 'problematic,' and a problematic property being either 'pure' or 'occasional.'

One word more on the problem of Essence. Once the intent or defining purpose is determined, and the content limited to what is strictly relevant to the intent, the meaning of Essence is logically clear. But in ordinary irreflective thought we are, as a rule, neither self-conscious of our defining purpose, nor do we consistently apply it to the deciphering of a given content. We are largely the slaves of suggestion and habit. When we habitually experience certain things together, we come, in accordance with well-known laws of mental association, to conceive them as inherently belonging to each other. Indeed, we show independence of mind just in proportion as we cease to be the slaves of such association. I quote the following from Dr. Watts's 'Logic':

'A court lady, born and bred amongst pomp and equipage and the vain notions of birth and quality, constantly joins and mixes all these with the idea of herself, and she imagines these to be essential to her nature, and, as it were, necessary to her being. Thence she is tempted to look upon menial servants and the lowest rank of mankind as another species of beings, quite distinct from herself. A ploughboy that has never travelled beyond his own village, and has seen nothing but thatched houses and his parish church, is naturally led to imagine that thatch belongs to the very nature of a house, and that that must be a church which is built of stone, and es-

pecially if it has a spire upon it. A child, again, whose uncle has been excessively fond, and his schoolmaster very severe, easily believes that fondness always belongs to uncles, and that severity is essential to masters or instructors. He has seen also soldiers with red coats, or ministers with long black gowns, and therefore he persuades himself that these garbs are essential to the character, and that he is not a minister who has not a long black gown, nor can he be a soldier who is not dressed in red. It would be well if all such mistakes ended in childhood.' I can add an instance from my own experience. I was taken as a child to see the Crystal Palace. From that day onwards right on to advanced boyhood I firmly believed that a palace was not a palace unless it was made of crystal. Palace and stone were two ideas that would not blend in my mind until my further reading gave the necessary shocks to this old superstition, and the power of reflective thought at length slowly dissolved it.

Real or Scientific Definition.

Of all the special purposes we have in view in framing definitions, one stands out pre-eminently above all others—that of meeting the requirements of Science. The logical function of Definition is here adjusted to the ideal of a systematized knowledge of Nature, and consists in the removal of all ambiguities which arise in the pursuit of this ideal.

It will be readily understood that the definitions which are required for ordering our meanings within the vastly complex network of relations which subserve the organization of Science cannot be reached in quite so simple a manner as can the occasional definitions which subserve our varied practical interests.

Thus, the mere process of comparing one concept with another will not in any way suffice to define a fundamental physical concept such as that of inertia, weight, mass, or gravitation. In each of these concepts we have the condensed expression of great scientific discoveries, the embodiment of highly elaborated theory; hence the path to definition here lies not in a process of simple comparison, but in a searching analysis of the interactions and interrelations of the facts of Nature.

In Geometry such analysis proceeds by the help of construction, and it is by ideally constructing its concepts—*e.g.*, those of straight line and circle—that the definitions of Geometry are reached. Here the specifying mark is genetic, a mark embodying a rule of construction. Thus, 'The circumference of a circle is a line traced by a point which moves in one plane at a constant distance from a fixed point in that plane.' *Cf.* also the definition of a circle as a section of a cone drawn square to its axis.

Outside Geometry the genetic definition is not usual, though it

is common in Chemistry, when we wish to define compounds as made up of their elements. The main interest which Science has in defining the terms it uses is in connexion with the problem of Classification. Order is here the dominating need, and the work of definition is therefore dominated by this general requirement of order.

Thus the relatively simple and schematic requirements of formal definition are quite inadequate for the purposes of Science: the distinction between formal and scientific definition is inevitable; but the main value of a distinction of this kind would be lost if, by insisting on it, we were in any way to obscure the essential unity of the defining process at whatever stage of thought we choose to consider it.

In formal and in scientific definition alike we have necessarily to define by relations, and in reference to a purpose stated or implied.

In formal definition the subjective reference to purpose is more conspicuous than the objective relatedness to a system of kindred meanings. But the connexion of the defined meaning, through its very definition, with a system of interrelated meanings is none the less present for not being so obvious. If, in the interest of some restricted purpose, we find it sufficient to define 'Man' as 'rational animal,' we have still three closely related meanings—those of humanity, rationality, and animality—systematically involved in the definition. Thus formal definition is essentially relational in character, though in some cases the relational reference is more apparent than in others. 'King' can hardly be defined without explicit reference to the relations in which Kingship stands to the government of the country ruled; and in a whole class of cases—the so-called class of *cor*relatives (*e.g.*, 'Whole and part,' 'Genus and species')—the definition of either term involves the statement of its relation to the other. In scientific definition, where meanings are so much more systematically interconnected, the relatedness of the defined meaning, as defined, to a system of kindred meanings is a much more patent characteristic of the definition than is the reference to purpose, which here comes more definitely under objective control. It is true that different sciences have different points of view, but the reference to purpose which this distinction in view-point involves is implied rather than expressed, whereas the relatedness of the meaning to be defined to a whole system of other meanings tends to enter more and more explicitly into the very structure of the definition itself.

The essential unity of the defining process, whether formal or real, practical or scientific, is perhaps brought out most clearly by the consideration that the process of 'comparison' through which our practical or occasional definitions are obtained is only a special, simple case of the more general procedure of analysis and synthesis, which we utilize in all definition processes of a scientific character.

To have defined a term or concept scientifically is to have analyzed its relations to other concepts characteristic of the same scientific system, and to have then synthesized these relations in the simplest and most relevant way possible. But this involves just those very processes of criticism and reconstruction which we shall find indispensable in formal definition when we endeavour to remodel certain given definitions in a methodical manner (*vide* Chapter III.).

Note on the Categories.

It has for long been customary to preface a doctrine of Terms with a statement about 'Categories.' There are forms of thinking about reality which are, in a certain important sense, irreducible. Activity is not passivity, time is not place, nor quantity quality, nor substance relation. If we add to these eight varieties the concepts of 'state' and 'situation,' we have before us Aristotle's complete list of Categories—that is, of 'predicates one or other of which must in the last resort be affirmed of any subject, if we ask what in itself it is' (Joseph, *ibid.*, p. 38).

In his excellent chapter on the Categories (*ibid.*, ch. iii.), Mr. Joseph insists on the importance of this ancient enumeration of the ultimate forms of being. Of the importance of a theory of Categories from the point of view of an analysis of knowledge there can be no possible doubt. The subject is, indeed, so important that, were the discussion once broached, a thorough-going treatment would be indispensable. Mr. Joseph has done invaluable service in so lucidly connecting the Aristotelian and Kantian doctrines of the Categories; but, by the very necessities of an 'introductory' discussion, the story of the Categories is made to end with this reconcilement of Aristotle and Kant, and the great attempt of Hegel to systematize the Categories afresh from the point of view of Thought's own logical development is completely ignored. But even Hegel has not said the last word. There are Neo-Hegelian improvements, post-Hegelian developments, and anti-Hegelian reactions; there are even some who choose to ignore Hegel altogether. The Categories are, in fact, 'living options,' and cannot be adequately discussed as monuments, however imperishable, of a past that has no longer any relation to the present.

It seemed better, therefore, not to enter upon any systematic discussion of the Categories. At this initial stage, at any rate, logical propriety required that the Categories should yield precedence to the Predicables, and that the discussion of terms 'according to the nature of their meaning' should make way for that more relevant discussion of them which is 'based upon the relation in which a predicate may stand to the subject of which it is predicated' (*ibid.*, p. 53).

[On the whole subject of Predicables and of Categories, Mr.

Joseph's masterly treatment ('An Introduction to Logic,' ch. iii. and iv.) cannot be too strongly recommended, even by those who venture to differ from the Aristotelian standpoint from which those two chapters are written.]

CHAPTER III.

II. (iii.) THE TESTING OF DEFINITIONS.

Rules towards securing Soundness in Definition.

1. We must distinguish definitions from *translations* and *derivations*. *E.g.*, if we have two equivalent symbols for one and the same idea, we do not define the one symbol by substituting the other for it. To say that 'dyspepsia' is indigestion, or that a laundress is a washerwoman, is not to state what dyspepsia or laundress means. Such statements are sometimes called circular definitions; but why call them definitions at all? They have as little title to be called definitions as have the statements, 'Anima is the soul,' 'Mère is mother.' So, again, such statements as 'Sycophant means fig-shewer' (σῦκον φαίνω) suggest mere derivations. They answer the question, 'What *did* the word mean once?' not, 'What *does* it mean now?' They derive but do not define the term. Still a derivation is in a sense a fossil definition, and so has more right to the name than a mere translation. It might be reasonable to refer to it as an etymological definition. The statement 'Assiduity is sitting close to one's work' is an etymological definition, so far as 'sitting close' is concerned.

2. We must see that the definition *fits*—that it is neither too narrow nor too wide, that it exactly expresses the meaning we wish to convey by the term we use. In other words, definiendum and definition must be *commensurate* with each other—*i.e.*, whatever can be relevantly predicated of the object defined must be predicable of the definition also, and *vice versa*. This is, perhaps, the most important rule of all, and can best be observed by always adopting the natural method of defining, which consists in comparing the word or class to be defined with those other words or classes which approach it most closely in sense. This natural method of defining by simple comparison of what is most allied in meaning ensures a proximate genus being reached instead of some remoter genus; and, further, the differentia can be so chosen as to cover just the one species, and exclude all the sister-species, the class-terms most liable to be confused with it. If the genus is not proximate, the definition is likely to be too wide. Suppose I wish to define

'square.' I compare it with 'rhombus,' and find at once, as genus, equilateral quadrilateral, and as differentia, rectangular; or I compare it with 'oblong,' and find at once, as genus, rectangular quadrilateral, and as differentia, equilateral. But if I reach my definition through comparison with terms less closely allied in meaning, the definition is less likely to fit well. Thus if I compare 'square' with 'circle,' the obvious genus is 'plane figure.' The square is then quite sufficiently distinguished from the circle by means of the differentia 'rectilinear.' But the resulting definition, 'A square is a rectilinear plane figure,' is very much too wide.

As an important corollary from this second rule we have the requirement that a definition should contain nothing superfluous. Thus, the following attempt at defining a 'tip' obviously needs pruning: 'A tip is an extra gratuity paid out of goodwill over and above what can be demanded by contract.' Here 'extra,' 'gratuity,' 'over and above' all involve the same idea. When reduced to the more economical and 'fitting' form, 'A tip is a gift paid out of goodwill,' the definition, though still faulty, is much improved.

When practically applying this rule, we may profitably guide ourselves by the following test-questions:

(i.) Do all the kinds of objects denoted by the term possess the differentia given? If not, the definition is, to this extent, too narrow. Example: A dog is a domestic animal. Are all kinds of dogs domestic? No; 'dingoes' are wild. Therefore, the definition fails to include all kinds of dogs, and is consequently too narrow.

(ii.) Having ascertained that the definition is not too narrow, we ask, 'Is it too wide?' This test-question we may state in two equivalent forms: (*a*) Are there no other terms that satisfy the same definition? (*b*) Is the definition simply convertible?—*i.e.*, given that A is B, is it equally true that B is A? Example: The house-dog is a domestic animal that barks. Is it equally true to say that a domestic animal that barks is a house-dog?

3. The terms of a definition must be of the same *order* as the term defined. They must not be figurative or metaphorical. A metaphor is* 'the use of a word in a transferred sense, the transference being from the order to which it properly belongs to some other order.' Thus, if I define 'faith' as 'the eye of the soul,' I am transferring to the spiritual order the word 'eye,' which belongs to the physical order, and primarily means an organ of the body. So in the definition of a camel as 'the ship of the desert,' the term 'ship' is transferred from the inorganic to the organic order. The definition, in fact, must be homogeneous throughout with the term defined.

Example.—Logic is the medicine of the mind.

This is metaphorical. Logic and medicine are not of the same

* Father Clarke, 'Logic,' p. 222.

order. One is a discipline to be assimilated by the mind, the other a drug to be absorbed by the body.

4. The definition must consist of terms more elementary than the term defined—*i.e.*, it must be such that no one can reasonably expect to understand the term to be defined without first understanding what the defining terms themselves mean. This rule must be applied with reference to the given interest—*e.g.*, that of Geometry. I may be quite right in defining a circle as follows: 'A circle is a plane figure contained by a line of which all the points are equidistant from a fixed point within it,' since the specific mark contains only such terms as 'line,' 'point,' 'equidistant,' all of which express more elementary geometrical ideas than that of the circle. Hence a definition is not invalidated because the untrained mind finds its terms less simple than the term it defines. 'Man' is to most people, no doubt, a much simpler and more familiar term, much easier to understand, than its definition 'rational animal,' but these defining words are more *elementary* than the more obvious term they serve to define.

Example.—A fine is a pecuniary mulct.

This is, scientifically, a correct definition, as a mulct is *any* forfeiture or penalty. But from the purely practical point of view it would be a breach of this fourth rule, or, in technical language, an 'ignotum per ignotius.'

5. A term should not be defined by the aid of terms which cannot themselves be appropriately defined without first defining the original term. To break this rule is to commit a 'circulus in definiendo' or 'vicious circle.' *E.g.*:

'Man is a human being.'
'The sun is the centre of the solar system.'
'Network is a reticulated system of cordage.'
'An archdeacon is an ecclesiastical dignitary, whose business it is to perform archidiaconal functions.'

Example.—Cheese is a caseous preparation of milk.

'Here caseous' means 'cheesy,' and we still want the definition of 'cheesy.' We wish to know by what kind of preparation cheese can be obtained out of milk. The differentia should indicate the recipe for transforming milk into cheese.

A vicious circle in definition is more than a mere blemish. It destroys not only the value of the definition, but the definition itself. The 'statement' that 'cheese is cheesy' is, in fact, no statement at all. It does not predicate anything of cheese, but stops at the concept which is to be defined. 'Cheese is cheesy' takes us no further than 'cheese.' The definition is, therefore, to this extent non-existent.

We must be careful, however, not to be too hasty in accusing a definition of involving a vicious circle.

Example.—A Lilliputian is an inhabitant of the island of Lilliput.

Taking 'Lilliputian' in its primary sense (in its derived sense it is a synonym for 'dwarf'), we should have to meet the objection that if Lilliput is defined as the land of the Lilliputians, then to define the Lilliputian as an inhabitant of Lilliput is to shut oneself up within a vicious circle. But if 'Lilliput' is defined in such a way that its definition does not introduce the Lilliputian—*e.g.*, by its geographical position—then there is no vicious circle at all, and the definition is correct. Such definitions as 'A sovereign is a gold coin equal in value to twenty shillings' and 'A day is a period of time consisting of twenty-four hours' are liable under similar limitations to the fallacy of vicious circle, the former if a shilling is defined as the twentieth part of a sovereign, the latter if an hour is defined as the twenty-fourth part of a day.

As a particular case of circular definition, we have the attempt to define a term by means of its correlative. In the case of correlatives—in the case, that is, of such terms as 'whole' and 'part,' 'genus' and 'species,' 'first' and 'second,' 'cause' and 'effect,' the two terms must be defined together. We cannot define one by the other. A whole cannot be logically defined as 'an aggregate of parts,' if by 'a part' we mean 'a fraction of a whole.' The definition here is, in fact, the definiendum itself. To define a 'whole' is to define a 'whole of parts.' It is a unity of some kind, of which the nature varies with the form of relation between whole and part. In specifying this form of relation, whether spatial, organic, or spiritual, we define the type of unity we have in mind, and *specify* the general meaning of 'whole.' We may therefore define a 'whole' or 'whole of parts' by means of the genus 'unity' and the indeterminate differentia 'possessing some kind and some degree of self-coherence.'

Mill, in his 'Logic' (Bk. I., ch. ii., § 7), clearly points out why it is that certain words go in pairs, as in the case of the instances mentioned above. It is because the meaning of both terms is derived from the same fact or set of facts. Thus, taking the relation of 'father' to 'son,' he writes: 'The paternity of A and the filiety of B are not two facts, but two modes of expressing the same fact.' The terms 'father' and 'son,' however, are not strictly correlatives as are the terms 'parent' and 'child.' They are semi-correlatives. Fatherhood does not necessarily imply sonship, though sonship implies fatherhood. 'Sheep' and 'shepherd' are semi-correlatives in a precisely similar sense. There can be no shepherd unless there are sheep to be herded and tended, but there can be sheep without a shepherd. So, again, a third implies a first and a second, but these do not imply a third. Hence no circle is committed by defining a shepherd as 'a person who looks after sheep,' for we may very well define a sheep without introducing its relation to a shepherd. But

we cannot, without a circle, define a sheep as 'the kind of animal which a shepherd looks after.'

6. A definition should not be given in a negative form if a positive idea is intended. As Professor Read reminds us, a natural historian would not define a lion by saying that it was not a vegetarian. So, in the positive interests of Geometry, it would be better to define a curve as 'a line that is always changing its direction' than to define it as 'a line in no part straight.' On the other hand, where the word to be defined stands for a distinctly negative idea, this form of definition—*i.e.*, negative definition—is to be preferred to any other. *E.g.*, 'An alien is a person who is not a citizen,' 'A bachelor is a man who is not married.'

Examples on the Testing of Definitions.

I. A circle is a figure of which all the points are equidistant from its centre.

Purpose of the definition: To give a *geometrical* definition of a circle.

Criticism of the definition as given.

(*a*) The word 'centre' is not more elementary than the term 'circle,' therefore should be avoided.

Correction: A circle is a figure of which all points are equidistant from a certain fixed point within the figure.

(*b*) It is not true that all points of a circular area are equidistant from the centre; one point of the area is, in fact, the centre itself.

Correction: A circle is a figure enclosed by one line, the circumference, of which all points, etc.

If by 'line' we understand 'continuous line,' this correction should quiet the suspicion that the circumference might be punctiform, a discontinuous aggregate of points.

(*c*) The 'one line' of this definition may still meander freely over any surface of which all points are equidistant from a certain point within the figure. In mathematical phrase, its locus may be the surface of a sphere.

Final Reconstruction: A circle is a plane figure, enclosed by one line (the circumference), of which all points are equidistant from a certain fixed point within the figure.

II. Work is the salt of life.

Verbal Division :* By 'work' we may understand either an activity or its product. The former sense is evidently intended here.

Purpose : To define Work as an activity having relation to moral life.

Criticism : The definition is metaphorical. We must get rid of the metaphor.

Reconstruction : (i.) Work is a type of purposive activity (genus) which stimulates, purifies, and sustains the life (differentia).

Query : Is 'work' here sufficiently distinguished from 'play'?

(ii.) Work is a purposive activity which, when regarded in the light of a moral obligation, stimulates, purifies and sustains the life.

III. A chair is an article of furniture with four legs and a back.

Purpose : To define a chair by the use to which it is put.

Criticism : (*a*) Proximate genus not given.

Correction : A chair is a seat.

(*b*) The differentia is not satisfactorily given.

Correction : If we compare a chair with a stool, we obtain as genus 'moveable seat,' and as differentia 'having a back.' If we compare a chair with a sofa, the differentia is 'intended to seat *one* person.'

(*c*) 'Four legs' is a mere 'accident' or problematical property of the occasional type.

Reconstruction : Proximate genus (of chair, sofa, stool)—'Moveable seat.'

Differentia : 'Intended to accommodate one person at a time, and having a back.'

IV. A cow is a ruminant with cloven feet and sweet-smelling breath.

Purpose : To define a cow zoölogically.

Criticism : (*a*) Proximate genus wanted.

Comparing 'cow' with 'bull,' we obtain as genus 'ox' (in the ordinary generic sense of that term), and as differentia 'female.'

(*b*) 'With cloven feet' is a characteristic property.

'Sweet-smelling breath' is a problematic property of the occasional type. A cow may or may not have sweet-smelling breath. The breath might deteriorate without the creature ceasing to be a cow.

Reconstruction : A cow is a female ox.

* By 'Verbal Division' we understand the division of an equivocal or many-meaninged word into its various alternative significations. Thus the division of 'box' into 'covered case, partition in a theatre, blow with the fist, shrub, or driver's seat' would be a verbal division.

V. A candle is a kind of light used before gas was invented.

Purpose: To define a candle from the point of view of its use as a light and the structure which subserves that use.

Criticism: (*a*) Genus inexact; a candle is not a *kind* but a *means* of light.

(*b*) The specifying mark leaves the definition in one respect *too wide*, for other things besides candles were used for lighting before gas came into use. In another respect it is too *narrow*, for candles are still used, though gas has been 'invented.' 'Used before' implies that candles ceased to be used when gas came into fashion.

(*c*) Further, gas was not 'invented' but 'manufactured.' The specific mark must, therefore, be cancelled as flat and irrelevant, and a radical reconstruction is called for.

Reconstruction: A candle is a means of lighting, consisting of a stick of fatty matter traversed by a wick.

VI. The Sun is the star that shines by day.

Purpose: To define the sun from the point of view of its *appearance* (Ptolemaic point of view).

Criticism: Can 'day' be defined without involving a vicious circle? Is not 'day' that time during which the *sun* is above the horizon?*

Reconstruction: Comparing the sun with moon and stars, which agree in giving forth no perceptible heat, we obtain:

The sun is a celestial luminary which warms the earth.

VII. 'A soldier is a brave man who is ready to die for his country.

Purpose: To define a soldier as such—*i.e.*, from the point of view of his military office.

Objection: (*a*) 'Brave' superfluous, as the essential kind of bravery that a soldier requires is implied in 'ready to die.'

Correction: (i.) A soldier is a man who is ready to die for his country.

Objection: (*b*) 'Man' makes the definition too narrow. *Cf.* Amazons and drummer-boys.

Correction: (ii.) A soldier is a person who is ready to die for his country.

Objection: (*c*) The definition is still too narrow. It excludes mercenaries, organized revolutionists, etc.

Correction: (iii.) A soldier is a person who is ready to die for country, cause, or material reward.

* This criticism, as Mr. Joseph points out (*ibid.*, p. 100), is given by Aristotle himself.

Objection : (*d*) Many units in an army are *not* ready so to die. In this respect the definition is again too narrow.

Correction : (iv.) A soldier is a person *pledged* to fight —to the death if need be—for country, cause, or material reward.

N.B.—In the above, it has been found convenient to merge the two stages of criticism and reconstruction under a single process of reconstructive criticism through successive objections and corrections. A tendency in this same direction may already have been noticed in connexion with the discussion of some of the preceding definitions.

CHAPTER IV.

II. (iv.) DEFINITION AND DIVISION: LOGICAL DIVISION.

THE process of comparison by which definitions are framed to meet the logical need of the occasion gives, as a result, a genus with two or more species included under it. In a word, the definition of the species through a process of comparison results in the division of a genus into two or more of its species. Definition and Division are thus closely connected from the point of view of logical *origin.* They are also closely connected from the point of view of logical *function.* Definition and Division are both necessary to the full understanding of the meaning of a word. Definition gives us—

(*a*) The more general class under which the class in question falls.

(*b*) A specific distinguishing mark.

Division continues the process of supplying information by giving us the alternative sub-classes.

The problem of meaning, then, covers both Definition and Division, and the principle of Non-Ambiguity is regulative of both processes. (*Cf.* the illustration of p. 18 borrowed from Mr. Sidgwick.) Hence, if we identify the Principle of Non-Ambiguity with the principle of Definition, we must understand the term 'Definition' in that wider sense of a complete definition of meaning which includes Division as well.

Logical Division.

The term 'Division,' which is the established designation of the procedure we have now to examine, is not happily chosen. We cannot appropriately speak of dividing a word, or the meaning of a word, for meanings are 'differentiated' rather than divided. The very term 'Division' (as also such other metaphorical expressions as 'parts,' 'joints,' etc.) seems almost to imply a physical division, a division of some individual thing into its component parts.* The use of the word has the further disadvantage of prejudicing the interpretation to be put upon the process in its logical aspect. For this process essentially concerns the relation between a genus and its species, and the term 'Division' in this connexion naturally suggests that logical Division consists in the *splitting up* of a genus into its constituent species. If this is the way in which we are to conceive the process, then the true formula for the relation between genus G and species S_1, S_2, S_3 is $G = S_1$ and S_2 and S_3. Plane triangles, we should have to say, are divided into equilateral, isosceles, and scalene. These are the parts of which 'plane triangle' is the whole. But when I say that ABC is a plane triangle, I certainly do not mean to say that it is an equilateral triangle *and* an isosceles triangle *and* a scalene triangle, that it is S_1 and S_2 and S_3; I mean that it is S_1 or S_2 or S_3. It is this *disjunctive* formulation which alone truly represents the nature of logical Division.

Logical Division is in no sense a splitting up of things into their parts. For the thing is not a genus, nor are its parts species. The division of an animal (mentally, of course) into head, trunk, and limbs, or of a book into parts or chapters, is a purely physical division. The part here does not stand to the whole in the relation of species to genus. We cannot say that the head or trunk or limb of an animal is itself a sort of animal. But in logical Division the genus divided must be predicable of each of the species into which it is divided. If we divide 'human being' into 'man or woman,' each of the two species into which the genus 'human being' is divided is itself a sort or kind of human being.

There is another species of non-logical Division usually referred to as 'Metaphysical Division.' This is the mental division of an object into its several attributes, as when I analyze 'organism' into its genus, differentia, and various properties. These are not *parts* of the concept 'organism' in the sense in which head, trunk, and limbs are parts of an animal, for the qualities could not really be separated from each other as head or limb could be separated from the trunk, nor are they collectively equivalent to the object divided.

* *Cf.* Plato's admonition that 'the philosopher must divide by the joints, and not hack anywhere like a clumsy cook'; and Seneca's remark that a genus 'should be divided, not cut into shreds.'

The true significance of logical Division can best be gauged by considering the relation of Division to Definition in connexion with what we may call the logical development of meaning. To this development, as we have seen, both processes are essential, and we may define their respective functions within this development by saying that Division serves to render determinate those elements of meaning in the definition which are still left indeterminate, and therefore capable of further specification. Division, in a word, is just the further differentiation of the definition in so far as it contains indeterminate elements. Given the definition of a plane triangle as a three-sided rectilinear plane figure, the relations between the three sides are not determined except to this extent—that we know, from the geometrical definition of 'figure,' that the three sides must include an area; there is otherwise an indeterminateness in the side-relations, an indeterminateness which is rendered determinate by the division or differentiation into equilateral, isosceles, or scalene.

Illustration of the Logical Development of Meaning through Definition and Division.

A *government* may be defined as the ruling power in a society consolidated through some dominating interest, the form of rule varying in every case with the structural character of the body wherein the ultimate authority is vested.

The consolidating interest may be either political or non-political. If non-political, it may be either ecclesiastical (Church-government) or non-ecclesiastical. We restrict ourselves to developing, through division, the meaning of a State-government.

In the case of a *State-government*, the structural character of the ruling body may take any one of three forms: it may consist of an individual,* or it may consist of a privileged class, or of the community itself. A State-government, that is, may be either an Autocracy, an Oligarchy, or a Democracy. If it is autocratic, the form of government will vary according as the 'rule by one' is limited or unlimited. An Autocracy, that is, may be either a Limited or a Constitutional Monarchy, or else an Absolute Monarchy or Despotism, passing, when degenerate, into a Tyranny.

If the government is a class-government, the form will vary according to the nature of the ruling qualification. If this is rank, the government will be an Aristocracy; if wealth, a Plutocracy.

If the government is a government by the people, its form will vary with the method of self-government. This may be direct, as in the Citizen-Rule of ancient Athens, or representative, as in the case of modern Democracies, the form of representative govern-

* Perhaps two or three, as in the case of the two Kings of Sparta, or of the Roman Triumvirates.

ment varying again with the conditions of the franchise and the number and nature of the representative bodies.

Thus we see that a logical division is not necessarily exhausted by a single division of a genus into its alternative species. The interest which prompts the division may require for its fulfilment the further division of the species into sub-species, and these, again, may require to be divided. These further divisions of species and sub-species would at the same time be subdivisions of the genus.

The conception or genus with which the division starts is known as the summum genus of the division; the ultimate subdivisions of this genus—ultimate, that is, in respect to the purpose of the division—are its infimæ species. The intermediate classes are sometimes called 'subaltern genera'—genera, because every species except the infima species is a genus to the classes into which it is divided. Just as the infima species is a species which is not also a genus, so the summum genus is a genus which is not also a species. Every subaltern class in a continued division is at once species and genus.

The logical interest which prompts and guides a division may be either formal or real. It is 'formal' (with a small 'f'; *vide* p. 16) when it is 'practical' and 'occasional' in character. It is 'real' when the divisions are drafted in the sole interest of scientific research. This distinction between formal and real may be applied to the divisions themselves. A real division might then be regarded as a Scientific Classification. There is, however, a reason for not identifying the two terms 'Classification' and 'Real Division.' Real Division proceeds always downwards from 'genus' to 'species.' In the process of Classification, on the other hand, we may move in either of two directions: we may move from the 'species' upwards, or from the 'genus' downwards.

Every separate classification has its own summum genus, so that a summum genus cannot profitably denote anything absolute, as the 'being' of Porphyry's tree is not unusually supposed to do. Thus the summum genus of the classification scheme in Zoölogy is the kingdom 'Animal,' and not 'Living Being,' which would include Plants as well, and might even be extended to Metals, if we may trust certain recent scientific research.

The 'infima species,' again, is by no means a fixed distinction in any given system of classification, but is relative to the limit of purposiveness in the making of class distinctions. The African Lion, which is classed as a 'variety' in Animal Classification, may be regarded as an infima species, but if it became useful to distinguish sub-varieties, these latter would in their turn become the infimæ species.

Logical Division must be carefully distinguished from Enumeration. Enumeration is a summing up of the individuals which

answer to a given class-designation, whether that class be a summum genus, subaltern genus, or infima species. It is therefore a process which runs parallel to the development of meaning through logical Division. At any stage of that development it may be purposive to turn from the conceptual ordering of fact to the counting up of the individual units which the concepts serve to include under classes. When we consider facts from the point of view of their number or quantity, the process is an Enumeration. From the logical point of view the interest in Enumeration centres mainly, as we shall see, in questions relating to its completeness or its incompleteness.

Basis of Division, or Fundamentum Divisionis.

Every division is based upon and guided by a *fundamentum divisionis*—*i.e.*, by some character of the group or genus which is a source of difference amongst its members. Thus, in the botanical division of Angiosperms into Monocotyledons and Dicotyledons, the fundamentum divisionis, or F.D., is the number of primary leaves possessed by the plant-embryo. It will readily be seen that the fundamenta divisionis are simply indeterminate attributes of the genus. If 'Man' is divided into 'White man,' 'Black man,' 'Yellow man,' 'Brown man,' 'Red man,' the F.D. is 'skin-colour.' But the genus 'Man' is here relevantly defined as 'a rational animal (det.) possessing a skin-colour of some kind (indet.).' The F.D. cannot be a determinate attribute of the genus, *qua* determinate, for the simple reason that, in so far as it is determinate, it ceases to be specifiable. At the same time, most so-called determinate attributes are only partially determinate, and, in so far as they are indeterminate, may serve as fundamenta divisionis or bases of division.

From the point of view of the interest we have in dividing or differentiating the meaning of a concept these fundamenta divisionis are essential characteristics of the concept, and must therefore be included within its definition. Thus, suppose we desire to define the statistical unit from the point of view of a statistical inquiry which purposes to class the citizens of a country according to means and occupation. The definition would take some such form as this: The 'statistical man' is 'a citizen of a certain means and occupation'; and the full meaning of this unit can be made clear only when we specify the divisions we intend to draw under these two heads. Thus the 'statistical man' may be regarded as (1) 'a citizen who has an income that is either under £50 a year or under £500, or over that amount'; and as (2) 'a citizen who is an artisan or is engaged in business, or is in a profession, or falls outside these three classes.'

The Rules of Logical Division.

I. There should be one fundamentum divisionis, and one only, for each complete act of division.

II. The species or alternatives into which a genus is divided must be mutually exclusive.

III. If the division involves more than one step, it should proceed gradually from the summum genus towards the infimæ species. *Divisio ne fiat per saltum.*

IV. The division, within the limits of relevancy, must be disjunctively exhaustive.

Rule I.—There should be *one* fundamentum divisionis, and one only, for each complete act of division.*

The division of a genus is complete when the genus has been differentiated, and the process of successive differentiation continued until the degree of distinction required by the purpose of the division has been precisely attained. In this process each sub-differentiation, or subdivision, should help to develop, more and yet more distinctly, that one indeterminate aspect of the genus of which the differentiation was the original aim in dividing.

The principle which is here involved is that the F.D. must be a mark of the meaning that we aim at developing through division. We may find it convenient to change the F.D. after a first division, and to carry out the 'subdivisions' upon fresh bases. But in this case our division is no longer a single process, but a chain of divisions, and the term 'subdivision' becomes a misnomer. For, in assuming a fresh basis, we have started a fresh division. A division of the species is therefore not necessarily a subdivision of the genus.

And yet we must not misinterpret the function of the F.D. in Division by insisting that it is itself incapable of any development. Discontinuity between one basis and another implies, indeed, a corresponding change of the interest which gives unity and direction to the dividing process, and so implies also a corresponding break in the division. But there is an important *via media* between discontinuity and a static continuity. The F.D. may legitimately be changed, provided the change is a change *within* its own original meaning. Thus, after dividing 'human being' into 'male or female,' the F.D. being 'sex,' we do not necessarily abandon this F.D. when we proceed to subdivide 'male' into 'man or boy,' and 'female' into 'woman or girl,' for the age-basis may be here brought forward in its bearing on sex differences. What is essential is that the sex interest should dominate the division into its most detailed differentiations, and that all variations in divisional basis

* It is, of course, possible (as in the last illustration) to divide a genus according to more than one principle of division, provided that we keep the divisions distinct. We then have what is called co-division. Thus, again, adopting the fundamenta of age and sex, we may co-divide 'human being' into 'young, middle-aged, or old,' and into 'man or woman.'

should be variations on the sex-theme. It is in this sense that the F.D. must be one and constant throughout the development of any given division. There may be many sub-fundamenta, but these must themselves be developed in the service of the original fundamentum.

In so far as the 'sub-fundamenta' are developed on their own account, each initiating a new interest, the division is broken up into component parts, which are only loosely and, as it were, externally connected with each other. The organic unity of the division is lost. Moreover, overlapping is almost certain to ensue, for the supreme preventive against the overlapping of the various parts of a division lies in making sure that the parts stand for the various modes in which a *single* general meaning—*e.g.*, the sex of a human being—can be differentiated or developed.

When two or more bases of division are *simultaneously* adopted and developed, the resulting overlapping is known as *cross-division*. The different divisions cross each other, and the confusion which ensues bears witness to the importance of the first rule of logical Division.

Rule II.—The second rule of logical Division follows naturally upon the first rule. It is directed against the errors which result in overlapping, whether of the cross-division kind or not. *The species or alternatives into which a genus is divided must be mutually exclusive*—*i.e.*, no part of the division must overlap or be included under any other part. The only security for observing this rule lies in holding to a single fundamentum. If we divide 'human being' into 'male or female or young or old,' employing simultaneously the two fundamenta of sex and age, we obviously break this rule. It is possible, however, to break Rule II. without breaking Rule I.—namely, through carelessness in the statement of alternatives. Thus I may divide 'man' (F.D. 'means') into 'rich, easy, or poor,' but may define 'easy' in such a way as to cause it to overlap with 'rich' or 'poor,' or both.

Rule III.—If the division involves more than one step, *it should proceed gradually* from the highest genus towards the lowest species. *Divisio ne fiat per saltum.*

In each step of the division the species must stand in the same order or rank of generality. Let G be divided into S_1, S_2, S_3 ; and S_2 again into S'_1, S'_2, S'_3. Were we to divide G into S_1, S'_2, S_3, we should have two ranks of generality under one and the same genus. The division would clearly be inadequate, since no account would have been taken of S'_1 or S'_3.

Consider the old-fashioned division of 'Digitigrade' into 'weasel, civet, hyæna, the cat-kind, fox, wolf, dog.' Here the species are not in the same order of generality. Thus 'fox' and 'wolf' are species of the genus Canis (the dog kind), just as 'lion,' 'tiger,' etc. are species of the genus Felis (the cat kind). Had we given the genus Canis, and thereby kept in the same order of generality the members of one step in the division, we should have been

secure against omitting the jackal, which would have been included as being under that genus.

Rule IV.—The division, within the limits of relevancy, must be disjunctively exhaustive.

We have already had occasion to point out the essentially disjunctive character of Division. When we divide G into S_1, S_2, S_3, we mean that G may be developed either into S_1 or into S_2 or into S_3; we do not mean that G may be developed into S_1 and S_2 and S_3. Hence, when we say that the division must be disjunctively exhaustive, we mean that S_1, S_2, S_3 must—within the limits of relevancy—exhaust the *alternatives*.

The meaning of the word 'exhaustive' can, in fact, be defined only in relation to the requirement of relevancy. When we say that a division of a genus into its species must be exhaustive, we mean that it must give all the differentiations of the genus which are at once *possible* and *relevant*. The limit of relevance will be given by the purpose of the division. In the case of the divisions which figure within the classification of the natural sciences, the exhaustiveness cannot be other than *provisional*, for further investigations may reveal new species, or call for the revision of divisions as previously carried out. Moreover, only those species would be relevant that are also actual, for scientific classifications are not concerned with the laying out of possibilities as such, but only with the ordering of such possibilities as Nature has realized. Thus a division of Man, according to skin-colour, which included blue man and green man, would include irrelevant items, since anthropological science studies not mere possibilities, but facts. It would be more than exhaustive, and break this fourth rule of Division just as much as a division into 'white man or black man' which would be under-exhaustive.

In Division by Dichotomy (*vide* p. 47) the division will be seen to be implicitly, though not determinately, exhaustive.

In connexion with this rule of exhaustiveness in Division Mr. Joseph (*ibid.*, p. 103) gives an instructive illustration which I take the liberty of quoting in full: 'Suppose that an income-tax is introduced; it is necessary that the Act imposing it should state what forms of wealth are to be regarded as income, and taxed accordingly. The rent of land and houses is clearly a form of income, and would be included in the division of that genus; but if the owner of a house lives in it instead of letting it, he receives no rent. Nevertheless, he enjoys an income, in the shape of the annual value of the house he lives in, just as truly as if he had let that house, and received for it a sum of money sufficient to hire himself another; and he ought to be taxed if he lives in his own house as much as if he lets it. But if the income-tax Act omitted to include among the species of income the annual value of houses occupied by their owners, he would escape payment on that head altogether. Such is the practical importance of making a division exhaustive.

Division by Dichotomy.

In the process known as Dichotomy (δίχα, in two; τέμνω, I cut) we divide the genus into two alternative species—'x or not-x': x is commonly called the positive, and not-x the negative species; but, as the negative species proves on analysis to be negative only in the name, we propose to substitute for the words 'positive' and 'negative' the words 'definite' and 'indefinite.' Thus we may divide 'Animal' into 'vertebrate or non-vertebrate,' when by 'non-vertebrate' we mean 'some animal other than vertebrate.' We then systematically subdivide on the same principle, and continue dichotomizing in this way until it ceases to be purposive to go further.

What is known as Porphyry's Tree* illustrates the process in that incomplete form in which only the definite terms are dichotomized.

```
                                  Being.
                                    |
                            ----------------
                            |              |
                       Corporeal.     Incorporeal.
                            |
                     ---------------
                     |             |
                 Animate.     Inanimate.
                     |
              ---------------
              |             |
          Sensible.    Insensible.
              |
       ---------------
       |             |
   Rational.    Irrational.
       |
 -------------------
 |        |        |
Socrates. Plato.   Etc.
```

* As Mr. Stock points out ('Logic,' ed. 1903, p. 94), the 'Tree of Porphyry' is 'a device added by later writers.' In Porphyry's treatise there is no division by dichotomy, but simply the logical development of the single category of Substance taken as summum genus:

```
              Substance.
                  |
                Body.
                  |
             Living body.
                  |
               Animal.
                  |
           Rational animal.
                  |
                 Man.
                  |
   ---------------------------------
   |              |                |
Socrates.       Plato.            Etc.
```

Mr. Stock adds the following interesting footnote: 'We might suppose that "thing" or "being" could be predicated of "substance," but Porphyry, following Aristotle, regards each of the ten categories as a distinct summum genus. He will not allow that "being" is predicable of them all in the same sense.'

This rejection of the indefinite term at each step of the division is technically known as an 'abscissio infiniti,' the 'infinitum' or 'indeterminate' being here the indefinite term.

The definite and indefinite terms *in their relation to each other* are sometimes referred to as Contradictory Opposites, Contradictory Relatives, or Contradictories. Thus 'cold' and 'not-cold' are said to be contradictory opposites. But the name is unfortunate and apt to mislead. A definite term and its counter-indeterminate are not contradictory in the sense of contradicting each other. It is only statements that can contradict or be contradicted. It is true that when such terms are predicated of the same subject in the same relation the assertions within which they thus function as the respective predicates contradict each other; but it is the opposition of the two statements, and not that of the two predicates as such, which constitutes the contradiction.

We shall, in fact, see, when we come to consider what we mean by an indefinite term, that these so-called contradictory opposites are complementary rather than antithetic. They should therefore be carefully distinguished from contrary opposites or contrary relatives, which may be defined as terms markedly opposed under the same head. We say 'markedly' and not 'most,' since under any given head—*e.g.*, that of temperature—we may have more than one pair of contraries. Thus 'cold' and 'hot' are contraries; but so also are 'freezing' and 'broiling.'* It will be seen that each of a given pair of contrary terms is itself a positive term with well-defined positive reference. 'Black' is just as positive in meaning as 'white,' 'miserable' as positive as 'happy,' 'hard' as positive as 'soft.'

A term is, of course, a 'contradictory' or a 'contrary,' not *per se*, but only in relation to its opposite. In particular the indefinite term 'not-x' is not in itself 'a contradictory term.' It is contradictory only in relation to the complementary definite term 'x,' and that only in the derivative sense already indicated.

The Meaning of the Indefinite Term.

The logical significance of Dichotomy depends primarily on the meaning we assign to the indefinite term. We must, therefore, carefully consider what this meaning may be.

An indefinite term is a term of the form 'not-x' or 'non-x.' It indicates what is other than x in a sense that we must now proceed to determine. Some logicians insist that it must be, in character, perfectly and illimitably indefinite. Not-x, they say, must surely take up all that is excluded from x. Out of the sum-total of think-

* This indefiniteness does not extend to contrary *propositions*. There the opposition exists unambiguously between 'all' and 'none,' between 'All S is P' and 'No S is P.'

able existence we subtract x : all that is left must be not-x. Not-x= everything $- x$. Thus if x= Europeans, not-x stands for 'the unlimited myriads of entities which people the heterogeneous domain' of 'everything − European.' It is in this sense that not-x has been called an Infinite Term. This, however, is a useless logical figment, and only worth mentioning as a warning concerning what not-x should not be made to mean.*

It seems clear that in the interests of logical science not-x cannot be indefinite in this illimitable sense. This brings us to what we may call the disjunctive, or the suppositional, use of the indefinite term not-x.

The ordinary use of terms is limited by some 'Suppositio,'† some Topic, some Universe or Subject of Discourse. In so far as a man's interest is not that of pure negation—in which case the denial will take the form 'S is not P,' and not the form 'S is not-P'—his mind is always moving within some assignable suppositio, and the significance of the indefinite terms he uses is limited by reference to this suppositio.

It is, moreover, important to realize that the term not-x requires to be *disjunctively* differentiated. Let us take, by way of illustration, the following division :

Colour.

Red. Not-red.

Here 'not-red' has the implicitly disjunctive meaning of 'some colour other than red'—*i.e.*, 'either blue or green or yellow,' etc. It does not stand conjunctively for the sum of colours other than red. Were this its meaning, not-red would be a term fulfilling a merely epitomic or abbreviative function, and 'red' and 'not-red' taken together would *conjunctively* exhaust the suppositio of colour. It is true that the division of colour into red *or* not-red is also exhaustive, but it is exhaustive in a disjunctive and not in a conjunctive sense.

This view of the implicitly disjunctive meaning of not-x in Dichotomy supports the more general view that we have taken of logical Division as the progressive differentiation of the meaning of a concept. On this disjunctive view the division of 'colour' into 'red' and 'not-red' precisely means that colour is either red

* 'Aristotle long ago pointed out that οὐκ-ἄνθρωπος was not properly a name at all; and he perhaps extended his countenance too much to it when he said that, if we were to call it anything, we must call it a "name indeterminate" (ὄνομα ἀόριστον) because, being the name of nothing positive and in particular, it had a purely indeterminate signification' (Joseph, *ibid.*, pp. 29, 30; *cf.* also footnote, p. 30).

† 'Suppositio' is an earlier name for 'the universe of discourse,' a name recently revived by Venn and Carveth Read. It means 'the range of subject matter about which we consider ourselves to be speaking.' Mr. Joseph, following De Morgan, prefers the term 'limited universe.'

4

or not-red. Hence, when we proceed to differentiate 'not-red' into 'blue or green or yellow, etc.,' we are simply carrying on the very same principle of disjunctive differentiation which we applied to the division of the concept 'colour.'

When the indefinite term is understood in the sense which we have attempted to define, the main objections which have been levelled against Dichotomy as a process of division fall entirely away. Thus Mr. Joseph (*ibid.*, p. 106, *sq.*) maintains, in the first instance, that in the subdivision of the 'negative' class or 'conception,' the essential nature of division as a process which exhibits its membra dividentia as 'alternative developments of a common notion' (*ibid.*, p. 107) is consistently violated. Mr. Joseph holds this objection to be fatal and decisive (*ibid.*, p. 109). But it depends entirely for its force on what we conceive to be a misinterpretation of the meaning of the 'negative' term. Mr. Joseph takes 'land' as the meaning or conception to be divided. He divides it by dichotomy into 'building-land' and 'land not used for building.' Each of these conceptions he subdivides. Thus 'land not used for building' is divided into 'farm-land' and 'non-farm-land,' and so on. He then points out (*ibid.*, p. 109) that 'to farm land is not a way of not building on it,' and, generally, that the division of a 'negative' conception is necessarily a division in which the species is no longer a specification of the genus—a division, therefore, which fails to respect the true logical relation between genus and differentia.

Now it is undeniable that 'to farm land is not a way of not building on it,' but the 'negative' term 'land not used for building' has, as we have seen, a certain positive meaning of an indeterminate predisjunctive kind. It stands for 'land used for some purpose other than that of building,' and the farming of land is precisely a specification of this indeterminate generic idea. A 'negative' conception affords, therefore, as sound a basis for subdivision as does a positive or definite conception. It is just as sound to specify 'land used for some purpose other than that of building' by 'farm-land' as it is to specify 'land used for building' as urban or suburban.

The objection may perhaps be raised that if we are proposing to divide the genus 'land' into the two alternative species 'land used for building purposes' or 'land used for purposes other than building,' we do not really carry out what we propose to do. For what we are so dividing, it may be said, is not 'land,' but 'land as subserving a human purpose.' Hence 'waste-land,' the land that subserves no human purpose, is excluded from the division, though it is as genuine a species of land as building-land or farm-land.

This objection has a certain point and directness which challenges close consideration. We must admit the justice of the plea that it is not 'land as such' which can be divided into 'building-land,' or 'land used for some purpose other than building.' This division

is undoubtedly a division of 'land as subserving a human purpose.' But, from our point of view, 'land as such' is not a suitable genus for logical division. Meaning is necessarily the meaning of an object for a subject, and can be made unambiguous or logically definite only when the subjective interest which goes to meet the object is first clearly specified. Indeed, we could go a step further, and maintain that the object which we propose to define and divide is first constituted as a logically definable and divisible object through the selective, abstracting activity of a subjective interest. The object to be logically divided is always a 'genus,' and, as such, its meaning will be variously differentiated according as the dividing-interest is variously specified.

We admit, then, that 'waste-land' is not included in our division of 'land as subserving a human purpose,' but hasten to add that, in so far as 'waste-land' means 'land that subserves no human purpose,' it would be irrelevant, and therefore logically meaningless, to include it.* The 'negative' term 'not used for building,' therefore, does not mean 'used for some purpose other than building, or else not used for any purpose at all'; for the addendum which the words 'or else' introduce is irrelevant to the genus we are dividing, and cannot therefore be included within the meaning of the 'negative' term. 'Not used for building' must therefore mean, as already stated, 'used for some purpose other than building.' By first defining the object to be divided, through the limiting activity of a definite subjective interest, we cut off from the outset, at one logical stroke, all differentiations of the object's meaning which do not positively subserve the development of that interest. Negating addenda of the type of that just considered have no longer any *raison d'être*. The whole race of them is excluded *ab initio*.

Mr. Joseph further accuses a dichotomic division of not proceeding on a single fundamentum. 'In the proper division of land,' he says (*i.e.*, the division of land into building-land, farm-land, forest, means of communication, pleasure-ground, waste), 'the basis taken was the use to which land is put, and that was retained throughout; but in the division by dichotomy, the basis taken was, first, the use of land for building, by which it was divided into building-land and the rest; and the rest was divided on a different basis—viz., the use of land for farming, and so on' (*ibid.*, p. 109). But once the indefinite term is understood in the sense we have adopted, it is no longer true to say that the first F.D. in the process by dichotomy was 'the use of land for building.' When we divide land into 'building-land or not-building land,' we are dividing it into 'building-land or land used for purposes other than building.' Our F.D. is therefore 'the use to which land is put,' just as in the case of

* In a division of 'land' from the point of view of scientific intent, such as that of the geologist, there would be no waste-land, just as to the botanist there is no such species as 'weed.'

'the proper division of land.' So, again, when we proceed to divide 'land used for purposes other than building' into 'farm-land or non-farm land,' we do not adopt a different basis of division. The basis still remains, as before, 'the use to which land is put.' When land is used for purposes other than building, it may be used either for farming purposes or for non-building purposes other than farming.

We would point out, in conclusion, that Dichotomy is by no means a purely Formal process, which can be carried out independently of material knowledge. As Mr. Joseph convincingly insists (*ibid.*, p. 110, footnote),* we have no right to divide x into the species a and *not-a* unless we know that, as a matter of fact, a is a species of x. Thus, it is absurd to divide circle into rectilinear circle and non-rectilinear circle, though we are, of course, perfectly justified in saying that every circle (here, as in Analytical Geometry, identified with its circumference) must be either rectilinear or not. We cannot develop the meaning of 'circle' by assigning to it as one of its species the rectilinear circle. In dividing G into S or not-S, S must be a possible and relevant differentiation of the genus G.

The Testing of Given Divisions.

In the testing of given divisions we have first to decide whether the division is logical or non-logical, and, if the latter, whether it is physical, metaphysical, or verbal.

If the division is logical in form, we must test its observance of the four rules. This we may conveniently do by means of the following test-questions :

I. (*a*) Is there more than one F.D. ?
   (*b*) If only one, is it appropriately chosen ?

II. (*a*) Is there overlapping of the classes ?
   (*b*) If so, to what cause is it due—to a confusion of fundamenta (giving cross-division), to careless definition, or to a confusion of the ranks of generality ?

III. If due to the latter cause—*i.e.*, if the membra dividentia are not 'cognate'—what is the remedy ? Answer : Subdivision.

IV. Is the division adequately exhaustive ?

Examples.—Test the following divisions :

(i.) 'Living being' into 'moral or immoral.'

We must begin by defining 'moral.' If we mean genuinely, actively moral, then the class of indifferents in morality is left out. If under 'moral' we mean to include all creatures capable of morality that are not positively immoral, then the division is sound. But in any case we have omitted the non-moral in the sense of

* *Cf.* also Mellone, 'An Introductory Text-Book of Logic,' ch. vi., § 10.

'moral incapables'—plants and animals, human infants, and pathological cases of adult human beings. The division, therefore, is not exhaustive (breach of Rule IV.). Moreover, the division as applied to 'living being' is unsatisfactory, as most men are sometimes moral, sometimes immoral. It would better apply to 'act.'

(ii.) 'Man' into—(*a*) 'timid or rash';
(*b*) 'avaricious or prodigal.'

The F.D.'s are not specified. They may be taken to be (*a*) 'behaviour with regard to danger,' and (*b*) 'behaviour with regard to money.'

Again, these divisions are not exhaustive. There is in each case a twofold mean to be introduced—

(*a*) 'cautious or valiant';
(*b*) 'economical or liberal.'

So long as we 'cut well at the joints,' the more distinctions we can relevantly make, the better. Indeed, it is well to avoid the habit of fancying that between two extremes there can only be one mean. In the case of (*a*) the objection must be raised that the division much more naturally applies to the act than to the man; for most people are timid in certain respects and not in others, much depending on habit. Further, the timid may, when their emotions are sufficiently roused, become rash, or even really brave (*cf.* the maternal instinct of protection in ordinarily timid women, or the moral courage of the convinced but naturally timid reformer, or the courage of the martyr for faith's sake).

(iii.) 'Students' into 'idle, athletic, and diligent.'

Criticism (1).—The dividendum is not expressed in logical form. The plural term 'students' necessitates an 'extensive' interpretation. In so far as Division is differentiation of meaning, we must adopt the singular form, and restate our dividendum as 'student.' A corresponding alteration must be made in the form of the division itself. We must substitute the disjunctive 'or' for the conjunctive 'and.' The division, then, which we have now to discuss is that of 'student' into 'idle, athletic, or diligent.'

Criticism (2).—The F.D. is twofold: work-status and games-status. A co-division is here required to remedy overlapping (Breach of Rules I. and II.), and to ensure adequate exhaustiveness (Rule IV.). The division according to work-status may be given briefly as follows:

Student.

Idle. Not-idle.

Perfunctory. Diligent.

The attempt to divide 'student' according to play-status raises, however, a fundamental difficulty. For the F.D. 'play-status' cannot be included, even as an indeterminate mark, in the definition of 'student.' A student cannot be defined as one who patronizes some form of play or takes some form of recreation. Problematic properties cannot, even at the call of the dividing-interest, be transfigured into differentiæ. There is certainly a difficulty here, but the logical remedy is simple and direct. The genus or dividendum may be altered so as to answer appropriately to the requirements of the case. We cannot accept 'play-status' as an F.D. of 'student,' but we can accept it as an F.D. of 'student who is interested in games.' It should not, however, be supposed that this procedure is a mere subterfuge or dodge. We are not infrequently asked to perform operations on inappropriate objects. We might be asked, for instance, to multiply 8 cows by 15 sheep, or to divide 15 sheep by 5 sheep. We might be asked to decide upon the specific spiritual quality of a ghost's body or a comet's tail. We may even be asked to convert an O proposition. Against all such questions as these we safeguard ourselves by pointing out that the requirement cannot be met, and that the nature of the object resents the subjective demand inconsiderately made upon it. A number can be divided by another number, but not a sheep by a sheep, nor so many cows by so many horses. A comet's tail cannot grow spiritual by the simple process of becoming sufficiently thin. Similarly a student cannot put on a games-interest in order to suit the caprices of a question in logical Division. A question in Logic may itself be illogical. When we are asked, then, to divide 'student' according to play-status, we answer that it is only the play-student that has a play-status, and that, from the point of view of play, the student who does not play must be cancelled, not, indeed, as a 'skulk,' a 'shirker,' or a 'book-worm'—for these pretty labels do not express feelings controlled by logical interests—but as an irrelevance—an irrelevance to the limited interests of the play-topic.

We may adopt, then, as our division according to play-status, some such classification as the following :

Playing student.

Athletic. Non-athletic.

Shaping badly. Shaping well.

(iv.) 'Quadrilateral figure' into 'square, rectangle, parallelogram, or rhomboid.'

We take 'quadrilateral figure' to mean 'plane rectilinear quadrilateral figure.'

The classes overlap, with breach of Rule III. The correction

needed here is therefore not that of co-division (the cross-division remedy), but that of subdivision. Species and sub-species are confused together, the division taking leaps along the predicamental line. We may correct the division thus :

- Quadrilateral figure.
  - Parallelogram (bi-parallel).
    - Rectangle.
      - Square.
      - Oblong.
    - Non-rectangle.
      - Rhombus.
      - Rhomboid.
  - Non-parallelogram (F.D. parallelism of sides).
    - Trapezium (mono-parallel).
    - Trapezoid.

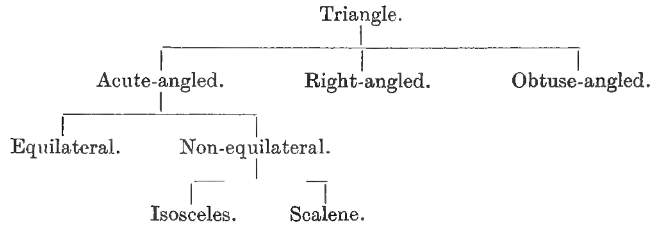
We now see very clearly that, in the original division, the classes are not mutually exclusive (Breach of Rule II.). The square is a rectangle, and the rectangle a parallelogram ; and, further, the rhomboid is a parallelogram. Thus, what the given division tells us is, briefly, that the quadrilateral figure is a parallelogram. No account, therefore, is taken of the non-parallelogram—the trapezium and trapezoid. The division, then, is not exhaustive (Breach of Rule IV.).

It may be worth while to consider this division more closely from the point of view of the fundamenta involved. Unless the first rule of Division is to be broken, the fundamentum must remain generically the same throughout. Now the division according to parallelism of sides and the subsequent division of the parallelogram—(1) according to angle-relations, (2) according to side-relations—introduce fundamenta which do not at once appear to be modifications of one and the same generic idea. But on closer scrutiny they are seen to be so. We may accept side-relation as the generic fundamentum, and characterize our three specific fundamenta as (1) side-parallelism, (2) side-inclination, (3) (relative) side-length. With regard to (2), we see that the angle-relation is itself a specification of the side-relation ; it is that relation of one side to another which is measured by the inclination of each to the other. According as the two sides which contain the angle are more or less inclined to each other, the angle itself is greater or less. We conclude, then, that Rule I. is not in any way broken, but rather legitimately applied.

(v.) '*Plane Triangle*' into 'equilateral, obtuse-angled, or right-angled.'

Identifying 'equilateral' with the commensurate term 'equiangular,' we notice that the division is not exhaustive. The acute-angled triangle which is not equiangular is not included (Breach of Rule IV.). There is no overlapping.

But the division, as given, involves two fundamenta, which, though not generically, are still specifically different—namely, 'side-relation of relative magnitude' and 'angle-relation.' And it is a breach of Rule III. (though not necessarily of Rule I.) to utilize simultaneously at any given stage two fundamenta that are *specifically* different. Proper subdivision, then, might seem to be the natural remedy, and we might present the corrected division as follows :



But the division, so framed, seems to require to be completed by the subdivision of the two remaining members of the first division, and the total result is unnecessarily complex. It would be simpler in this case to institute a co-division which, when completed, would run as follows : Triangle into equilateral, isosceles, scalene (F.D. relative side-length), and into acute-angled, right-angled, obtuse-angled (F.D. side-inclination).

(vi.) '*Yorkshire*' into 'North, East, and West Ridings.'

This is physical division.

(vii.) '*Lemonade*' into 'fluid, acid, sweet,' etc.

This is an incomplete metaphysical division.

(viii.) '*Accident*' into 'misadventure or irrelevant predicable.'

This is verbal division, the discrimination of the possible meanings of an ambiguous term.

CHAPTER V.

II. (v.) CLASSIFICATION.*

THE first main object of Classification is to keep control over facts by marshalling them in order ; and the general principle which guides

* As already indicated in the last chapter (*vide* p. 42), the term 'Classification' is more comprehensive than the term 'Real Division' ; for, in the first place, it includes not only the downward movement from summum genus to infimæ species to which we are restricted in Real Division, but also the upward movement from the lowest species to the highest genus. In the treatment of Classification here

every such endeavour is that of bringing together those things which are most alike and separating those things which are most unlike. Thus, to take the case of animals, we have here an immense and bewildering variety of individual beings. A sufficient knowledge of Anatomy enables us to detect within this maze of life certain relatively permanent types of structure by the aid of which we form zoölogical species. When these are compared together, some will be seen to have characters in common by which they resemble one another and differ from all other species. These we group together into what is here technically called a genus. From genera we pass by similar steps to families, orders, classes, and finally to sub-kingdoms.

The words 'class,' 'genus,' 'species' have here acquired meanings quite different from those involved in stating that the definition of a class or species is given by stating genus and differentia. In this latter statement all the terms are general and relative. From the point of view of the predicables, the word 'class' is used generally for any group of objects resembling each other in certain characteristics.* Thus, a sub-kingdom, or an order, or a class proper, or a genus, or a species is a class in this sense of the word. So, again, if we take any two successive groups in the scheme of classification, the first will stand to the second as genus to species in the predicable sense of these terms. 'Class' is the genus of which 'Order' is the species. But in these classifications the words 'class,' 'genus,' 'species' have fixed specialized meanings. A 'class' comes between a sub-kingdom and an order, a 'genus' between a family and a species, a 'species' between a genus and a variety.

TYPES OF CLASSIFICATION.

Classifications are of two kinds: they may be either *real* or *formal.*

When we state that classifications are governed by the paramount consideration of *order*, our primary meaning is that classifications arise in response to a dominating subjective purpose, the need for order. But there are two main ways in which this subjective purpose realizes itself: it may either develop in whole-hearted conformity to the nature of the material studied, or it may show a divided adherence, conforming partly to the requirements of the

---

given we have used the term almost exclusively in that sense in which it cannot be mistaken for Real Division—*i.e.*, we have considered the up-building of a classification rather than its explication from the most general concepts downwards. But even where the direction of Classification coincides with that of Real Division, the two processes remain distinct. For Classification includes processes of Definition as well as of Division; whereas Division and Definition, as we have defined them, are mutually exclusive.

* The extension-import of a class is here assumed, as the more convenient for our purpose (*vide* p. 146).

material, but partly also to one or other of the specialized demands for order which the subject makes in the interests of his own practical life and culture. In the former case the classification may be called real; in the latter, formal. In each case the dominating factor is the subjective interest in order, and here, as well as there, the interest may be 'disinterested.' But in the one case this interest is fixed on the discovery of the material's own order imposed upon it by the laws of its own nature; in the other—whether through choice or necessity—it is bent on arranging the material in a selective spirit by the help of such of its characters as happen to be relevant to the classifier's specific requirements.

All the classification-schemes of the Natural Sciences are real in the sense above defined.

There are two main types of Real Classification, respectively known as *natural* and *diagnostic.* But they do not stand on the same level, for the diagnostic type of Classification has its sole *raison d'être* in the service of the natural. As the distinction between these two types of Real Classification is particularly important, we proceed to consider it at some length.

Natural Classification.

In classifying according to Nature, scientists have been guided by the following important clue, which may be regarded as the guiding-thread of true Natural Classification—to wit, that it is characteristic of the ways of Nature that, when she makes a difference in any single fundamental particular—*e.g.*, possession or lack of a spinal cord—she correlates with this difference a large number of other differences. In the case of the Genetic Sciences, which view their object-matter from the standpoint of its development, this characteristic admits of a ready explanation. Given two species, one with and the other without the rudiment of a spinal cord, it is obvious, from the point of view of Evolution, that they will develop in very different ways, and acquire very different properties. Such classes as are formed of things which agree among themselves and differ from others in a multitude of characters were called by J. S. Mill 'natural kinds.' A classification is natural only in so far as it keeps to natural kinds throughout.

A natural classification, then, may be defined as one in which, roughly speaking, the divisions are so constituted that the objects included in any one of them resemble each other and differ from all others in many significant respects.

In Natural Classification the more important characters—*i.e.*, those which are accompanied by the larger number of correlated differences—are selected for determining the higher groups, and thus the kinds classified will, on the whole, be arranged, from the primary divisions downwards, according to the principle of 'sub-

ordination of characters.' In this arrangement, the higher the place which any class holds in the classification, the more important are the characters which constitute it. This arrangement will prevent any widely dissimilar groups from being brought together in the lower divisions. The ox and the frog will be held apart in the classification, as in Nature. Thus, if we are considering flowering plants, we notice that plants in which the ovules are enclosed in a protective structure resemble one another (and differ from those whose ovules are unprotected) not only in this particular, but in a large number of other points as well, such as the structure of their vascular tissue, the form of the stamens, the germination of the pollen-grain, and the development of the endosperm. In classifying flowering plants, we therefore divide them first of all (according as they have protected or unprotected ovules) into Angiospermæ or Gymnospermæ. In subdividing the Angiospermæ, we choose the character of the presence of two primary leaves, or of only one, and thus form the two alternative sub-classes, Dicotyledons and Monocotyledons. After this we go on to other characters in descending order of importance, and so form our Orders, Sub-orders, Genera, and Species.

The characterization of Angiosperms, according as they are dicotyledonous or monocotyledonous, admits of being stated in a relatively untechnical way. Thus :

(i.) Dicotyledons have the following characters :

(1) The embryo has two seed-leaves or cotyledons.
(2) The first or primary root of the embryo branches *after* it leaves the seed.
(3) The stem branches repeatedly.
(4) The stem, when perennial, has a distinct pith, continuous rings of wood, and separable bark. The stem increases in thickness by the formation of fresh rings of wood outside those already formed and inside the bark. The hardest wood is inside.
(5) The outer parts of the flower are most commonly in fives—*i.e.*, have five members in each whorl.
(6) The leaves are net-veined.

(ii.) Monocotyledons have the following characters :

(1) The embryo has only one seed-leaf.
(2) The primary root branches *before* it leaves the seed.
(3) The stem, as a rule, shows little branching, and in the monocotyledonous trees (such as Palms) it may be quite unbranched, growing only from a bud at its apex, the buds produced in the axils of the leaves remaining undeveloped.

(4) The stem is without any distinct pith, continuous rings of wood, or separable bark. The wood consists of bundles of fibres and vessels, which are separately embedded in cellular tissue. The hardest bundles are outside.
(5) The outer parts of the flower are in threes.
(6) With few exceptions, the leaves are straight-veined.

It is to be noticed that the most important characters are by no means (usually) the most obvious. Our natural groups seem, at first sight, to include extremely heterogeneous kinds. To an unbotanical mind, the yellow cowslip, the scarlet pimpernel, and the purple cyclamen would seem as unlike as flowers could be; yet these three species are closely related, and we class them all in the Natural Order Primulaceæ. So also the daisy, the goldenrod, and the thistle belong to one Natural Order, the Compositæ; and two flowers so unlike as the blue cornflower and the purple knapweed belong, not only to the same family, but even to the same genus (Centaurea). We do not, in Classification, give the preference to the most obvious, but to the most significant and the least variable characters. Thus, in both Zoölogical and Botanical Classification, Analogy (resemblance arising from adaptation to similar functions) is of far less importance than Homology or morphological identity. Hence the paramount necessity, for purposes of Classification, of the study of *Development.* In classing any organism, we must consider not only its characters at any one moment of observation, but also those exhibited by its past history; for thus alone is it possible to ascertain the homologies of structure upon which Comparative Morphology is founded.

The importance to Classification of a close study of Development has been tenfold increased by the discovery of the connexion between ontogeny and phylogeny, the establishment of the theory that each individual organism (at least among animal forms) 'recapitulates' in its development the whole history of its race. If we were to meet, for the first time, a full-grown hen, we might be uncertain of her exact place in the Animal Kingdom; but when we have watched day by day the development of the chick, in the egg, from the single cell which represents some protozoan ancestor, through the fish-like stage which exhibits a swimming tail and conspicuous gill-slits, and again through the reptilian form with its four limbs and hands, each with its five digits distinctly shown by the microscope, on to the first emergence of the characteristic bird-like form, then we have no difficulty in relegating our adult fowl to her proper position in our zoölogical classification.

Thus, an ideal natural classification of animals or plants would represent, not only the present affinities, but the whole ancestral history of the organisms dealt with. It would indicate no mechan-

ical arrangement of isolated types, but an organized *continuum* in which some of the missing links would be supplied by palæontological research, and others would be ideally reconstructed with more or less of probable exactness. Our scheme of classification would thus become a genealogical tree, showing the vital relation of each kind to all the others, and thus making evident the organized unity of the whole.*

Definition in Connexion with Natural Classification.—The Problem of Classification involves the necessity of defining the names which constitute the Nomenclature. In the case of Natural Classification, Definition takes the form of Characterization—*i.e.*, of giving an inventory of the known characteristics common to all the typical members of the class indicated by the term to be defined—a result to be obtained only through those thorough-going analyses and syntheses which are called for when we study Nature, with reconstructive intention, as a complex and developing system of which all the parts and aspects are intimately interrelated. In a natural classification, as we have seen, every group, from the primary divisions downwards, will possess a number of common characteristics. Thus, the definition of the term 'Dicotyledon' might be stated somewhat as follows:

The term 'Dicotyledon' stands for a plant possessing the distinguishing characters of the Angiospermæ (genus), and further characterized by the following marks:

Embryo with two cotyledons.

Stem, when perennial, having a distinct pith, continuous rings of wood, and separable bark, and branching repeatedly.

Leaves net-veined.

Parts of the flower usually in fives.

So, again, the definition of the term 'Vertebrate' in Zoölogy would be somewhat as follows:

The term 'Vertebrate' stands for a multicellular animal (genus) characterized by the following marks:

1. The possession (at some stage of the animal's development) of a smooth, elastic, dorsally placed rod (the Notochord) lying ventral

* As a particularly important and impressive instance of Natural Classification in the realm of *inorganic* Nature, we may mention the classification of the chemical elements according to Mendeléeff's Periodic Law. This same instance is also an excellent example of Classification by Series (*vide* Professor Duncan's 'The New Knowledge,' ch. iii.; Hodder and Stoughton, 1906). As another, perhaps still more important, instance of Natural Classification—this time in the realm of spiritual values—we would refer to an article on 'The Classification of the Virtues,' by H. W. Wright (*The Journal of Philosophy, Psychology, and Scientific Methods*, vol. iv., No. 6, March 14, 1907). However, Mr. Wright does not so much furnish the classification itself as the principles for making it. 'As the species are classified according to the part they play in the process of organic evolution, so the virtues are classified according to the office they discharge in the organization of conduct. Thus our ideal of a principle of classification organic to the field of its application is realized' (*ibid.*, p. 160).

to the nerve-cord. This may be replaced by a cartilaginous (*i.e.*, gristly) rod, or by a column of distinct 'vertebræ.' These, again, may either remain cartilaginous or be replaced later by vertebræ of bone. (These vertebræ grow round and protect the nerve-cord.)

2. The possession, at some stage of development, of gill-slits in the anterior part of the alimentary canal.

3. An unpaired dorsal nerve-cord, which is tubular, having a central canal, and is protected by the notochord or the vertebræ. In the more advanced forms the brain and sense organs are highly developed, the latter being paired.

4. A highly organized circulation. The heart is always ventral to the alimentary canal.

5. Symmetrical segmentation.

Definition by characterization tends, in the case of the developmental sciences, to take the form of Definition *by Type*, a type being defined as 'an example of any class—for instance, the species of a genus—which is considered as eminently possessing the character of the class' (Whewell). Thus, Dr. P. Chalmers Mitchell says that Morphologists 'are slowly coming to some such conception as that a species is the abstract central point around which a group of variations oscillate, and that the peripheral oscillations of one species may even overlap those of an allied species' ('Enc. Brit.,' 10th edn., vol. xxviii., article on 'Evolution,' p. 343). Definition by Type is no doubt to this extent logically defective—that it does not provide ideally against ambiguity; and, in its insistence on a central as distinguished from a peripheral definiteness of characterization, it resembles Description rather than Definition proper; but it is none the less the Definition natural to classificatory Science.

There is good reason why, in Botanical and Zoölogical classification, the reference to organized reality should call for definition by type. Typical structures possessing a complete fitness for existence survive, and the intermediate forms tend to disappear, though there may be many deviations from type that are not important enough to interfere with that fitness to survive upon which the persistence of the type depends. Hence, in the developmental sciences, Real Definition—the definition of a class or concept that is framed to bear the searchlight which Science throws upon Nature—is essentially *central* in character. The central qualities and tendencies determine the definition; but in its application the definition takes in all variations which show a more marked approximation to the central requirements in question than to those of any other definition. This Definition by Type, we may add, forms a transitional link between a rigid peripheral definition, or definition by *boundaries*, and the more inward and vital definition by *ends* or ideals. In this latter kind of Definition, the defining marks, far from being possessed in common by all the members of the class defined, may be

possessed, in strictness, by none. If 'Man' is defined by the ideal his nature is capable of reaching, it is not necessary that any single individual man should possess the marks in question.*

Real Definition, finally, is essentially provisional and progressive. The widening of knowledge implies the remodelling of scientific principles and scientific classifications, and this implies that the definitions of essential concepts and of natural kinds undergo a sympathetic renewal. Where the aim of Definition is to characterize according to Nature, and the knowledge of Nature is continually deepening, the definition must adapt itself if it is not to stultify the very reason for its existence.

Diagnostic Classification and Definition.

A natural classification, in order to be really useful, should be accompanied by an analytical key. Such a key is a diagnostic classification, its function being to serve as an index or searcher for the corresponding natural classification. The essential distinction between these two types of Classification is that, whilst the marks which serve to locate a given species within the system are, in the case of Natural Classification, fundamental, in the case of Diagnostic Classification they may be merely superficial. The marks here are external and salient, and easily recognized. A diagnostic classification meets an important practical requirement —that of easy diagnosis—diagnosis being the method of determining the place of a natural kind in a classification, finding the correct name or label for the object by means of its characteristics.

The botanical system of Linnæus is essentially of the diagnostic type. It has been called artificial, because its classification-connexions do not stand for natural affinities. This is true, but it was not the author's intention that his classification should be natural. He intended that it should serve as a complete practical index or catalogue. 'It is an index to a department of the book of nature, and as such is useful to the student. It does not aspire to any higher character, and although it cannot be looked upon as a scientific and natural arrangement, still, it has a certain facility of application which commends it to the tyro. In using it, however, let it ever be remembered that it will not of itself give the student any view of the true relations of plants as regards structure and properties, and that, by leading to the discovery of the name of a plant, it is only a stepping-stone to the natural system. Linnæus

* As a suggestive illustration of what is involved in a philosophic definition—a definition, that is, which is framed under the control of such categories as those of *development* and *personality*—see Edward Caird's 'Evolution of Religion,' Lecture II. For the so-called 'pragmatic' definition, see C. S. Peirce, 'Illustrations of the Logic of Science,' in *The Popular Science Monthly*, vol. xii., under the two headings, 'The Fixation of Belief' (November, 1877), and 'How to Make our Ideas Clear' (January, 1878).

himself claimed nothing higher for it. . . . Besides his . . . index, he also promulgated fragments of a natural method of arrangement' ('Enc. Brit.,' 9th edn., vol. iv., p. 80).

This distinction between Natural and Diagnostic Classification puts in a clear light the relative character of what is 'essential' for Definition. The need for a diagnostic classification shows that for scientific purposes the salient mark is often more essential than the structural or the functional mark. If we wish to identify a plant, we don't, as a rule, need to examine its microscopic characters in order to class the specimen by some minute peculiarity of structure. It is, therefore, not true to say that in Diagnostic Definition the specifying mark may be non-essential. For the purposes of Diagnosis and of Diagnostic Definition its saliency is just what is essential. Definition in connexion with Diagnostic Classification is Definition with the purpose of identification. Hence it naturally takes the form of diagnostic characterization. It is a definition giving the salient, easily tested marks. Thus 'Iodine' may be diagnostically defined as a substance that colours starch blue. Where the absence of a mark forms the most striking means of identification, the diagnostic definition includes a negative characteristic :

Manx cats are cats that have no tails.

An apetalous plant is a flowering-plant in which the corolla is absent.

Formal Classification.

As we have already stated, formal classifications are characterized by the intimate relation in which they stand to the specific requirements of the individual classifier. There are two main types of formal Classification to which we may conveniently give the names of Conventional Classification and Index-Classification, the latter existing solely for the sake of, and in the service of, the former.

Conventional Classification, again, may be either Appropriately Conventional or not. In the latter case, the Classification may suitably be called Artificial. A conventional classification is *appropriately* carried out when there is no maladjustment between the nature of what is classified and the specific nature of the purpose which directs the classification. The kinds here classified must therefore not be those proper to any natural science, for in that case a subjective, conventional ordering of them would not be proper to their nature. They must be drawn from products of human art and thought, such as statues or books. Here, more especially in the case of books, no valuable end would be gained by attempting to group the types or kinds after the complete manner characteristic of Natural Classification, in which the full resemblances of the types classified are taken into account. It is here more purposive,

and therefore more logical, to fix on an attribute or group of attributes which happens to be of importance for the purpose, and to construct and classify the types in strict relation to it. On the other hand, where the types are natural kinds—*e.g.*, species of plants —and yet are not classified according to their nature, but according to a specifically subjective principle of selection and arrangement chosen without regard to the real nature of the material in question, the classification may conveniently be termed 'Artificial.' This appears to be a right and proper use of the word, though it gives a sense more restricted than that implied in the ordinary contrast between Natural and Artificial Classification. Thus the various kinds of garden plants might be formed and arranged in the light of some subjective interest (*e.g.*, the decorative interest, which may find colour-distinctions essential), and such an arrangement, when contrasted with the true, genetic order in which the various species of plants stand in relation to each other, might suitably be called 'Artificial.' Artificial Classification would then be a kind of Conventional Classification, but would not be identified with Conventional Classification in general.

Conventional, like Natural, Classification needs the co-operation of a key-classification subordinated to its own special requirements. But this key-classification will not, of course, share the objective character and intention of the analytical keys proper to Diagnostic Classification. As a rule, it will be found to be strictly alphabetical, as in the case of all indexes and catalogues. Thus, a librarian, in constructing and classifying types of books, will do so according to some subjective plan for which his own convenience rather than the nature of the object is the dominating standard. But the librarian's classification of the books is one thing, the cataloguing of the same for the convenience of readers is another thing. The latter classification is a mere finder to the former as that is represented by the arrangement of the books on the shelves, and stands to it in a relation closely analogous to that in which a diagnostic stands to a natural classification. And yet there are differences. Thus, a diagnostic classification yields in itself a certain superficial knowledge of the nature of what is classified, and can be translated into other languages, the arrangement not being alphabetical; but a catalogue, *qua* alphabetical arrangement, yields no knowledge of what it classifies. To be aware that *Punch* and the 'Principia' have a common initial letter hardly constitutes a knowledge of the books in question, and any attempt at translation would involve such a complete transformation of the original arrangement as to be equivalent to the construction of a new catalogue.

Finally, we would draw attention to the fact that spatial grouping, such as that of the books on the shelves of a library, or the arrangement of a collection of butterflies in a cabinet, is in no sense a logical classification. It is an arrangement of *specimens*, and not

a classification of *species*. Again, the assigning of individuals to their respective classes, though in itself a logical operation, is not a *classification of species*, but only a *classing of objects*. We may class specimens, but we cannot classify them.

- Classification.
  - Real.
    - Natural.
    - Diagnostic (Analytical key).
  - Formal.
    - Conventional.
      - Appropriate.
      - Artificial.
    - Index-Classification (Alphabetical key).

CHAPTER VI.

II. (vi.) SCIENTIFIC TERMINOLOGY AND NOMENCLATURE.

Technology.

LET us first say a few words about Technical Terms. Words, as we have seen, are sensitive, and pass through a process of growth. Various associations cluster round the original nucleus of meaning, making it extremely hard to define sharply even the usual meaning of the word. Now Science, in its anxiety to escape the dangers arising from these clinging associations, often takes the extreme step of inventing symbols which, since they can never be current, can never gain an associative meaning. Hence a great advantage in clearness and precision. On the other hand, we must face the fact that, by using technical terms, we cut Science off from ordinary life. To meet this objection Political Economy has, on the whole, adopted the plan of using popular words—*e.g.*, 'rent,' 'wages,' etc. But it defines these strictly for its own accurate purposes, and is therefore misleading to the uninitiated reader. And yet this plan has the great advantage of keeping the student in close touch with fact. The use of technical terms is, of course, justifiable only on the ground that accurate distinctions are needed. To be technical in one's language and inaccurate in one's thought is to make oneself ridiculous. The two essentials of the technical language of Science are (*a*) a good descriptive Terminology, (*b*) a good Nomenclature.

(*a*) *Terminology.*—By descriptive Terminology we mean a collection of words which will enable us to describe natural kinds. Of all the sciences, perhaps Botany has the best descriptive terminology.

Every part of a plant and every variety of plant structure has been so exhaustively named that the plant can, so to speak, be drawn in words. The flower has its calyx, consisting of sepals, its corolla of petals, its stamens, with their filaments and anthers, its pistil with its carpels, style, and stigma, etc.

Again, among all the various forms of the leaves of plants there is not one which cannot be accurately described. Thus, when a leaf is long and very narrow, it is said to be linear ; when the length is three or more times as great as the breadth, and the broadest part is below the middle, while the summit is tapering, the leaf is described as 'lanceolate' ; when the broadest part is above the middle, and the blade tapers towards the base, the leaf is called 'cuneate' ; and when the blade is broadly cuneate with a rounded top we say that it is 'flabelliform.' A leaf that approaches the form of a spoon or ladle is called 'spathulate' ; and other forms of leaves are known as 'ovate,' 'obovate,' 'orbicular,' 'oval,' 'oblong,' 'elliptical,' 'rhomboidal,' 'oblate,' and 'falcate.'

(*b*) *Nomenclature.*—A descriptive terminology must be carefully distinguished from a nomenclature. The nomenclature of any classification consists of the names for the groups or kinds which the classification systematizes ; the words by which these groups are characterized constitute its terminology.

Nomenclature, like Definition, tends to vary with the point of view from which names are considered. The purpose of Science being steady—that of naming in accordance with principles of Classification—we have, of course, a corresponding steadiness of nomenclature. With variety of interest comes variety of nomenclature, as the following extract from Watts's 'Logic' (quoted by Dr. Gilbart, 'Logic for the Million,' pp. 66, 67) clearly shows : 'Most of all [flowering] plants agree in this—that they have a root, a stalk, leaves, buds, blossoms, and seeds ; but the gardener ranges them under very different names, as though they were really different kinds of beings, merely because of the different use and service to which they are applied by men—as, for instance, those plants whose roots are eaten shall appropriate the name of roots to themselves, such as carrots, turnips, radishes, etc. If the leaves are of chief use to us, then we call them herbs, as sage, mint, thyme ; if the leaves are eaten raw, they are termed salad, as lettuce, purslain ; if boiled, they become pot-herbs—as spinach, coleworts ; and some of those plants which are pot-herbs in one family are salad in another. If the buds are made our food they are called heads or tops, so cabbage-heads, heads of asparagus, and artichokes. If the blossom be of most importance we call it a flower, such as daisies, tulips, and carnations, which are the mere blossoms of those plants. If the husks or seeds are eaten, they are called the fruits of the ground, as peas, beans, strawberries, etc. If any part of the plant be of known and common use to us in medicine, we call it a physical

herb, as carduus, scurvy-grass; but if we count no part useful we call it a weed, and throw it out of the garden; and yet, perhaps, our next neighbour knows some valuable property and use of it; he plants it in his garden, and gives it the title of an herb or a flower. Now, when things are set in this clear light it appears how ridiculous it would be to contend whether dandelion be an herb or a weed, whether it be a pot-herb or a salad, when, by the custom or fancy of different families, this one plant obtains all these names, according to the several uses of it and the value that is put upon it.'

In an ideal nomenclature each name would indicate the place occupied by the class named in the classification. This would be done by relating the class-names to each other, instead of allowing each group to name itself independently of the rest. Definition *per genus et differentiam* is thus represented, in a very simple form, in the systems of scientific nomenclature. In Botany and Zoölogy, for instance, each kind takes the name of the genus of which it is a species, and adds to it a differentia giving, usually, some characteristic or salient mark. Thus, the zoölogical name of the Rabbit is *Lepus cuniculus;* that of the Common Hare, *Lepus timidus*. The Red Deer is *Cervus elaphus*, the Wapiti Deer *Cervus canadensis*. The Brown Bear is *Ursus arctos*, the Grizzly Bear *Ursus ferox*. So also Botanists call the Field Rose *Rosa arvensis*, the Dog Rose *R. canina*, the Sweetbriar *R. rubiginosa*. The Marsh Violet is *Viola palustris*, the Sweet Violet *V. odorata*, the Hairy Violet *V. hirta*, the Sand Violet *V. arenaria*. The Common and the Creeping Buttercup, the Hairy Ranunculus, 'Goldilocks,' and the Lesser Celandine all belong to the genus *Ranunculus*, and are distinguished respectively by the specific names *acris*, *repens*, *hirsutus*, *auricomus*, and *Ficaria*.

In view of the great efficiency secured by making the name itself a sort of condensed definition, we may feel considerable sympathy with Mr. Garden's protest against what he calls the evil fashion, once so prevalent amongst naturalists, of paying compliments by naming genera and species after each other. 'What am I the better,' he asks, 'for hearing a rare moss called *Hedwigia hornschuchiana*, beyond being led to infer that Germany has, or had, two botanists, one called Hedwig and the other Hornschuch? On the other hand, when I am told that such a moss is called *Trichostomum lanuginosum*, I am, on supposition of previous knowledge of *Trichostomum*, presented with a definition, *lanuginosum* ("woolly") expressing the differentia of this species in the genus *Trichostomum*, even as *Trichostomum* does that of the genus when viewed as species of the higher genus which contains it.'

Chemical Nomenclature is peculiarly efficient. The names of the Elements, indeed, have, for the most part, been arbitrarily chosen, and are of historic interest rather than of scientific value; but the names of compound substances are assigned on systematic principles. Those of simple binary compounds (*i.e.*, substances composed of

two elements only) are formed by combining the names of their component elements, and as in many cases the same two elements combine in different proportions, the different compounds so produced are distinguished by means of the addition of terminal syllables or (more usually) of prefixes. Thus, among the compounds of Sulphur and Oxygen we have Sulphur dioxide, Sulphur trioxide, Sulphur sesquioxide. So, too, we have Potassium monoxide and Potassium dioxide, Lead tetroxide, Arsenic pentoxide, etc. Similarly the names of acids indicate their places in the classification by means of significant prefixes and suffixes. Thus Sulphur, in combination with Oxygen and Hydrogen, forms a whole series of acids known respectively as hyposulphurous acid, sulphurous acid, sulphuric acid, thiosulphuric acid, pyrosulphuric, and anhydro-sulphuric acid. In combination with other elements Sulphur forms a series of sulph*ides*, sulph*ites*, and sulph*ates*, the termination showing in each case the position in the series of the compound indicated. Further, the symbolic nomenclature of Chemistry is even more efficient and precise than the verbal system. Each element is symbolically represented by the initial letter (or *two* of the letters) of its Latin name, and the symbolic names of compounds are made up of the symbols of their component elements with the addition of numbers which indicate the proportions in which these elements are combined. Thus H is the symbol of Hydrogen, S of Sulphur, and O of Oxygen; and, in the series of acids cited above, hyposulphurous acid is symbolically represented as H_2SO_2, sulphurous acid as H_2SO_3, and the others in order as H_2SO_4, $H_2S_2O_3$, HS_2O_7, $H_2S_2O_7$.

In Astronomy, also, we find both verbal and symbolic names. The verbal name is framed, as a rule, on the 'genus et differentia' principle. The system is analogous to our method of designating persons by a family name and a Christian name. The family name is represented by that of the constellation, the Christian name by a letter of the Greek or Roman alphabet, or a number. Thus, α Lyræ, β Pegasi, Z Herculis, T Coronæ, 113 Herculis.

Frequently the number of the star in some given catalogue is used as its designation—*e.g.*, Lac(aille) 7215, Brad(ley) 3077.

As the constellations, especially those in the southern hemisphere, have been variously mapped out by different astronomers, and as different astronomers, again, use different catalogues, there is still a good deal of uncertainty as to the naming of stars. The same star may thus belong to more than one constellation, and be differently numbered in different catalogues.

Hence the great advantage in star-naming of using the symbolic name. The symbolic name is a formula or rule for finding the position of a star, and so identifying it. The formula consists in giving what are called the co-ordinates of the star—its latitude and longitude, so to speak (technically its right ascension and declina-

tion). Thus, the symbolic name of α Lyræ is: 18 hours, 33′, 6″ R.A.; + 38° 41′ Declination.

It would seem, then, that, since the purpose of naming a star is to be able to identify it by means of its name, and since a star can always be identified by its right ascension and declination, nothing but this statement of the star's position is really necessary. The symbolic name should be completely sufficient. 'Unfortunately, the position constantly changes through the precession of the equinoxes and other causes, so that this designation of a star is a variable quantity.'* The true symbolic name of a star is, therefore, given by the formula noting its R.A. and declination + all the rectifications required for precession, refraction, errors of instrument, personal equation, etc.

CHAPTER VII.

II. (vii.) CONNOTATION AND DENOTATION.

DEFINITION and Division are the two fundamental methods for developing or making explicit the meaning of a term. The results of these two processes have special names given to them;—Definition gives the *connotation*, Division the *denotation* of a word. Thus, if we define 'Man' as 'rational animal,' this is its connotation. If we divide 'Man' into 'Aryan, Semitic, or Turanian,' we are giving its denotation.

The *connotation* of a term, then, consists of the defining marks which the name implies; the *denotation*, of the alternative class-distinctions into which the meaning of the name can relevantly be divided. Let us consider more closely each of these two aspects of a word's meaning.

1. *Connotation* may be either formal or real. The connotation of a name will be *formal* when the name is used in the service of an interest that is more or less subjective and occasional; it will be *real* when it is the connotation of a scientific term. But in either case connotation is essentially definite, being the product of definition. And just in so far as an object possesses the attributes or the characteristics formulated in the connotation does it merit and obtain the name.

From the two types of *logical* connotation above referred to, the formal and the real, we must carefully distingush a type of non-logical connotation usually referred to as 'subjective intension.'

The 'subjective intension' of a class includes only such marks as

* Simon Newcomb, 'The Stars,' pp. 36, 37.

happen to be suggested on any occasion to any individual when a given word is mentioned. Thus, in the instance given on p. 29, 'crystal' would have been to the author, as a child, part of the 'subjective intension' of the word 'palace.' The ideas which a word serves to bring before the hearer's mind constitute its individual or 'subjective' intension (in the usual sense of these words), but *not* the logical connotation. The psychological suggestions of a word are one thing, the logical implications of a word quite another.*

2. The *Denotation* of a class or class-name depends upon its connotation, and in the same sense in which the division of a class depends upon the definition given to it. As division serves to differentiate and to specify what is indeterminate in a definition, so the denotation differentiates and specifies what is indeterminate in the connotation. The distinction between these two ways of stating the matter is that, in the former case, when we are speaking of Definition and Division, we have logical *processes* in mind, and, in the latter case, the *results* of these processes. It is quite customary, however, to use the term 'Definition' in the sense here reserved for 'connotation.'† Denotation will be formal or real according as the connotation is formal or real. The formal denotation of the term 'horse' includes, as alternatives, all the different kinds of horses that come logically under the class 'horse' as formally defined. The real denotation of the term includes, in a similar disjunctive sense, all the kinds of *Equus caballus* that come within the central or 'typical' definition of that species recognized by classificatory Zoölogy.

3. Interpreting 'Denotation' as a stage in the logical development of a term's meaning, we manifestly need another term, quite distinct from logical denotation, for specifying the enumerative relation between a class and the individuals to which the class-concept applies. The term '*Extension*' appears to be the most appropriate for this office. This would involve a specialization of the function which Professor Keynes has suggested for this very term. The problem, as it presented itself to Professor Keynes ('Formal Logic,' 4th ed., p. 30), was to fix a type of extension (in our sense of the word) which, in reference to the concept 'horse,' say, should include not only the horses that breathe and eat grass, but the steeds of fiction and imagination—such creatures as Pegasus, the Wooden Horse of Troy, the white horse of the fine lady at Banbury Cross, the horses of our dreams and of our desires.

* *Vide* Chapter I., p. 17.

† It would greatly conduce to clearness of nomenclature were the terms 'Definition' and 'Division' exclusively applied to the *propositions* which represent the defining or the dividing process. Thus, 'Man is a rational animal' might pass as the definition of 'man,' 'Man is Aryan, Semitic, or Turanian' as a corresponding division of the term. 'Rational animal' would then be the connotation of 'man,' 'Man is a rational animal' its definition, 'Aryan, Semitic, or Turanian' its denotation, 'Man is Aryan, Semitic, or Turanian' its division.

It is for this comprehensive office that Professor Keynes has suggested the term 'extension.' 'By the . . . *extension* of a general name . . .,' he writes, 'we shall understand the whole range of objects, real or imaginary, to which the name can be correctly applied, the only limitation being that of logical conceivability.' But if the term 'extension' is adopted in the sense here defined by Professor Keynes, it should not be overlooked that in so far as its range is not limited by a corresponding connotation, either in the formal or in the scientific sense, it cannot rank as a *logical* type of extension. Considered apart from any reference to a definite connotation, there would, for logical purposes, be the same objection to the term 'extension' as there is to the so-called infinite term 'not-x' (*vide* pp. 48-52). It is logically essential that extension should in all cases be determined by connotation.

We must carefully distinguish between the *specification* of meaning, through Division, and the *application* of meaning, through Enumeration. We propose to stamp this distinction by a correspondingly distinct use of the two terms 'denotation' and 'extension.' Denotation we define as differentiation of meaning, to be interpreted disjunctively through the formula '$G = S_1$ or S_2 or S_3, etc.' Extension we define as application of meaning to individual objects, to be interpreted conjunctively by means of the formula 'G applies to (or indicates) the individuals I_1 and I_2 and I_3, etc.'

Corresponding to this use of the term 'Extension' we would suggest the term 'Intension.' Let 'Intension' stand for the full relevant development of the meaning of a concept through Definition and Division. The intension of a term will then be equivalent to its connotation and denotation combined, and we have the formula:

$$\text{Intension} = \text{Connotation} + \text{Denotation}.$$

Our use of the term 'Intension' in the sequel will always be in the inclusive sense here indicated.

The 'Inverse Relation' of Connotation and Denotation.

It has been customary to formulate the relation between Connotation and Denotation by pointing out that, as we pass from summum genus to infima species in a classification, at each step increasing the determinate connotation by at least one differentia, we *tend* to diminish the number of kinds denoted by the concept. Thus, if we specify 'ship' as 'steam-ship,' the word no longer denotes the mere sailing-vessel. If we further qualify it as 'screw-steamship,' the species paddle-steamer is ruled out.

But, substantially correct as is this view of the relation in question, it is none the less superficial and misleading in its emphasis. It obscures the fact that the fundamental relation between connotation

and denotation is not 'inverse,' but 'complementary'—that connotation and denotation are, in fact, complementary stages and co-factors in the logical articulation of meaning. Moreover, the relation by no means implies that the total meaning or *intension* of a genus is poorer than that of a subordinated species. This remains an open question. Let G stand for any given genus, S_1, S_2, S_3 for its species (F.D. f) ; let d_1 d_2 d_3 stand for the determinate connotation of G, and let f_1, f_2, f_3 stand for those specifications of f which give the differentiæ of S_1, S_2, S_3 respectively. Then the full meaning or intension of G, as relevant to the present comparison between G and its species, is given by

$$\begin{tikzcd}[column sep=small] & d_1+d_2+d_3+f \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ f_1 & f_2 & f_3 \end{tikzcd}$$

and the corresponding intension of S_1 is given by $d_1+d_2+d_3+f_1$. The question before us, then, is whether (f_1 or f_2 or f_3)—for this is what

$$\begin{tikzcd}[column sep=small] & f \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ f_1 & f_2 & f_3 \end{tikzcd}$$

amounts to—is richer or poorer in meaning than f_1. The difficulty which besets the solution of this conundrum suggests that genus and species are more profitably studied as mutually indispensable links in the development of meaning than as rival claimants for some monopoly of meaning which shall enrich the one at the expense of the other.

Connotative and Denotative Names—The Limits of Definition and Division.

A name may appropriately be called 'connotative' in so far as it possesses a connotation, 'denotative' in so far as it possesses a denotation. The expressions are useful as helping to give precision to the important inquiry concerning the limits of definability and divisibility.

If we consider a given conceptual system, such as a natural classification of animals or plants, it is at once clear that all the subaltern genera, the classes between the summum genus and the infimæ species, are both connotative and denotative ; they are, we may say, conno-denotative. But it is not so clear that either the summum genus or the infima species is conno-denotative. The summum genus—'Animal' in the case of the zoölogical, 'Plant' in the case of the botanical classification—cannot, relevantly to the system which it represents, be defined *per genus et differentiam.* The summum genus, being the 'highest,' cannot be the species of a

genus higher than itself. It might seem, then, that summa genera were indefinable. So, again, it would seem that the infimæ species, being 'lowest' in the ranks of a division, were not relevantly subdivisible.

There are, moreover, other difficulties—those, namely, which are associated with the ambiguity of the term 'infima species.' If the infima species is a class-concept, it should be definable in a sense precisely similar to that in which a subaltern genus is definable. If, on the other hand, the proper name, or the singular meaning, is to be identified with the infima species as its limiting form, we are confronted by a new set of difficulties which centre round the question: 'Can a proper name be defined?'

Having thus briefly stated the difficulties, we pass on to consider how they can best be met.

(i.) *Is the Summum Genus Definable?*

It is manifestly true that in the case of the summum genus 'Animal' there is, *within the classification*, no higher genus by the help of which it could be defined. But this simply points the moral that the zoölogist cannot develop the full meaning of his leading concept, 'Animal,' without connecting it with the leading concepts of other sciences—*e.g.*, the botanical concept 'Plant'—and recognizing a superordinate genus, 'Organism,' which dominates both interests alike. At the same time, even *within* the limits of the specific classification which it represents, a summum genus must admit of a partial definition through an indeterminate attribute, the primary F.D., of which all subsequent fundamenta divisionis are the specifications. Thus, taking as our summum genus one of the two primary groups into which the Animal Kingdom is divided, the Sub-Kingdom 'Metazoön,'* this summum genus is definable as 'an animal organism *possessing an anatomical structure of some kind.*' This last-named characteristic, though indeterminate or indefinite, supplies, none the less, a perfectly unambiguous mark, and, as we have seen, an indefiniteness which does not amount to ambiguity is no disqualification for the purposes of Definition.

* Protozoa cannot unambiguously be said to possess anatomical structure. If we have shirked the definition of 'Animal,' it is because scientists do not yet seem to have discovered a satisfactory differentia between 'animal' and 'plant.' But if this should not be obtainable, the logical course would be to absorb the so-called 'Animal' and 'Plant' Classifications within the single Classification of Organic species. The Summum Genus 'Organism' might thus be defined as 'a cellular structure of some kind,' or, better still, 'a protoplasmic structure of some kind.' The essential point is that no classificatory system can be developed without a primary F.D., and this primary F.D. supplies an adequate differentia of the Summum Genus, distinguishing it unambiguously from all other Summa Genera. The reader who is interested in the attempt to fix the distinction between 'plant' and 'animal' will find an excellent treatment of the problem in Prof. Bergson's 'L'Évolution Créatrice' (deuxième édition, pp. 115–130).

The disjunctive specification of this dominating fundamentum gives the division of 'Vertebrate or Invertebrate'; and all the subsequent fundamenta—*e.g.*, 'Dentition'—are so many modifications of this original attribute of the summum genus—the possession of some kind of anatomical structure.

But a fresh difficulty arises when we conceive the process of abstraction, whereby summa genera are reached, carried to its limit, and culminating in a concept like 'Being' or 'Existence.' Such a concept or meaning can have no more general concept beyond it, since it is posited as ultimate for our thinking. We cannot, therefore, bring it under any superordinate genus, nor can we connect it with any co-ordinate species fulfilling a function logically similar to its own, as we can connect 'Animal' with 'Plant.' The ultimate summum genus cannot be defined *per genus et differentiam.* We cannot compare this unique definiendum with any co-species; we therefore cannot sift agreement from difference, and so distinguish a genus from a differentia. We must look elsewhere for a solution of the problem.

It might be urged that this arch-concept is self-defining. But if so, in what sense? It cannot be self-defining in the sense in which connotations are self-defining. The ultimate concept does not tell us its own meaning as do the expressions 'rational animal' and 'the mother of the two Gracchi' (*vide* p. 80). But if not self-defining in this determinate form of self-definition, may it not still be self-evident, and therefore in last resort self-definable?

There is no logical justification for supposing this. The ultimate abstraction can make no appeal to immediate experience; it therefore does not proclaim its own meaning, in however vague and undeveloped a form, by the easy way of unreasoned intuition.

But it might conceivably be self-evident in another sense. It might proclaim its meaning indisputably to the trained insight of the logical reason, though it failed to impress the exoteric consciousness. Can it be self-evident in this esoteric sense? In order to test this point we apply the well-known logical criterion of intuitive certainty; we ask whether it is impossible to deny the self-evidence of Pure Being without falling into self-contradiction.*

Let us first consider the argument in favour of the logical self-evidence of the statement that 'Something, *qua* pure being, is.' We take as our model Dr. McTaggart's defence of the indubitable certainty of Hegel's dialectical starting-point, the Category of Pure Being stated in the form 'Something is.' Hegel's Pure Being differs in some respects from the summum genus we are here considering, but the differences do not affect the present argument, and our proof of the non-self-evidence of the 'Being' which gives the summum genus tells equally well, in our opinion, against Dr. McTaggart's

* The 'self-affirmation' of Being—namely, the affirmation *that* it exists—is, in a sense, a statement of *what* it is, and to this extent implies its definability.

defence of the self-evident character of the 'Something is,' which gives the leading category of the Dialectic.

To deny the self-evidence of 'Being'—so runs the argument—is to deny the self-evidence of the assertion that 'Something is.'* But this assertion cannot be denied without being at the same time reaffirmed. For the denial at least 'is.' And to *doubt* the assertion is as conclusive in its favour as to deny it. For our doubt must be either genuine or not. If it is genuine, then we do not doubt that we doubt; we hold that something *is*—namely, our doubt: and if it is not genuine, then we are all the while admitting the truth that 'Something is,' while we pretend to doubt it.

Now if this argument were sound we should have to admit the self-evidence of Pure Being. But the argument is surely fallacious. Suppose I deny the self-evident character of Pure Being. I assert my denial, certainly, but not in the sense of 'pure being.' I assert it in a much less abstract sense. I may, therefore, without any logical inconsistency, deny that 'Something *qua* pure being is,' for the assertion of my denial is the assertion not that 'Something is' *qua* pure being, but that 'Something is' for me as an immediate experience. In the two propositions 'Pure Being is' and 'My denial of the existence of Pure Being is' the word 'is' has two quite different meanings.

We therefore cannot admit that the ultimate summum genus is either self-defining or self-evident; nor, as we have seen, can it be defined *per genus et differentiam*.

It would, no doubt, be convenient if at this point we could cut the knot with the short sharp word '*indefinable*.' The stroke would, however, be suicidal, for it would cut at the root of the whole logical theory of Definition. If a term is 'indefinable' in the strict sense of the word, it must remain permanently infected with ambiguity, should ambiguity ever come to cleave to it; for, the remedy of Definition being unavailable, the ambiguity must remain to tease logicians to the end of time. But no one will pretend that the term 'Pure Being,' that 'x' which is the ultimate summum genus, is free from ambiguity. Moreover, if an incurable ambiguity attaches to the summum genus, there is no root of soundness in any classificatory system developed on the *genus et differentia* principle. For in such a system there is no class-term of which the meaning does not rest ultimately upon the summum genus. 'Man,' we say, 'is a rational animal'; but both rationality and animality are in last resort specifications of the wholly indeterminate concept from which the development of all meaning initially flows. If the summum genus is indefinable, our definitions are, one and all, illusory, and we can never ultimately know what we really do mean. Our definitions will all be more or less remote specifications of 'that we know not what.' If x_n be the ultimate concept, and x_{n-1} a penulti-

* *Vide* McTaggart, 'Studies in the Hegelian Dialectic,' § 18, p. 21.

mate concept—a species of x_n with differentia δ_n—we say that x_{n-1} is x_n qualified by δ_n. But what is x_n? Similarly x_{n-2} is x_{n-1} qualified by δ_{n-1}. Thus, x_{n-2} inherits the vice of x_{n-1} which originated in x_n; and so we might go on to the limit of the infima species.

At this point we may be met with the impatient reply that the whole difficulty is surely gratuitous, and that the ultimate concept is not only definable, but definable in many ways. When we call 'Being' the 'summum genus,' are we not defining it? Or we may define 'Being' as the ultimate category or concept, the wholly indeterminate meaning. And are not these meanings, it will be added, unidetermining? Can 'the wholly indeterminate' be confused with any other meaning?

To these plausibilities the sufficient response is that, from the point of view of Definition *per genus et differentiam*, they all involve a *circulus in definiendo*. 'Wholly' and 'indeterminate' must themselves be specifications of the wholly indeterminate concept they profess to define, for the ultimate concept is, by hypothesis, ultimate, the ultimate generality whence proceed all the determinate forms of conceptual being. Every term of every definition presupposes this ultimate summum genus.

We have, then, still before us the task of showing in what sense, if any, the summum genus is definable.

Our first step must be to name the summum genus more precisely. From the standpoint of a merely objective view of the nature of 'meaning' there might be good reason for retaining 'Being' or 'Pure Being' as the ultimate concept. But we have already repudiated this abstractly realistic conception of 'meaning,' and have adopted an inclusively idealistic conception, which claims that meaning, whatever else it is, must always be the meaning of an object for a subject. This fundamental conviction, which has been stated rather than defended, or defended only through its power to reconcile conflicting views, calls for a correspondingly idealistic framing of the ultimate concept. Shall we, then, call it 'Knowable Being'? If it were urged that this concept suggests a further-lying concept, 'Being as such,' which may be specified as either knowable or unknowable, we should reply that 'unknow*able* being' is, from our point of view, meaningless. We might, indeed, content ourselves here with Professor Ferrier's simple argument that ignorance is relative to knowledge, and that where no knowledge is possible there can be no possible ignorance. If there be an unknowable, it is out of all relation to Consciousness, and we cannot logically refer to it as 'it.' We cannot say that we are ignorant of *it*, or that we don't know what *it* is. It must be nothing for our ignorance as well as for our knowledge.

Adopting 'Knowable Being' as our ultimate concept, we abandon, as intrinsically unreasonable, the attempt to define the ultimate concept except in relation to the logical interest through which we

know it, and we begin to realize that the search for the hidden sources of meaning must take us beyond the limits of any conceptual tree which does not, throughout its ramifications, involve this reference to interest and to knowledge. Nor is the appeal from 'object' to 'object of interest' an appeal to the *deus ex machinâ*. For the reference to logical interest is involved in the very notion of a concept. A concept is a concept-in-relation-to-logical-interest. If the addendum be torn away, the concept itself vanishes. Hence the appeal from the concept *per se* to the concept as known is simply the demand to have made explicit what is already implied. But to bring out implications is precisely to do that which renders superfluous all unintelligible assistance from a *deus ex machinâ*.

The problem now presents a somewhat different aspect. The ultimate genus, the genus that has no superordinate,* is seen to have as its correlative a thorough-going logical interest. It therefore cannot get its whole meaning from itself. If it is definable at all, it must be definable through the relation in which it stands to such an interest.

This relation is, in brief, the subject-object relation. Though bereft of all determinate content, the summum genus is still an object, the object of a logical interest. Hence the problem of its definition becomes the problem of deciding what we mean in general by an object of logical inquiry. The attempt to cope with this problem would bring us to the fundamental question of Kant's whole critical inquiry: 'What are the conditions of a possible object of experience?' It is not necessary for us to reconsider this problem, or to gauge the value of Kant's solution of it. It is not necessary because our aim is not to give the definition of 'Object,' but to consider, and in some sense to answer, the question as to its definability. We, therefore, content ourselves with noting the following points: (1) That there is no meaning more ambiguous or more in need of careful definition than the term 'object of experience'; (2) that the problem of its definition—*i.e.*, the question of its function and significance within the unity of experience—is a fundamental problem of the Theory of Knowledge; (3) that the ultimate logical postulate which this defining-process presupposes is the postulate of the radical intelligibility of experience. We have only to add that this postulate is not optional. We cannot think at all without making it. For to think is the same thing as to think what is not self-contradictory (*vide* Chapter X. *c*); the self-contradictory cannot be thought. But there is nothing ultimately meaningless save the self-contradictory (*vide* pp. 103, 104). Hence, to think at all is to think what has meaning and is *pro tanto* radically intelligible. This ultimate postulate, then, may be accepted as a ποῦ στῶ. Not only is it true to say that if the universe

* It makes no difference to the general argument if, with Porphyry and Aristotle, we hold that there are as many ultimate genera as there are categories.

were in any respect meaningless we could never know it, being unable to think what is meaningless ; we may go further, and say that if there were any such unknowable, we should not be suspecting the possibility of its existence. To be conscious of a limit is to have already transcended it ; the barrier that closes upon our knowledge closes also upon our ignorance.

(ii.) *Is the Infima Species Divisible?*

The reference to logical interest which has determined the nature of our answer to the question, 'Is the summum genus definable ?' suffices to give a definite answer to the further question, 'Is the infima species divisible ?' For, if a given species is 'lowest' relatively to a stated interest, any further division of the species would be necessarily irrelevant, and would involve that over-distinction and false subtlety which is the inevitable penalty of irrelevance. If we know our purpose and point of view, it must always be possible to fix the critical point beyond which refinement and the differentiation of meaning degenerates into irrelevant subtlety. Where a classification is dominated by a scientific interest, the extent to which a division is carried will depend not only on the degree of accuracy required, but on the progress of scientific knowledge. Thus 'giraffe' (*Camelopardis giraffa*), which used formerly to stand as an infima species, is now known to include eight or ten distinct varieties. We conclude, then, that while the infima species, as such, should be regarded as logically indivisible, yet fresh advances in scientific knowledge are always promoting infimæ species to a higher grade, and thus making them divisible. Division, like Definition, is essentially progressive.

(iii.) *Can Proper Names be Defined?*

Let us once more consider the celebrated conceptual Tree of Porphyry, the tree that grows downwards through a series of successive divisions from the heights of independent Being, through the distinctions between Animate and Inanimate, Sensible and Insensible, Rational and Irrational, to Socrates, Plato, Aristotle, etc. at the terminus (*vide* p. 47). This tree of concepts is made to include individual elements, and the point which here concerns us is the question whether this inclusion can be logically justified. Can the infima species of a conceptual tree be a proper name ?

It seems reasonable, and perhaps necessary, to suppose that it can. A division of the term 'Man' might conceivably be so carried out as to pass from Man to Statesman, thence to Prime Minister, then to the Prime Minister of England in May, 1907. This last concept would necessarily be singular in its reference, and would be represented by the proper name, 'Henry Campbell-

Bannerman.' Similarly we might pass by successive subdivisions through the subaltern genera Greek, Athenian, Philosopher, etc. to the Socrates and Plato of Porphyry's tree. So long as the single individual represented by the proper name is not considered as he is *for himself*, but in the same external way as are the many individuals indicated by any ordinary class-name such as 'Man' or 'Athenian,' there seems to be no ground for differentiating between the status of the general name and that of the proper name in regard to the problem of the development of meaning. The latter becomes simply a limiting case of the former.

With the above-mentioned proviso, we seem to be justified in including individual elements in a conceptual tree and in regarding them as intrinsically general or conceptual in character, though sharpened—intensively—to such a point of conceptual fineness and determinacy as to have—extensively—a unique reference. They are then singular names, the function of a singular name, from the point of view of Extension, being to refer us unambiguously to some single individual object or person.

Regarded in this light, the singular name or meaning belongs, by natural birthright, to the organized system of logical concepts. There is no opposition between the class-concept and the singular or individual concept, nor is the phrase 'singular concept' a contradiction in terms. The individual meaning is simply the concept in its limiting form. Where the only *relevant* element in this meaning is the *individuality* of its reference, the name which represents it is known as a *proper name.*

The singular concept is that limiting form which the infima species of a division tends in last resort to take, though whether this terminus is actually reached or not depends entirely on the answer to the question whether it is logically relevant or advantageous to reach it. The singular concept, or the proper name which represents it, we hold to be definable by means of a significant singular name—a name which, by virtue of its very significance, is singular. This appears to be the natural connexion between the proper name and the significant singular term. 'Rational animal,' which defines the general concept 'Man,' is a significant general expression; 'The mother of the two Gracchi,' which defines the singular concept 'Cornelia,' is a significant singular expression.* Significant singular expressions, precisely alike in this to significant general expressions, are self-defining. Thus, 'The highest mountain on the Earth at this present stage of our planet's geological history' defines itself.

* Significant singular expressions are sometimes more briefly called 'designations.' Thus Mr. Joseph ('An Introduction to Logic,' p. 21) defines a designation as 'a phrase which by a pronoun or what not serves to indicate an individual otherwise than by a name of its own.' This briefer title, however, does not render the longer one superfluous. A singular symbol may either designate or signify. It may designate an individual (Extension), or it may signify a meaning (Intension).

Its significance consists in the meaning of the marks that make up the term. The very expression tells us that the single object referred to is a mountain in our planet, and the highest of all at the present time. As other instances of the significant singular term we may mention 'The centre of the Earth,' 'The first Emperor of Rome,' 'The town at present situated at the mouth of the Dart.'

Our attempt to maintain the definability of the individual may be met by certain objections :

1. It may be urged that proper names do indeed *suggest*, but do not *imply*, attributes. The distinction between suggestion, according to psychological laws of association, and logical implication is, indeed, fundamental, and there is a great difference between unregulated 'subjective' meaning and logical connotation. But once we recognize that such 'implication' is relative to purpose—that it is only through knowing what our defining purpose is that the implication can be relevantly fixed—the objection loses its point. 'Common' natures, no less than individual natures, suggest a great deal more than is relevant to the purpose for which their definition may chance to be required. But the defining interest once clearly stated, those attributes or marks can be selected which, in the most direct and economical way, suffice to render our meaning and reference unambiguous. The unselected marks that are not required for the fixation of our meaning will then assume the status of propria, and will remain propria so long as our defining interest remains the same. It may be sufficient, from a given point of view, to identify Mr. Balfour with 'the British Prime Minister in the year 1900,' or with 'the philosopher who wrote "The Foundations of Belief."' These identifications will give just such meaning as the name relevantly, and therefore logically, implies in respect to the interest which dominates the definition. They will be definitions of a proper name. Proper names, like general names, require as many definitions as there are points of view from which they can serviceably be used.

2. An objection of quite another kind may be raised against the definition of the proper name. It may be contended that it is just a meaningless mark. 'The only names of objects which connote nothing,' writes Mill, 'are *proper* names; and these have, strictly speaking, no signification. . . . A proper name is but an unmeaning mark which we connect in our minds with the idea of the object, in order that whenever the mark meets our eyes or occurs to our thoughts, we may think of that individual object.'*

In the use of the phrase 'unmeaning mark' it is evident that Mill does not intend us to understand that the name is bereft of all meaning whatsoever. It must at least retain a meaning as a complex of sounds or letters, for if the proper name were bereft of this meaning it could not function as a mark at all. By the phrase

* J. S. Mill, 'A System of Logic,' Book I, ch. ii., § 5.

'unmeaning mark' he seems to mean a sign which stands for the 'that' and not for the 'what' of the object which it indicates. But is this possible? Is the 'that' intelligible apart from its development into a 'what'? A case might conceivably be made out for the pure thatness of a summum genus, but not of a proper name. Mill compares the imposition of a proper name to the marking of Ali Baba's house by the robber in the 'Arabian Nights.'* But the chalk-mark affixed to the house in order to distinguish it from the other houses in the row did not represent a mere 'that.' It was a sign which meant 'The house of Ali Baba,' and to this extent it stood for and signified *what* was the object to which it was attached. It therefore cannot be regarded as 'an unmeaning mark.'

3. The best (though, in our opinion, still unconvincing) defence for the indefinability of the proper name is based upon the conviction that an individual's *principium individuationis* is not what the individual is for an outside spectator—or logician—but what it is for itself, the contention being that the individual, *qua inwardly* individual—*qua* experient—is strictly indefinable. But is this so? If we cannot here have Definition *per genus et differentiam*, may we not have Experience-definition reflecting the point of view of the experient himself? Such definitions would present in systematic form the meaning of our own immediate experiences stamped with the impress of our own relevant reflection upon them. Concepts like 'Thou' and 'I,' concepts which mark immediate experiences of feeling or sensation, such as 'blue,' 'hot,' 'angry,' 'tired' (considered not as concepts of Physics or Physiology, but as genuine psychological meanings grounded in immediate experience)—may not all these gain a definite meaning through a reflective reconstruction of what is primarily self-evident? As experience-concepts they could not belong to any conceptual tree which, in its arrangement, failed to recognize either that meaning is 'for a subject' or that the subject is also an experient. 'Now,' 'here,' 'ultramarine,' 'I,' and all proper names, *qua* personal, belong neither to Porphyry's tree nor to any branch of strictly scientific Classification. They belong, as perhaps all concepts ultimately do, whether directly or indirectly, to the tree of self-knowledge which is rooted in immediate experience.

The difficulties which beset the attempt to define experience-concepts apart from their relation to the experience which they signify are excellently illustrated by the following passage from an article by Professor Stout.†

'When I refer to the present Lord Chancellor, I determine him as the particular Lord Chancellor existing at the time at which I am speaking; when I refer to this table, I determine the table as that

* J. S. Mill, 'A System of Logic,' Book I., ch. ii., § 5.

† Proceedings of the Aristotelian Society, N.S., vol. vi., pp. 360-362.

which is near me or at which I am pointing, or as that of which I have just been talking. The application of proper names is determined by the particular occasions on which they have been uttered—*e.g.*, in the baptismal service. And so, quite generally, we can never mark off in language the particular existence we mean except by its relation to some other particular existence presupposed as already known. But this process obviously involves a vicious circle unless there is ultimately some direct apprehension of particular existence which supplies a point of departure for thought. If we attempt to reach this ultimate basis by a regressive process, we find ourselves approaching nearer and nearer to our own psychical life as the final centre of reference through which all other particular existence is determined. The limit of this regress is marked by such words as "now" and "I." In such words we indicate a particular existence which is not determined by the thought of relation to some other particular existence, but by the direct apprehension of particular existence just as it is actually existing. For this reason it is impossible without a logical circle to define adequately in language what it is we refer to when we say "now" or "I." This is impossible because we can only express in language the relatively complex cognition of which immediate apprehension is an element. What is immediately apprehended cannot be so detached as to become by itself a distinct object of knowledge. It is not nameable except as being an element of a relatively complex object. Thus, if I am right, when the application of words to particular existents is directly determined by immediate experience, it ought to be impossible to explain what is meant without a vicious circle. And, as a matter of fact, this is so. Let anyone try to explain what time it is which he refers to when he says "now." It is not enough to say that "now" means the time at which a person is speaking, for persons speak at different times, constituting a great many *nows*; but in saying "now," the reference is to only one particular time. How is this particular time distinguished from the others? It is circular to say that by "now" I mean the "time at which I am now speaking." Yet anything short of this is inadequate. Again, we cannot define the time meant by assigning its relation to past or future time. For the "now" forms the ultimate starting-point from which we determine temporal position in the past or future. The future is what follows the "now," and the past is what precedes it. Thus, any attempt to determine the meaning of the now merely by its relation to the past or the future involves a vicious circle. The "now" must be stamped by a peculiar signature of its own—a peculiar character intrinsic to it. What is this peculiar character? We may attempt to express it by saying that the "now" is the moment of actual experience. We may say that it is the moment in which sensations, pleasures, pains, etc. are not merely being thought but actually existing. But, again,

we have to press home the old question. The "now," it is said, is a moment of actual experience. But which moment of actual experience is it? For there are an indefinite multiplicity of these; the mental life of each of us from the cradle to the grave includes an incessant succession of moments of actual experience. How is the particular one which we refer to in saying "now" singled out from the others? Evidently no general conception of actual experience, and no mere thought of there being particular instances of actual experience, will help us in the least. Mere thought leaves us moving round in the old circle. The moment of actual experience referred to is the present moment; the "now" is the time of that actual experience which is *now* existing. If there is any way out of this *impasse* except the one I propose, I should be exceedingly glad to know what it is. The only escape that I can discover lies in frankly admitting that there is a direct apprehension of particular existence as it is actually existing. The application of the word "now" is determined, not by any mere thought of it, but by our immediate experiences in the way of sensation, sensuous imagery, pleasure, pain, etc., directly cognized in the moment of their existence as they cannot be cognized at any other moment.'

With the fundamental contention of this passage I should be in cordial agreement. But I would put in a plea against the suggestion that certain ultimate meanings are indefinable. 'It is impossible,' says the writer, 'without a logical circle to define adequately in language what it is we refer to when we say "now" or "I."' Is this strictly true? Has not the passage just quoted shown us rather that it is impossible without a logical circle to define in adequate language what it is we refer to by such terms as 'now' and 'I' when they are used as symbols of *immediate personal experience?* But is it, we ask, justifiable to *attempt* a definition of the self-evident *qua immediately self-evident?* And if we do not attempt this, deeming the task uncalled for, may we not still live in the hope that the self-evident, when sufficiently reflected on, so as to reveal an inner spiritual structure, may prove in the end to be self-defining?

The adequate definition of these experience-concepts is, no doubt, a philosophical *ideal*, but, as such, it is surely a problem of supreme interest and importance. With regard to the nature and conditions of such a problem, we cannot do more than throw out one central suggestion. Is not the essential condition this—that the definitions can be grasped only in proportion as the experiences are experienced? Just as the definitions of 'here' and 'now' must be formulations of actual experience, so there can be no genuine definitions of religious concepts except such as express genuine religious experience; no genuine definitions of art-concepts which do not express genuine artistic experience. The definer who would understand his definitions will, in such case, be compelled to ex-

perience and, in some sense, sympathetically to live out what his definitions may subsequently attempt to formulate and express. Experience must, in his definitions, relive in conceptual form, otherwise the definitions are mere deceptive formulæ which cannot possibly mean what they say. An adequate definition of the self-evident 'now' of immediate experience will, at any rate, not be given till our human nature has realized in its own time-experience how past, present, and future stand related not only to each other, but also to the eternal Present which in some way transcends them.

CHAPTER VIII.

II. (viii.) CONCRETE AND ABSTRACT TERMS.

A TERM may be said to be concrete when it refers us to concrete objects, abstract when it refers us to an abstract object. Whether a term is concrete or abstract depends, then, on the nature of its objective reference.

Now, an object is concrete when it is regarded as possessing an immediacy either for sense or for feeling, or individuality in time or space, and is thought of as thus 'immediate' or 'individual.'

Abstract objects are derivative. They are derived from concrete objects through a process (1) of discrimination, (2) of analysis.

Discrimination alone cannot give us an abstract object. It can only differentiate the concrete object. In a yellow orange we may discriminate the yellowness and the roundness. But the yellowness of the orange is not an abstract yellowness; it is a quality, a discriminated quality, of the concrete orange.

Whether we use the adjectival or the substantival form makes no difference in the nature of the discrimination, and therefore no difference in regard to the question of abstract and concrete. 'A yellow orange' and 'the yellowness of an orange' are equally concrete in their reference, and accordingly are both concrete terms.

But if, after having discriminated the yellowness *as a quality or feature of the orange*, we proceed to *analyze it out* of its concrete context, so that it can no longer be said to be an inherent feature of the orange, but is withdrawn by the abstracting power of thought and brought under the conditions of thought-existence, we have yellowness as an abstract object—an object which has immediacy for thought, but no longer any relevant sense-immediacy, nor yet any immediacy for feeling in the ordinary acceptance of that term.

An abstract conception or meaning is thus a common quality as such, a universal, detached from all reference to individual objects, and considered solely *qua* universal.

Terms which indicate or otherwise refer to these abstract objects, these products of mental analysis, are called abstract. They are the names of detached attributes and relations, of attributes and relations mentally isolated, of attributes and relations *per se*. The merely discriminated attribute or relation which is not mentally severed from the concrete object which it qualifies or relates is, as we saw, still concrete in its reference. (See 1, *b* below.)

We obtain, then, the following distinctions :

1. A term is concrete—
   (*a*) When it serves to indicate* individual existents (things, persons, events, etc.) ;
   (*b*) When it serves to indicate an attribute presented as actually qualifying individual existents, or a relation considered in connexion with the individuals related.
2. A term is abstract—
   (*a*) When it serves to indicate an attribute considered apart from the individuals (things, persons, events, etc.) of which it is the attribute ; or a relation considered in severance from the individuals related ;
   (*b*) When it serves to indicate an attribute or other qualification of an abstract object.

As illustrations of the distinction between Abstract and Concrete Terms regarded from the point of view of objective reference, let us consider the subject-terms of the following propositions :

'*A tyrant's hate* is a thing we need not fear.'
Here the subject-term is concrete (1, *b*).
'*The hate of my fellows* is a force I dare not face.' Concrete (1, *b*).
'*Hate* is old wrathe' (Chaucer). Abstract (2, *a*).
'*The hate of hate* is the Poet's dower.' Abstract (2, *a*).
'*The hatefulness of sin* is an eternal fact.' Abstract (2, *b*).
'*The hatefulness of Mr. Hyde* is a haunting horror.' Concrete (1, *b*).
'*A hateful thought* is a crime.' Abstract (2, *a*).
'*That hateful thought of yours* is a disgrace.' Concrete (1, *b*).
'All *the Virtues* are personified qualities.' Abstract (2, *a*).
'*Virtue* is a self-rewarding activity.' Abstract (2, *a*).
'*A man's virtue* is to be truly a man.' Concrete (1, *b*).
'Some *virtuous people* are not pious people.' Concrete (1, *a*).
'All *virtuous activities* are forms of happiness. Abstract (2, *a*).
'*The virtue of suffering* is the fostering of sympathy. Abstract (2, *b*).
'*This man's thoroughness* is an acquired characteristic. Concrete (1, *b*).

* Thus 'Being' or 'Person' is, from this point of view, as concrete as 'Welshman' or 'John Jones.'

'*Thoroughness* is a test of efficiency.' Abstract (2, *a*).

'*The thoroughness of this work* is an admirable quality.' Concrete, (1, *b*).

'*Thoroughness of work* is a praiseworthy quality.' Abstract (2, *b*).

Question.—Examine whether the italicized terms in the following propositions are abstract or concrete :

1. 'Truth is not *mercy.*'
2. 'What I ask of you is *mercy.*'
3. '*Mercy* is the sister of Justice.'
4. '*A light* is a characteristic of the glow-worm.'
5. '*Light* is a necessity of plant-life.'
6. '*Light* is a mode of motion.'
7. 'This figure is *a square.*'
8. '*This square* is not *that square.*'
9. '*The square* is a rectangle.'

1. 'Truth is not *mercy.*'

'Mercy' is here used abstractly, apart from any reference to a stated individual possessing it as a quality of character. Abstract (2, *a*).

2. 'What I ask of you is *mercy.*'

'What I ask of you is *your mercy.*' 'Mercy' is here used as qualifying the person referred to by 'your.' Concrete (1, *b*).

3. '*Mercy* is the sister of Justice.'

The personification of 'mercy' only makes its abstract use more evident. If the proposition had been 'Mercy is the sister of Charity,' with a reference to 'Martin Chuzzlewit,' 'Mercy' would have been used as a concrete term. The personified virtues that are not of flesh and blood are the sheerest abstractions. Abstract (2, *a*).

4. '*A light* is a characteristic of the glow-worm.'

Here 'a light' is a concrete term. It refers to individual things. Concrete (1, *a*).

5. '*Light* is a necessity of plant-life.'

'Light,' as a necessity for plant-life, indicates radiant energy manifested at definite times and places. The term is therefore concretely used. Concrete (1, *a*).

6. '*Light* is a mode of motion.'

It is true that 'Light,' *as a mode of motion*, can be neither sensed nor felt. But the word has reference to undulatory movements passing through definite spaces at definite times. Concrete (1, *a*).

7. 'This figure is *a square.*'

Here 'a square' is used abstractly. Despite the fact that a particular figure is indicated, what is implied is that this particular figure stands for an ideal square. However badly it may have been drawn, it still *represents* that ideal form. Abstract (2, *a*).

8. '*This square* is not *that square*.'

Here both subject and predicate terms are concrete. The meaning is that the given square ABCD, as drawn or constructed, without reference to the ideal mathematical square, is not identical with the given square EFGH. Of course, if the 'squares' indicated are of the 'Trafalgar Square' kind, the concrete nature of the terms is still more obvious. Concrete (1, *a*).

9. '*The Square* is a rectangle.'

Here the reference is to the ideal square, not to any particular representation of it. Abstract (2, *a*).

Since it is the nature of the objective reference which decides whether a term is abstract or concrete, it would seem to follow that the degree of generality which a class-term as such may possess, though it involves a certain kind of abstraction—namely, 'generalization'—has nothing to do with its being ranked as an abstract or concrete term.

The objection may, indeed, legitimately be raised that class-meanings are as such abstract, and that one class meaning is more abstract than another. 'Animal,' 'mammal,' 'lion,' it may be urged, are abstract in the sense that they are general classes and not individual things; and, again, 'animal' is more abstract than 'mammal,' 'mammal' more abstract than the typical 'lion.' As we pass from lion to carnivore, from carnivore to mammal, from mammal to vertebrate, each higher class—higher in the sense of being more general—is at the same time more abstract, and the *summum genus* in any system of classification will be the most general and most abstract of all.

That the distinctions here referred to are real enough cannot be denied. The presence of degrees of generality among concepts is essential to the existence of a classification. But the word 'general' is surely good enough to enable us to dispense, in this connexion, with the word 'abstract.' There is, moreover, a more appropriate word still—namely, 'indeterminate.' This word is, in fact, forced upon us by the construction we have put on 'meaning' and its logical evolution from the indeterminate to the determinate, or involution from the determinate to the indeterminate. From this point of view, every concept has its place within a certain conceptual system. What this system precisely is depends on the interest which controls our use of the concept in question. This system we may call the logical topic, or universe of discourse, and in so far as it is allowed to develop itself logically—*i.e.*, in the direction determined by the growth of the logical interest itself—it takes its start from a *summum genus* as its indeterminate fountain-head, and differentiates itself in an orderly conceptual way, till it reaches the limit of determinacy in the infimæ species. The concept 'Carnivore' represents, from this standpoint, a stage in the logical

development of the Topic of the Animal Kingdom, and it is the aim of Definition and Division, as combined in the single process of Classification, to indicate precisely the nature and position of this stage in relation to the whole Topic.

Where class distinctions are understood in this way, the use of the term 'abstract' is unnecessary and uncalled for. It is simpler to say that there are degrees of determinacy or indeterminacy among concepts, and to interpret this statement in the way above suggested.

Logicians are accustomed to raise the question as to whether abstract terms are connotative or not. The question is legitimate, but should include the question whether abstract terms are denotative or not.

There seems to be no reason for dealing differently in this matter with concrete and with abstract terms. For the concreteness or abstractness concerns the objective reference only, and in no way the conceptual interrelations of any system of meanings, whether these be abstract or concrete, whereas it is precisely with these interrelations within a conceptual system that the distinctions of connotation and denotation are concerned. If by 'connotation' we mean the product of definition *per genus et differentiam*, then in a system of abstract concepts the *summum genus*, relatively to the system, will be non-connotative; the infimæ species—*e.g.*, mathematical equality—will be non-denotative; all the other abstract concepts will be both connotative and denotative. Thus the term 'Insanity,' abstractly used, might have a connotation 'morbid mental state,' and a denotation 'mania, or monomonia, or melancholia, or dementia, or amentia.' So the connotation of 'Roundness,' abstractly used, might be 'spatial form having a curved surface or outline,' and this connotation admits of being differentiated after the ordinary manner of denotation. There are at least two main kinds of roundness: roundness of line and roundness of surface. To the former type belong circularity, the roundness of the oval, of the cycloid, of the catenary, etc.; to the latter all the varieties of three-dimensional roundness—the roundness of the sphere, the ellipsoid, etc.

III.

THE LOGICAL PROPOSITION.

(i.) The Judgment or Proposition. Introductory Statement (ch. ix.).
(ii.) The Laws of Thought (ch. x.).
- (*a*) The Law of Logical Identity in its relation to the Proposition.
- (*b*) The Laws of Non-contradiction and Excluded Middle.
- (*c*) The Inviolability of the Laws of Thought.

CHAPTER IX.

III. (i.) THE JUDGMENT OR PROPOSITION.

A JUDGMENT, in the simplest logical sense of the word, is a meaning which admits of being characterized as true or false, or at least as self-consistent. Where such characterization is out of place, the expression cannot rank as a judgment. Thus, optatives, imperatives, and ejaculations fall, as such, outside the sphere of logical judgments. An optative expresses a wish, and we cannot say of mere wishes that they are either true or false; they are merely reasonable or unreasonable. Similarly imperatives call, not for belief, but for obedience; they announce commands, but do not communicate truths.

The relation of Proposition to Judgment or Assertion may be defined by saying that the proposition is the judgment in a purposively fixed form. It is the judgment in that form in which it first becomes available for logical purposes. The proposition is not to be understood as a mere drapery of words which the judgment, as a synthetic act of thought, can put on or put off as it pleases. It is quite true that we can think without words. It is not at all essential that the sensory assistance so indispensable to thought should take the form of a conventional *verbal* sign. But whatever the sensory symbolism may be, it is only as a purposive fixation of meaning that it has any logical significance. The grammatical, or, to speak more correctly, the philological and phonetic interest in verbal structure as such is non-logical. In Logic we are interested in words only as the visible or audible forms in which thought fixes and controls its own meaning. The proposition, *qua* logically serviceable, is therefore indistinguishable from the judgment. As in Definition the distinction between verbal and conceptual definition was found, on closer inspection, to have no logical *raison d'être*, so, in the matter of assertion generally, the distinction between proposition and judgment, so far as logical interest is concerned, is a distinction without a difference. The terms 'proposition' and 'judgment' are logically interchangeable.

On the other hand, we must distinguish between proposition and sentence. Every proposition is a sentence, but not every sentence a proposition. For the sentence is the unit of speech generally. Any

syntactical expression is a sentence, whether it expresses a truth or a mere wish, a command or a supposition. A proposition is only a certain kind of sentence—a sentence in which an affirmation or denial is appropriately expressed. It has been called an *indicative* or *declaratory* sentence.

We have referred to the judgment or proposition as the logical unit. The reason for this is that the proposition is the simplest form in which we can state our meaning, the simplest form to which the verdict 'true' or 'valid' can be applied. Ideas in themselves —'horse,' 'leaf,' 'man'—are neither true nor false; nor can we predicate truth and falsity of mere combinations of ideas. We can no more say that such expressions as 'high horse,' 'lingering leaf,' 'miserly man,' are true than we can say that 'hexagonal horse,' 'league-long leaf,' 'microscopic man' are false. They are not judgments, and therefore do not admit of truth or falsity. Nor can such complex ideas be self-contradictory or invalid. 'Human horse,' 'limping leaf,' 'mineral man' cannot contradict themselves, because they express no assertions they can proceed to deny. Before I can maintain anything to be either true or false, I must express it as an affirmation or a denial. 'This horse is not high,' 'These leaves are lingering,' 'This man is a miser.'

The statement of meaning which we find in every proposition as such implies an objective reference to fact. For 'meaning,' as we saw, is always the meaning of an object for a subject. But it is by no means essential that we should take this implied objective reference explicitly into account. Whether we do so or not will depend on the nature of our interest. Where the interest in putting forward a proposition lies in ascertaining whether the interpretation of fact, as stated in the proposition, does or does not tally with the more authoritative interpretations based upon experimental analysis of sensible fact, the proposition has a truth-import and calls for proof or disproof. The reference to fact is here of the essence of the proposition. The discussion of such propositions will occupy us at a later stage, when we come to treat of Inductive Method.

The preliminary inquiries of logical science into the nature of Judgment presuppose a more abstract view of propositional import. We require not only, as in inductive inquiry, to consider the bearing of statement upon fact; we must also consider the statement itself, the interrelations of its elements, and the relations in which it may stand to other statements. We have also to consider how the implications of a statement, or combination of statements—implications of quite another kind from that of objective reference—may be made clear and explicit. In a word, we have to consider, not only the *truth-import* of a proposition, but also, and from the various points of view above specified, its *statement-import*, its import as a statement of meaning.

The theory of the proposition, from the relatively abstract standpoint of its statement-import, consists essentially in an elaboration and application of two fundamental principles, respectively known as the *Principle of Logical Identity* (or, more briefly, the Law of Identity) and the *Principle of Non-Contradiction* (or, more briefly, the Law of Contradiction). Of these, the Principle of Identity is logically the more fundamental. It is implied in the very stating of a proposition, and is therefore presupposed in the very enunciation of the Principle of Non-Contradiction, which, in a form still to be more closely specified, concerns the relation in which an already stated proposition stands to other propositions.

CHAPTER X.

III. (ii). THE LAWS OF THOUGHT.

(*a*) THE LAW OF LOGICAL IDENTITY IN ITS RELATION TO THE PROPOSITION.

THE fundamental principle of the logical development of meaning is the Principle of Logical Identity.* Its essential purport is that a meaning remains identical with itself through all its manifold developments, that these developments are *its* developments, and therefore belong to one and the same single system of meaning. A meaning-identity in its most undifferentiated form is known as a topic or subject of discourse. It is the identity of some total interest, *qua* total and undeveloped. We have but to remember that meanings are constantly changing and developing to see that that type of meaning which logicians refer to as the Concept is just some more or less determinate specification of some such topic or universe of discourse. The topic itself is the concept in its most indeterminate and total form, and the first and simplest formulation of the Principle of Logical Identity is the statement that the conceptual system in which all the implications of a concept are made systematically explicit is a single, identical unity. A classification-scheme is a conceptual system of this kind. As such it is a logical Identity, which finds its incipient expression in that most indeterminate of concepts, the *summum genus*.

Such a view of identity has its roots in the very nature of the process according to which our minds habitually work. We always think within an interest. What interests us may be a question or

* The terms 'principle' and 'law,' *as applied to the logical nature of thought*, may be treated as equivalent expressions. The Principle or Law of Identtiy is a fundamental Law, or Principle, of Thought.

a problem, a desire, hope, or aim; and the intellectual movements of our life, however fleeting and evanescent they may be, gain their identity or organized unity through relation to some such interest, and through working the interest out to some kind of ending or consummation: the answering of a question, the solution of the problem, the reasoned fulfilment of the end we had in view.

We do not propose at present to discuss in its full bearings the Principle of Logical Identity. Its relation to Inference is as close as its relation to Judgment. We shall see later on in what sense an Inference is an Identity. We restrict ourselves here to considering the principle of Logical Identity in its relation to the unit of all our thinking: the judgment or proposition.

It has been customary, in this connexion, to state the Principle in the unpromising form 'A is A,' or 'A is not non-A.' This is the form of statement natural to a view of the significance of 'meaning' which takes no account of development. It is the Principle of Statical Identity. As such it can only endlessly affirm the separate self-identity of logical concepts. It circles about the concept as a starting-point, impotent to move beyond it. It cannot regulate the development of a concept, or take us from the concept, through the proposition, to inference and a system of inferences. For to say that A is A is precisely equivalent to saying 'A.' If we are told that Logic is Logic, we still hold the undifferentiated concept of Logic before our minds; 'Logic is Logic' means 'Logic,' but we have said nothing at all *about* Logic. This so-called principle of judgment is not itself a judgment at all. 'The propositional form itself contradicts it: for a proposition always promises a distinction between subject and predicate, while the present one does not fulfil what its form requires' ('The Logic of Hegel,' tr. Wallace, p. 213). If we were really guided by this so-called principle, we should never make any assertions whatsoever. We should be continually positing concepts, and never developing them; proposing topics, and then revolving helplessly around the mere idea of them.

To state a proposition, we must, to put the matter quite generally, *specify* our meaning. If we wish to make a definite statement of fact, we must first specify that aspect of the total topic which we wish particularly to speak about; this will give us the subject of our statement. We have then to specify this subject by predicating something about it that is other than itself. This whole specification of meaning will have taken place within a topic whose unity dominates the whole procedure, so that the specifications will be all specifications *within* the topic; and the Principle of Identity will be a Principle of Identity in relation to Differences, a Principle of Identity in Difference.

Identity, then, can be fruitfully understood only in the light of difference. Hence, when identity is conceived out of all relation to difference, it is barren, and has no logical value whatsoever. A

'principle' of Identity so conceived is a mere principle of Tautology. Thus, the empty insistence that 'A is A,' or that 'Whatever is, is,' is worse than meaningless. Many who despise truisms in ordinary discourse seem to think that in Logic they are very suitably housed, fulfilling there some kind of mysterious function which must anyhow be very satisfying to the initiated. Moreover, the greatest truths, we are told, are the simplest. But the very reverse of all this holds good. A truism is worse in Logic than anywhere else, for it is there paraded, not as a mere truth of fact, but as a truth of the very reason.

The inanity of the old formula 'Whatever is, is,' or 'A is A'—distinctions of formal statement are here of no consequence—was first clearly exposed by Hegel in his 'Logic.' 'The main thing,' he urges, 'in connexion with thought, is not to confuse the true Identity . . . with an abstract identity, identity of bare form. All the charges of narrowness, hardness, meaninglessness, which are so often directed against thought from the quarter of feeling and immediate perception rest on the perverse assumption that thought acts only as a faculty of abstract identification' ('The Logic of Hegel,' tr. Wallace, p. 215). 'It is asserted,' he writes on a previous page (*ibid.*, p. 214), 'that the maxim of Identity, though it cannot be proved, regulates the procedure of every consciousness, and that experience shows it to be accepted as soon as its terms are apprehended. To this alleged experience of the logic-books may be opposed the universal experience that no mind thinks or forms conceptions or speaks in accordance with this law, and that no existence of any kind whatever conforms to it. Utterances after the fashion of this pretended law (A planet is—a planet; Magnetism is—magnetism; Mind is—mind) are, as they deserve to be, reputed silly. That is certainly matter of general experience. The logic which seriously propounds such laws, and the scholastic world in which alone they are valid, have long been discredited with practical common sense, as well as with the philosophy of reason.'

It is extremely important, however, that this lawful contempt for the impotent tautology 'A is A' should be precisely limited to its lawful object. What is logically inane is an identity that is out of relation to all differences. *Identity in relation to difference* still remains the basic and the guiding conception of logical Science.

The logical unit—the Statement or Proposition—is an identity in difference. A statement is a movement of mind—a purposive, ideally directed movement of mind*—and this movement is a de-

* The purposive character of all assertion whatsoever is implied in its being a statement of meaning; for 'meaning,' as we have conceived it, 'tells us what an object *is* in relation to a specified interest or purpose' (*vide* p. 23). Meaning, in so far as it has any logical significance, is always *intended* meaning. The teleological character of Judgment can, however, be adequately grasped only in relation to the problem of Experience, and the further discussion of this central point is therefore postponed.

velopment through which different elements are evolved within the unity of a single total meaning. Nor, as we have already said, is the scope of the Law of Identity limited to a single statement. Whatever we think together must be knit together through the constitutive activity of the Identity Principle, otherwise it is illusory to suppose that our thoughts really cohere.

The question now suggests itself: Does the Law of Identity indicate, not only a necessary, but also the *sufficient* condition of sound thinking? If this were so, the Law of Identity would be *the* Law of Thought. There is good reason for maintaining that this is really the case, and that, in guarding itself against degenerating into a meaningless formula of difference without identity, the Principle would naturally take the limiting, or rather the self-limiting, form of a Law of Self-Consistency. If Σ and Σ' are accepted as contributing to the development of one and the same idea or topic, Σ cannot affirm what Σ' denies. There is, however, at this point no reason for pressing thus far the claims of a logical Monism. For the present we may conveniently regard the Law of Identity as limited in its application by a second law, which insists that difference cannot intelligibly be carried to the point of inconsistency; for, in so far as the integrity of our thinking is broken, our statements must cease to mean anything at all.

(*b*) The Laws of Non-Contradiction and Excluded Middle.

The Principle of Non-Contradiction, which supplies the criterion or test of Logical Consistency, states the conditions under which a given statement can be held consistently with the primary requisite of Intelligibility. This involves the relating of a proposition Σ, or, more specifically, 'S is P,' to what is called its contradictory $\overline{\Sigma}$, or '$\overline{\text{S is P}}$.'*

The Principle may be formulated as follows:

> If the statement 'S is P' is accepted (*i.e.*, accepted as an understood statement of meaning), the statement '$\overline{\text{S is P}}$' must be rejected (a).

And—

> If the statement 'S is P' is rejected, the statement '$\overline{\text{S is P}}$' must be accepted (β).†

* A convenient symbolic designation for the denial of 'S is P.' The 'bar' —— placed over the affirmation is a recognized logical device for expressing the corresponding denial. The device is a natural one, for we have only to place the bar a little lower down, thus: ~~S is P~~, and 'S is P' is then appropriately cancelled in the ordinary way. We shall, therefore, use the symbol $\overline{\text{S is P}}$ in place of the more awkward expression 'Not (S is P)' whenever the substitution proves convenient.

† To reject a statement is to accept its contradictory.

These enunciations, it will be observed, include an explicit reference to the thinker's attitude of acceptance or rejection. This reference is essential. If it were dropped, the Principle would cease to be a Consistency-principle. To insist simply that 'If "S is P" is true, "$\overline{\text{S is P}}$" must be false' is, indeed, to postulate the Intelligibility of Reality; but the principle so enunciated has no direct bearing on the Consistency of the Thought which is exercised upon, or within, Reality. If Σ and $\bar{\Sigma}$ could be true together, Reality would not be intelligible; but there is no inconsistency in our thinking until we attempt to think these two statements together. Σ and $\bar{\Sigma}$ must be entertained as true together before our thought can be said to be inconsistent and to run into contradiction.

It must not be supposed, however, that the principle 'If Σ be true, $\bar{\Sigma}$ must be false' is not, in some sense, a Law of Thought. The Principles which express the necessity we are under of holding Reality as intelligible are of fundamental significance for our thinking. Still, they concern not statement as such, but statement only in relation to fact. They concern the *truth-import* of the Proposition, not its *statement-import*.

This consideration will serve to explain the omission of the Law of Excluded Middle from the list of principles upon which the intelligibility and consistency of our statements depend. The Law of Excluded Middle may be formulated as below. The statement implies that no third alternative, no mean, no middle course is possible.*

Either Σ is false or $\bar{\Sigma}$ is false (α).
Either Σ is true or $\bar{\Sigma}$ is true (β).

Here, as before, (α) and (β) are mutually complementary.

It is plain, moreover, that the attempt to interpolate a reference to the thinker would destroy the self-evidence of the principle. It is by no means necessary that 'Either Σ or $\bar{\Sigma}$ must be accepted as true,' or that 'Either Σ or $\bar{\Sigma}$ must be rejected as false.' We may suspend our judgment. Σ may be a mere unverified hypothesis, in which case it would be an affront to Reality either to accept it as true or to reject it as false. It is, then, only on the supposition that Σ is actually accepted as true that we are bound, in the interests of consistency, to reject $\bar{\Sigma}$ as false.

The Law of Excluded Middle, then, does not concern the statement-import of the Proposition, and we can have no occasion to *apply* it until we come to deal with propositions from the point of view of their truth-import. There is, however, an important misconception to which the law itself is liable—a misconception affecting its very statement; and this it will be convenient and suitable to consider at once.

* For a criticism of the pseudo-form 'S is either P or non-P,' *vide* p. 191.

The Relation of the Law of Excluded Middle to Time.

It is frequently stated that the formulation of the Law of Excluded Middle should include a reference to time. If this were so, the Law of Excluded Middle should run as follows:

'The two propositions "S is P" and "S is-not P" cannot both be true (or false) *simultaneously*.'

The point is important, but it is not necessary to appeal to its importance to justify the following discussion of it, which I owe to the kindness of Professor Stout.

'We sometimes speak as if a proposition and its contradictory could both be true—not, indeed, at the same time, but at different times. Thus we may say, It was true that Jones was a bachelor, but the truth now is that he isn't a bachelor. . . . But closer examination shows that truth and falsity are in no way affected by the flux of time. Whatever reference there may be to time is already included in the *meaning* of the propositions which are true or false. These temporal conditions cannot, therefore, be used over again to qualify the proposition as a whole. "The Battle of Waterloo was fought in 1815 A.D." is a proposition concerning a temporal event and its date. But the truth of the proposition is not an event, and has not a date. The truth of the proposition is not something which occurred in 1815. The occurrence of the battle in 1815 will be a fact a thousand years hence; and if a thousand years before 1815 some one had assigned 1815 as the date of the battle, he would have been stating a fact.

'To bring this out clearly, we may compare with the analogous case of spatial relation. The same argument which may be used to show that truth is subject to time conditions may also be used to show that it is subject to spatial conditions. But in the case of space the fallacy is more obvious. Suppose three persons, A and B and C. A is on one side of the street; B and C are on the other. A calls to B: "Where is C?" B answers: "C is here." A rejoins: "C is not here, but there." Thus we have two statements: "C is here," and "C isn't here," and both of these are true. Are we, then, to say that truth is locally variable? Are we to say that a proposition may be true in one place, whereas it is false and its contradictory true in another only a few yards off? If this were so, the topography of truth would be a most interesting study which has been strangely neglected. Of course, the whole supposition is nonsense, and the apparent difficulty is easily removed. The word "here," although it has the same sound in the mouth of A and in the mouth of B, has a different meaning. In the mouth of A it indicates the place where A is at the time, and its immediate neighbourhood; in the mouth of B it indicates the place where B is at the time, and its immediate neighbourhood. Thus the statements "C isn't here"

and "C is here" are not the same statements in the mouths of A and B respectively. Hence we may admit that when B says "C is here," he is affirming a true proposition; and also that when A says "C is not here," he is affirming a true proposition; we can, I say, admit this without admitting either that two contradictory statements may both be true, or that truth and falsehood have position in space.

'The relation of truth to time is quite similar. Propositions are not true at one time and false at another any more than they are true at one place and false at another. If it sometimes looks as if this were not so, the appearance is illusory, and is due to the fact that the same verbal formula may on different occasions express different propositions. The variability in the meaning of the words "here" and "there" attaches also to the tenses of verbs, and to such terms as "now" and "then," "yesterday" and "to-morrow," "past" and "future" and "present." We say, for example, that "to-morrow never comes," but this only means that when it does come we no longer call it "to-morrow," we call it "to-day"; but when we call it "to-day" we mean the very same day which we had previously called "to-morrow." On the other hand, if we said "to-morrow" instead of "to-day," we should not be referring to the same day, but to another. Thus, if a man say on Monday "I shall play golf to-morrow," he does not contradict the statement by saying on Tuesday "I shall *not* play golf to-morrow." He contradicts the proposition that he will play golf on Wednesday; he does not contradict the statement that he will play golf on Tuesday. The tenses of verbs, and more especially the present tense, fluctuate similarly in meaning with the occasion on which they are used. In the year 1900 the words "Jones is now a bachelor" may express a true proposition; in the year 1905 the very same words may express a false proposition, for Jones has married in the interval. But we do not give a right account of what has taken place if we say that the proposition which was true in 1900 has ceased to be true in 1905. On the contrary, the truth of this proposition is absolutely unaffected by the lapse of time. What looks like a change from truth to falsity is really only a change in the meaning of the word "now" and of the present tense of the verb. The truth expressed by the verbal formula "Jones is a bachelor" as used in 1900 cannot be expressed by the same formula in 1905, and it cannot be contradicted by saying in 1905 "Jones is not now a bachelor." If a person speaking in 1905 wishes to contradict it, he must say "Jones was not a bachelor in 1900." Similarly, if we now say "Jones will be a widower in 1908," we express exactly the same proposition which would be expressed by some one speaking in 1908 in the form "Jones is a widower." In general, what is true does not become false, and what is false does not become true. Wherever this may appear to be so, the appearance is an

illusion. What has really happened is that a form of speech has changed its meaning because of a shifting of the point of view from which we determine time-relations.'

The relation of the Principle of Non-Contradiction to that of Non-Ambiguity may be briefly expressed as follows :

The Principle of Non-Ambiguity *logically* presupposes that of Non-Contradiction, though the practical *application* of the latter principle presupposes the former. For what is Ambiguity ? Not all variation in the meaning of terms, but only such as *may* lead to Inconsistency. And Ambiguity is to be avoided just for this reason. Without reference to the Principle of Non-Contradiction we can hardly distinguish Ambiguity from other variation in the meaning of the same term.*

We have said that in the *application* of the Principle of Non-Contradiction the Principle of Non-Ambiguity is presupposed. This may be shown as follows : If there is not between disputants complete agreement as to the meaning of statements and expressions, it is impossible for the one, with any certainty, to accuse the other of contradicting himself. Similarly, if there is not perfect correspondence between what I mean and what I say, two of my statements may be *apparently* contradictory—contradictory in form—and yet my meaning may not be so. The existence of a genuine thought-contradiction can be clearly pointed out only after we have first seen that the Principle of Non-Ambiguity is adequately satisfied. Meanings can be criticized as consistent or inconsistent with each other only in so far as they have been adequately defined. Thus, to borrow Father Clarke's† illustrations, the two propositions 'This man is wise,' 'This man is not wise' may be accepted together, in spite of their being verbal contradictions, if by the first we mean that the man is prudent and canny in business matters, and by the second that he holds many foolish opinions on speculative questions. So, again, an event may be said to be impossible or not impossible according as we use the word to signify moral or absolute impossibility ; and similarly, in reference to an unwelcome visitor, I may say without self-contradiction "I am at home, and yet I am not at home."

Before we can decide, then, whether two propositions are contradictory or not, we must agree as to their meaning. In this sense the observance of the Law of Non-Ambiguity is presupposed in all discussions as to whether one proposition is or is not consistent with another. But ambiguity does not necessarily in itself imply a contradiction. Two interpretations of one and the same statement may or may not be inconsistent, and therefore contradictory. A priori, they are as likely to prove adjustable and complementary as they are to prove mutually inconsistent.

* For this whole observation I am indebted to Professor Stout.

† 'Logic,' p. 37.

The following are two main ways in which the application of the Principle of Non-Contradiction presupposes the Principle of Non-Ambiguity :

1. The reference in both the opposed propositions must be to the same standard. Thus, the rate of five miles an hour is both fast and not fast—fast for a man walking, but not fast for a man running. Hence the two propositions 'This is a quick pace' and 'This is not a quick pace' are contradictory only when the standard is kept constant.*

2. Two apparently contradictory propositions are not really so if the objective reference of subject or predicate differs in the two cases. Thus, the two propositions 'All infants are under two,' 'Some infants are not under two' (*e.g.*, over twenty) are perfectly consistent if 'infants' is used in the ordinary sense in the first statement, and in its legal sense in the second. So, again, a child may be fair and yet not be fair—*i.e.*, he may have a fair complexion, but hair that is not fair ; a man may be strong and yet not strong—muscularly strong, but not strong constitutionally.

Self-Contradiction.

There are certain so-called statements that are self-contradictory. These are cancelled by the Principle of Non-Contradiction as logically meaningless ; *e.g.*, 'The statement of Epimenides the Cretan that all Cretans are always liars is perfectly true.' As Professor Stout says, 'When Epimenides utters the words "All Cretans always lie," then, if he tries to include the statement which he supposes himself to be making, he is not making a statement at all. For the meaning of the supposed statement cannot be determined without a vicious circle—*i.e.*, cannot be determined at all.

All statements by Cretans, including this one of mine, are false.

Including *what* statement of yours ?

Answer : Including the statement that all statements by Cretans, including that one of mine to the effect that . . . etc., are false.

And so on *ad infinitum*, with no πoῦ στῶ.'†

By way of further illustration we may consider the following example : 'Contradictories may be true together. For, admitting that every rule has an exception, then, since this statement is itself a rule, it follows that we must also admit that there is at least one rule without exceptions.'

This means that if we grant the truth of the rule 'Every rule has an exception,' then the rule which tells us this, being itself a rule, must have an exception. There is an exception to the rule that every rule has an exception as there is to every other rule—*i.e.*, there must at least be one rule that has no exception. The solution

* *Cf.* Father Clarke, 'Logic,' *ibid.*

† For a further treatment of this ancient puzzle, see p. 182.

of this puzzle lies in the consideration that the given proposition is self-contradictory. What we have just proved, then, is not that contradictories may be true together, but that if a proposition is implicitly self-contradictory, it is possible by a little reasoning to make its self-contradictory character explicit.

We conclude our treatment of the Laws of Thought by the discussion of a certain fundamental characteristic shared by all the Laws alike in virtue of their common function as Principles of Intelligibility. As such, the Laws of Thought are inviolable, and we now propose to discuss the precise significance of this striking property of inviolability.

(*c*) The Inviolability of the Laws of Thought.*

The Laws of Thought are so constitutive of our rational faculty that it is impossible to violate them. The deliberate attempt at violation results not in falsehood, but in a blank failure to think. What we say is unthinkable, and has no meaning. Thus, the attempt to override the Principle of Identity, in its relation to the proposition, is the attempt to treat an assertion as being void of any principle that holds its various elements together. Subject and Predicate are treated as mutually independent entities, whereas the assertion has no meaning apart from their dependence. Hence, an inward contradiction which, *qua* contradiction, is unthinkable. And if, further, the hypothetical violation of the Law of Contradiction reduces the 'proposition' that attempts the violation to the status of a mere meaningless formula, the attempt at violating the Law of Excluded Middle leads to similar results. The Law of Excluded Middle safeguards the intelligibility of fact. If fact is not intelligible, it cannot be intelligibly thought of. In so far as the object of our (would-be) thought—*e.g.*, a round square—is meaningless, the 'thought' is simply non-existent.

In this fundamental respect of inviolability, the Laws of Thought are unlike such other kinds of laws as laws of the land, and laws which regulate the relation between ends and means. Laws of the land are presumably violated by every criminal who finds his way into prison or reformatory. Laws of health, and all other laws which prescribe the conditions of success in the attainment of ends, are constantly infringed by all of us. But we cannot so speak of the Laws of Thought: a violation of these can never occur. If we are found accepting two apparently contradictory statements, it is either that they are contradictory in form only and not in substance, or else that, in accepting them, we accept them without thinking, or, at any rate, without recognizing that they are contradictory. Let us consider different classes of cases in which the Law of Contradiction is thus apparently violated.

* The following section owes much to the help and inspiration of Professor Stout.

The most common instance of a supposed violation of a law of thought is the case in which we are deceived by the ambiguities of the language we make use of—ambiguities which themselves reveal a lack of precision in our own thinking. Thus, to take up an illustration already cited, we may accept the two statements 'This man is strong' and 'This man is not strong' as contradictories, whereas a little further reflection will show us that we are using the word 'strong' ambiguously. Until we have made the distinction between 'strong muscularly' and 'strong constitutionally,' we have not said what we intended to say; but so soon as our intention is expressed in appropriate language, the contradiction disappears. *Distinction* is the first remedy to apply to that state of mental disease which we may call the illusion of Contradiction.

It is a well-known fact that when we set ourselves to think over the fundamental problems of our life—God, Freedom, Immortality, the World—we soon find ourselves reaching apparently contradictory conclusions. In their most fundamental form, these conclusions are known in Philosophy as Antinomies. Kant discusses their significance in his 'Critique of Pure Reason.' Perhaps the most familiar of these Antinomies is that between Necessity and Free-Will. Sincere reflection on the problem may lead to the two opposite conclusions that we are free, and yet that we are not free. But an Antinomy of this kind is not a Contradiction. If it were this, it would not be so mentally stimulating as it is. It is not actual Contradiction that stimulates—what is meaningless cannot fail to be depressing—but the necessity our thinking nature is under to get beyond the illusion of Contradiction. Kant solves the Antinomy by making a *distinction*. He qualifies the 'we' differently in the two cases. As members of a phenomenal world, he concludes, we are not free; we can be free only as members of an 'intelligible' or spiritual world.

In another class of cases we run into apparent contradiction through very failure to think at all. Thus it is conceivable that a student might conclude from the two statements 'No French are not Europeans' and 'All Parisians are French' that some Parisians were not Europeans. But it cannot be assumed that the conclusion of an argument is always reached by thinking out the premisses. A mere association or an ill-digested suggestion may very well relieve the tension of our thought at the critical moment, without our realizing what has happened. But in this case we have not violated a law of thought, but simply the salutary maxim that a result for which the reason is to be held responsible must have been reached under the sole direction of the reason.

Most people entirely fail to systematize their thinking. They live mentally in a number of isolated compartments, and are quite indifferent if an 'A is B,' which can be justified within its own abstract sphere, is irreconcileable with an 'A is not B,' which in its

own context, again, can be similarly justified. But there is no contradiction here. For if the two spheres in question are labelled S_1 and S_2, then S_1 (A is B) and S_2 (A is not B) are not contradictories. S_1A is not the same subject as S_2A, nor is S_1B the same predicate as S_2B. There is only the Illusion of Contradiction.

However we look at this matter, the Law of Contradiction is seen to be inviolable. When we imagine that we have violated it, we have simply ceased to think; and where there is no thought, we cannot violate its laws.

To think inconsistently, then, is strictly impossible. It would necessitate our thinking the gaps in our thinking. We cannot go on thinking through a contradiction, though we can think on to the very edge of the gap and then allow ourselves to be borne asleep on the wings of forgetfulness over the unthinkable itself.

But the metaphor is misleading. It implies that the unthinkable is something, whereas it is nothing. The only unthinkable is that which breaks a constitutive law of our thinking; but if these laws are inviolable, the unthinkable cannot exist in any form. It is not Contradiction that exists, but the baffled thought that realizes the restraining pressure of that which makes Contradiction impossible—the very Reason itself.

It is, then, impossible, we repeat, to think inconsistently. Where the inconsistency occurs there will be a bare blank of thought, showing up not, indeed, the inconsistency of our thought, but its *fragmentariness*. We shall thus appear to have violated the Principle of Identity which insists on the meaninglessness of logically connecting two judgments that are logically discontinuous. The Illusion of Identity here consists in the assumed unity and continuity of the reasoning. In treating what is fragmentary as though it were coherent, we have not, however, violated the Principle of Identity, for we have not been able to *think* it as one and continuous.

The difficulty which besets the conclusion that the Laws of Thought are inviolable is that, on this doctrine, it seems hard to account for the fact of Inconsistency. Inconsistency of Thought can mean no more than Discontinuity of Thought; in committing the Formal fallacy, we must have ceased to think. But if we are not inconsistent as *thinkers*, in what capacity is it that we are inconsistent? Moreover, even if we replace the conception of Inconsistency by that of Discontinuity, and are content to label our fallacies as mere breaches of Continuity, we have only shifted the difficulty from one word to another. We have still to reconcile the fact of discontinuity with the inviolability of the Laws of Thought. If a breach of continuity is not a violation of a law of thought, of what is it a violation? To hold that nothing is violated is to deny the value of logical distinctions, and reduce the distinction between Consistency and Inconsistency, Continuity and Discontinuity, to a difference that is purely psychological. But if we hold, as we must

if Logic is to have any distinctive meaning, that our inconsistencies are, in some sense, breaches of rational obligation, we are once more confronted with the old persistent difficulty: How can they be violations of logical requirement if they are not violations of the laws of our thinking?

Perhaps the simplest answer to this really perplexing question is the best. It may be enough to say that the fallacies which appear to be infringements of the Laws of Thought are, in truth, infringements of the duty to *think*, of the moral (or religious) obligation we are under to think faithfully—*i.e.*, to faithfully *think* whenever we profess to be thinking.

It may, indeed, be argued that though we cannot violate a law, we may yet break a rule, and that fallacies in Logic are no more than breaches of logical rules. This would no doubt be a happy solution did not the breach of the rule imply the violation of the law. But strictly logical rules embody the requirements of the Laws of Thought. How could they otherwise be qualified for the guidance of our thinking? A fallacy cannot therefore be a violation of logical rules as such. Again, though the Rules of the Syllogism may no doubt be used to test the logical satisfactoriness of our reasoning, it would seem that in thus testing its validity we are, at bottom, testing our own *fidelity* to thought. For if we have really been *thinking*, we must have been thinking validly. We are thus brought back to the view that 'logical' fallacies are breaches of intellectual morality or of the religion of the Intellect, violations of the duty to think faithfully whenever we are pledged to think. Logic would thus seem to be rooted in Morality and Religion.

It may seem strange that in a crucial difficulty of this kind there should have been no appeal to the insight of Hegel. The whole perplexity arises from the conviction that Contradiction is meaningless, whereas Hegel is insistent that Contradiction is not meaningless. 'Contradiction,' we read ('Logic,' tr. Wallace, p. 223), 'is the very moving principle of the world: and it is ridiculous to say that Contradiction is unthinkable.' I am, however, obliged to confess—though the fault may very well be mine—that I have found Hegel's conception and treatment of Contradiction irrelevant to the solution of the precise difficulty under consideration. In so far as Hegel is insisting that a certain Negativity, or negative movement, is essential to the vitality of spiritual experience, and that such Negativity, under one aspect or another, is a permanent element of such experience, even in its most harmonious developments, I believe that he is indicating and establishing in its abstract form the most distinctive conviction of all genuine Idealism. What I do not see is that he is in any way redeeming or transcending the unintelligibility of a contradiction.*

* For a profound and suggestive treatment of the problem of Negativity, *vide* Professor Bosanquet's article on 'Contradiction and Reality,' *Mind*, N.S., No. 57, January, 1906.

IV.

ANALYSIS OF THE LOGICAL PROPOSITION AS STATEMENT OF MEANING.

CHAPTER XI.

IV. (i.) KINDS OF PROPOSITION.

THERE are three main kinds of proposition :

1. Categorical : ' S is P,' ' S is-not P.'
2. Disjunctive : ' Either A is B, or C is D, or E is F.'
3. Hypothetical : ' If A is B, then C is D.'

If we examine the structure of these three main types of proposition, we notice that the elements out of which each type is constructed are different in the three cases.

1. The Categorical Proposition is a synthesis of the two elements which, in their verbal form, are called Terms*—the Subject-Term and the Predicate-Term. The subject of a categorical proposition is that about which something is being said ; the predicate is that which is being said or stated about the subject of the proposition.

Examples : (The next examination) is (held in June).
S P

It is (a tiresome thing) (to fail in an examination).
P S

No one need fail.
= (The possibility of passing) is (in the hands of
S P
everyone).

As the proposition is the true logical unit, and the term a mere derivative or abstract from the proposition, it follows that the term has meaning only in relation to the part it plays in a proposition. But this does not mean that we are logically precluded from discussing terms except in so far as they actually exist within given propositions. A word, or a word-complex such as ' The Science of Logic ' or ' The Cat and Fiddle,' may justly be called ' a term,' provided it *can* play the part of subject or predicate in a proposition. A term is, in fact, just a potential subject or predicate, so that even in abstraction from the proposition it still remains intrinsically related

* On the etymological significance of the word ' term,' *vide* Joseph, ' An Introduction to Logic,' ch. ii., p. 13, footnote.

to it. Though it need not be doing actual service within a proposition, it must at least belong to the reserve.

2. In the case of the Disjunctive Proposition—*e.g.*, 'Either the earth moves or Copernicus was mistaken'—the main elements are clauses, not terms; and they stand to each other in the relation of *alternatives*—a relation very different from that of Subject to Predicate.

3. In the case of the Hypothetical Proposition, again, it is evident that we cannot call its main elements 'terms.' Take, for instance, the proposition 'If the weather is fine, we shall go for a picnic.' Here we have two main elements; but these do not stand to each other as Subject to Predicate, nor are they terms. They are *clauses*, and stand to each other as Antecedent to Consequent, the antecedent being represented by the 'if' clause.

We conclude that propositional elements are of three kinds:

Terms:	1.	Subject and Predicate in Categorical propositions.
Clauses:	2.	Alternative possibilities in Disjunctive propositions.
	3.	Antecedent and Consequent in Hypothetical propositions.

The types of proposition that we have been considering may be connected together as different stages in the process through which the human mind passes on its way from question to answer. Every process of reflective activity which is guided, however vaguely, by the logical ideal of clear and consistent thinking is a process of which the essence is to give a more determinate form to what, at the outset, is relatively indeterminate. 'The action of thought,' it has been said, 'is excited by the irritation of doubt, and ceases when belief is attained.'* So, we might add, an act of judgment is excited by the discomfort of a question to which we do not at once foresee the answer, and it crystallizes into the form of a proposition only when some answer, however partial, has been definitely formulated. As Belief is to Doubt, so is the Answer which a judgment expresses to the Question out of which it arises.

Now, a question, if it is to be more than a vague unprogressive state of wonder or curiosity, is already, by the very fact of its being a question, partially determinate. It takes its start from a more or less vague conception, and the motive which prompts a more determinate answer is the pressure which the idea of the determinate necessarily exercises upon the undetermined whenever the logical interest is in any degree awake. This pressure exerts itself most naturally in the work of transforming the vague possibility of a solution which is implied in the existence of the question into a set of definite *alternative* possibilities. If I am told that a happy

* C. S. Peirce, 'Illustrations of the Logic of Science,' *Popular Science Monthly*, xii., p. 289.

event has raised me to the dignity of an uncle, but am not further informed, the knowledge I have is, in its indeterminateness, itself a question which tends to determine itself forthwith in the form of the disjunctive proposition: 'Either it is a nephew or it is a niece.'

A further stage of determination is reached when I make an assumption as to the sex, and proceed to define my mental attitude, prospectively, on that assumption. 'If it is a boy, my brother-in-law will perhaps be sorry; but my sister, I know, will be pleased. If it is a girl, there is no reasonable possibility of its being called after me.' And so forth. The second stage is thus that of the Hypothetical Proposition.

The third stage in the process is reached when the news arrives that my sister is pleased, and that the infant is to be called after me. The question 'What sex?' with which the whole thought-process started now receives the definite answer 'The child is a boy.' The disjunctive proposition passes into a *categorical* proposition which asserts the one alternative to the exclusion of the other.

It must not, however, be supposed that the Disjunctive Proposition, which is logically prior to the Hypothetical, is logically prior to the Categorical as well. It is certainly prior to the more determinate form of categorical which asserts one out of many alternative possibilities; but it is posterior to the more indefinite categorical out of which the question and the disjunction sprang. Thus, the proposition that the being is a human child is the necessary preface to the question 'What sex?' and to all that follows between the question and the determinate categorical answer, 'The child is a boy.'*

In many cases, indeed, a categorical statement possesses no indeterminateness at all, hence suggests no question, and therefore no disjunction as the first step towards a more determinate answer. Moreover, there is another class of cases in which we have a categorical proposition which, though determinate, cannot have been reached through the cancelling of alternatives in a disjunctive proposition. The statement 'The part is not greater than the whole' cannot have been preceded by a disjunctive proposition stating an alternative to the assertion in question. The proposition is self-evident, and in the case of all self-evident propositions the categorical statement cannot be considered in the light of an answer at all. Where there can be no doubt there can be no question, and therefore no answer.

* The Categorical Proposition is, in fact, implied all through. It mediates the transitions from the question to the disjunctive, and from the disjunctive to the hypothetical. Thus the statement 'Either it is a nephew or it is a niece' presupposes the categorical 'A baby is either a boy or a girl'; and the statement 'If it is a boy . . . my sister . . . will be pleased' presupposes the categorical 'My sister has a preference for boys.'

We conclude, then, that the logical order of the propositional forms is :

1. Categorical ;
2. Disjunctive ;
3. Hypothetical ;

and we propose to adopt this order in the following discussions on the nature of Proposition and Inference.

But before we proceed to the detailed discussion of the Categorical Proposition, it is essential that we should emphasize a distinction of fundamental importance arising out of the relation in which a question stands to the reality in reference to which the answer is sought.

The logical interest that inspires a question may be either occasional or systematic. It may be conventionally restricted within the limits of some definite topic, or it may aim at an answer that shall respect systematic connexion of facts apart from any such conventional restriction. In the former case the answer has a 'formal' value ; in the latter case it has a 'real' value. Where it has only a formal value, the judgment or proposition in which it is expressed may be designated as 'formal' ; where it has a real value, as 'real.'

Thus, if I inquire 'Is my friend in a good temper this morning ?' my question may express the merely occasional interest of ascertaining whether our meeting is likely to be pleasant or not. On the other hand, if I should chance to be a doctor, I may be scientifically interested in the effects on his nervous organization of some special condition such as neuralgia or a sleepless night. In the former case the question refers to formal reality, and expects a formal answer ; in the latter both question and answer have a real or systematic character.*

The distinction between a formal and a real logical interest may be aptly illustrated by the difference in meaning acquired by the Singular Proposition (the proposition which has for its subject a singular term) according as it is considered in the light of the one interest or of the other.

In the service of our unorganized everyday experience as well as in the service of Science the singular proposition plays an important part. In all our talk about individuals, be they persons, things or events, as well as in scientific observations and verifications, our direct, explicit reference is to the individual fact. In the case of Science, however, the sense-individuality of observed fact derives all its significance from the scientist's belief in natural law. A fact, for natural Science is a fact under law, and a species, instance, or example of a common nature. The reverence for fact which is so

* The reality of Nature, as interpreted by Science, is not the reality of Spiritual Experience. The same question might express a spiritual interest—*e.g.*, in my friend's power to be cheerful in difficult circumstances. The logical discussion of this more personal point of view belongs, however, to a Philosophical Logic.

characteristic of the scientific attitude is ultimately rooted in reverence for this common nature. This attitude towards the facts of observation gives to the singular proposition as understood by Science—*i.e.*, to the 'real' singular proposition—a peculiar import. The reference to individual fact is not to the individual *qua* individual, but to the individual as symbolic of a universal or common nature. In the case of the formal singular proposition, though this reference to a common nature must always be implied—the isolated individual being at bottom unthinkable—it is not implied in the same systematic sense. The fact is not conceived as a fact under law, but as having an individual importance relative to a certain topic. If I say 'This chair has only three legs,' my topic is the possible uses of a chair, and my meaning is that, whatever chair I may choose to sit on, I must avoid the chair in question.

It is convenient to distinguish from formal and real propositions alike the strictly *verbal* proposition. The Verbal Proposition states the meaning of a word or verbal sign *qua word.* 'Man means rational animal' and 'Man is a verbal symbol which stands for "rational animal"' illustrate the two forms in which a verbal proposition may be expressed. In either case the meaning we define is that of the word as such, and not that of the object signified by the word. Thus, the proposition which states a definition is not usually verbal, since what we wish to define is, as a rule, not the meaning of the word *qua* purposive combination of letters or sounds, but the meaning of that significate of which the word is but the verbal sign. The proposition 'Man is a rational animal' is a definition of the common nature conventionally symbolized by the sign 'Man,' and not a definition of the sign itself. It is therefore material (formal or real), and not verbal.

CHAPTER XII.

IV. (ii.) ANALYSIS OF THE CATEGORICAL PROPOSITION.

1. *Subject and Predicate in the Categorical Proposition.*

WE have defined—or, rather, distinguished—the Categorical Proposition as one which states something about something else, as a proposition, therefore, including a subject as well as a predicate term.*

* Whether these two terms stand for existents or not, and if so, in what sense, are further questions which concern what is called the Theory of the Existential Import of propositions. Prof. Keynes, in particular, has given an elaborate discussion of this problem in his 'Studies and Exercises in Formal Logic,' 4th edition, Part ii., ch. viii. But on this point see p. 189.

There are certain embryonic forms of the Categorical Judgment—viz., the Exclamatory and the Impersonal Judgments—in which the Subject-Predicate relation, though present, is still not obviously expressed. By Exclamatory Judgments we mean such ejaculations as 'Fire!' 'Man overboard!' where 'Fire!' is equivalent to 'There is a fire,' or 'A fire has broken out'; Man overboard!' to 'There is a man overboard,' or 'A man has fallen overboard.'

Impersonal Judgments are, from this point of view, abbreviations of a more perplexing kind. They are sometimes instanced as categorical propositions without a subject, but there seems to be no adequate reason for this view. In such cases as 'It is too bad,' 'It isn't fair,' the 'it' is quite properly regarded as an indeterminate subject-concept, which through its very indeterminateness truly represents some unnameable and unlabellable subject-thing, some total impression in its natural vagueness and indeterminacy. Such propositions as 'It snows,' 'It is foggy' hardly seem to be true impersonals. Grammatically, indeed, they have an indefinite 'provisional' subject (*cf.* Fr. *Il-y-a*); but as soon as they are reduced to the form of logical propositions—'Snow is falling,' 'Fog is prevailing'—we see that the subject-terms are really definite.

By the 'subject' of a categorical proposition we mean, of course, the '*logical* subject.' The question which term is subject and which predicate must be decided, not by grammatical structure or by the respective positions of the words, but by the meaning of the sentence. At the same time, the subject is usually the less emphatic, the predicate—that which is asserted—usually and naturally the more emphatic term. Hence in ordinary conversation we have the help of the speaker's voice and intonation; and in written work, where the right stress can be given through our knowledge of the meaning of the whole context in which the proposition occurs, we may frequently be helped out by emphasis. Thus, in reading the ancient proverb 'In the multitude of counsellors there is safety,'* we feel that the emphatic part of the proposition is 'in the multitude of counsellors.' 'Safety' is not what we are asserting about 'in the multitude of counsellors,' but 'in the multitude of counsellors' is what we are asserting about 'safety.'

But if the sentence occurs quite alone, and we have not the requisite data for unambiguously specifying the logical subject, we are reduced to stating the various alternatives which the logical subject may assume. Take, for instance, the isolated sentence:

'The Novum Organum of Bacon was not intended to supersede the Organon of Aristotle.'

* Proverbs xxiv. 6.

Here the meaning may be—

1. The *Novum Organum* of Bacon was not intended to supersede the Organon of Aristotle.
   I.e. (The work of Bacon which was intended to supersede the Organon of Aristotle)—Subject—
   is-not
   (his Novum Organum)—Predicate.
2. The Novum Organum of *Bacon* was not intended to supersede the Organon of Aristotle.
   I.e. (The Novum Organum that was intended to supersede the Organon of Aristotle)—Subject—
   is-not
   (Bacon's work of that name)—Predicate.
3. The Novum Organum of Bacon was *not* intended to supersede the Organon of Aristotle.
   I.e. (The statement that the Novum Organum of Bacon was intended to supersede the Organon of Aristotle)—Subject—
   is-not
   (true)—Predicate.
4. The Novum Organum of Bacon was not *intended* to supersede the Organon of Aristotle.
   I.e. (The superseding of Aristotle's Organon by Bacon's Novum Organum)—Subject—
   is-not
   (a result that was intended by Bacon)—Predicate.
5. The Novum Organum of Bacon was not intended to *supersede* the Organon of Aristotle.
   I.e. (The relation which Bacon intended his Novum Organum to bear to the Organon of Aristotle)—Subject—
   is-not
   (the relation of superseding)—Predicate.
6. The Novum Organum of Bacon was not intended to supersede the *Organon* of Aristotle.
   I.e. (The work of Aristotle which Bacon intended that his own Novum Organum should supersede)—Subject—
   is-not
   (the Organon)—Predicate.
7. The Novum Organum of Bacon was not intended to supersede the Organon of *Aristotle.*
   I.e. (The Organon which Bacon intended to supersede by his own Novum Organum)—Subject—
   is-not
   (the work of Aristotle)—Predicate.

The main conclusion to which we are driven by the foregoing analysis is that a proposition depends as much on context for its

true meaning as do the terms which are its elements. Everywhere we find the part incomplete and pointing beyond itself.

Now, it is of fundamental importance that we should not interpret this reference of part to whole in abstraction from the limiting or defining reference to purpose or interest; for, apart from this defining reference, we cannot hope to fulfil the logical requirement of relevance. A logical whole is objective reality as defined and, as it were, individualized through the selective agency of some specific interest. What Logic here demands as a requirement of right thinking is recognized in Art as a canon of right feeling. If, for instance, in looking at a picture we are to feel æsthetically, we must be able to feel the full appeal of the picture within the frame of the picture itself. As Professor Stout has somewhere said,* 'Whatever content enters into the work of art must be so connected with the whole as never to divert attention from the whole: picture or poem must be apprehended as a world in itself, the whole interest being gratified within it; hence to think of the real landscape whilst looking at its picture is to slip away from the artistic unity of the picture, and the enjoyment is no longer æsthetic.' So it is with a logical whole, the object that can *satisfy* a given logical interest. Its natural framework is defined by the limitations of the interest. What in respect of that interest is extra-marginal is logically irrelevant: it cannot enter into the whole within which the definite interest finds its definite satisfaction.

It follows that, when we say of a proposition that its true, or logically ultimate, meaning is given to it by its logical context, we understand by 'logical context' that limited topic or 'universe of discourse' within which the interest which inspires the statement in question lives and moves and has its being.

The relation in which the *subject* of a proposition stands to the relevant universe of discourse may, perhaps, be made clearer by the help of an illustration. Let the subject (or 'universe') of discourse (Σ) be the wanderings of Odysseus, and the proposition in question 'Ulysses (S) bends the bow that no other could bend.' *Here the true and logically ultimate subject of the sentence is S as interpreted in the light of* Σ. It is not Σ itself, but S as qualified by Σ. We are speaking of Ulysses, the hero of a hundred adventures already detailed, but now in Ithaca once again, and just about to reassert himself as lord of his own house and country. The true and ultimate subject of the proposition is therefore 'Ulysses'—Ulysses as we have come to know him through the story of his past adventures, not the bare 'Ulysses' severed from all reference to a past which alone gives to the present action, the bending of the bow, its critical significance. As for the predication 'bends the bow that no other could bend,' its relation to the subject 'Ulysses' may be defined by

* I quote from notes, but the statement substantially reproduces Professor Stout's point.

saying that it serves to develop, through its additional item of information, the meaning which the subject has already acquired through the previous development of the topic.

The universe or subject of discourse implies, as we have seen, a reference to reality, of which the character will vary according as the judgment is *formal* or *real.* Where the judgment is formal, the subject of discourse is conceived in isolation from the objective systems of Nature and History.* Thus the wanderings of Odysseus constitute, in the connexion already cited, a *formal* subject of discourse. There the bending of the bow is considered as an incident in the old Homeric story, which is itself uncritically accepted. We have in mind the Odysseus of the legend, and are quite unconcerned with what historical science may have to say about his existence or his wanderings. We say 'Ulysses bends the bow that no other could bend' under the implicit reservation that the relevant context and topic of interest is the legendary and not the historical context. On the other hand, the Σ, as the ultimate context of a *real* proposition, is Nature as interpretable by that department of Science within which the assertion falls. A real proposition is synonymous with a scientific proposition—a proposition stated, as it were, in the hearing of Nature, and open at every point to Nature's own correction.

This, of course, does not imply that the real or scientific proposition is a statement unlimited by any reference to purpose or direction of interest. A scientific proposition about a plant may be stated from the mere chemist's point of view, and imply no vitality whatsoever in the organism which it considers. Thus limited, the statement would be abstract enough, and the more faithfully it fulfilled the logical requisite of relevancy to purpose, the more abstract would it be. At the same time, it would be none the less *real* as measured by its significance for a scientific reconstruction of Nature. Reference to purpose, as we have seen (*vide* p. 5), is as essential to the *real* proposition as reference to reality is to the *formal* proposition.

Hence, when Dr. Bosanquet seeks to give a deepened significance to the meaning of Judgment by pressing 'subject of discourse' back upon 'reality as a whole' or 'the real world as a whole,' and the real world as a systematic whole is taken to be the ultimate or absolute Subject, the same for every judgment, there is involved in these interpretations a clear disregard of reference to purpose; and this essentially differentiates his theory of propositional import from that which is here defended. Students interested in the question will find Dr. Bosanquet's view developed in a masterly way in the first three lectures of the 'Essentials of Logic.' We content ourselves here with the remark that, from the standpoint

* By History I here understand the ordered connexion of events in time and space, as treated by methods of scientific criticism.

adopted in the present work, reference to context, whether that context be formal or real, is always conceived as limited by an involved reference to *purpose* or *interest;* and that we take considerations of relevancy to be logically ultimate.

The relation of Predicate to Subject within the unity of the Judgment suggests, on closer view, a distinction which, though based on a confusion of thought, is sufficiently widespread and popular to require special notice.

When the information supplied by the predicate is compared with the definition of the subject-term, the result—so the ordinary statement runs—will show either that the predicate gives no information not supplied by that definition, or that it takes us beyond the content of the definition. In the former case the judgment is said to be *analytic* or *explicative;* in the second case *synthetic* or *ampliative.* The words 'analytic' and 'synthetic' are usually adopted to express the distinction in question; but Kant, who is responsible for giving to the distinction between Analytic and Synthetic Judgment its great historical importance, himself suggested as alternative expressions the words 'explicative' and 'augmentative or ampliative.'

An Explicative Judgment ('analytic,' in Kant's sense of the word) is, then, a judgment which, on reference to the definition of the subject-term, turns out to be a mere verbal truism, telling us nothing about the subject that was not already given in its definition. Thus, if 'man' be defined as a 'rational animal,' it is a mere verbal truism or tautology to assert that 'All men are rational,' for no information is thereby given to anyone who knows the meaning of the terms which are being used.

An Ampliative Judgment ('synthetic', in Kant's sense) is a judgment which predicates of the subject something which is not already stated in the definition of the subject-term. Thus, if 'man' is defined as a rational animal, it is an amplification of this original meaning to say that man has adopted the habit of wearing clothes. So, again, if we take the proposition 'Man is man' (*i.e.*, Man is truly human in the sense of being master of his fate), we see that the predicate amplifies the meaning of the subject-term. The meaning of the statement seems to be this: 'Man, a natural being, is also a free agent.' The proposition, through its subject-term, introduces man as a natural being, and the predicate informs us that this 'natural' being is also 'spiritual.'

This distinction between explicative and ampliative judgments is often stated in a more precise form by reference to the system of predicables. It is argued that if, in the judgment 'S is P,' P is the genus to which S belongs or the differentia of S, or at once both the genus and the differentia, it simply repeats, in whole or in part, the meaning which the species S bears in virtue of its definition,

and thus the judgment is explicative. But if P gives a property or 'accident' of S, the judgment is ampliative; for these predicables can be assigned only after the meaning has been unambiguously fixed through definition. The statement that an equilateral triangle is equiangular—an equilateral triangle being defined as a plane rectilinear figure having three equal sides—is neither a truism nor a tautology.

Having stated this familiar distinction in its usual form, we now proceed to consider the question of its logical value.

As we have already pointed out, the distinction is illogical. It is, indeed, open to a fatal objection which deprives it of all value. We state this objection when we maintain that the explicative proposition, as above defined, is no proposition at all. If, in the statement 'Man is a rational animal,' 'man' is already understood to mean 'rational animal,' we have fallen back upon the tautology 'Rational animal is rational animal,' which, as we saw, takes us no further than the bare concept 'rational animal,' and is therefore no statement at all. If the principle of Identity-in-Difference is to rule the logical Proposition, the predicate must not repeat the subject in whole or in part. Otherwise we have a *circulus in proponendo*. Every proposition is therefore essentially ampliative. If I define 'man' through the statement 'Man is a rational animal,' the meaning of 'man' which I undertake to define is the as yet unspecified common nature which, when analysed and reconstructed, is specified as 'rational animal.' A proposition which states a definition involves as genuine a development of meaning as does any other proposition, the meaning growing in clearness as we pass from the as yet indeterminate meaning of the subject-term to the determinate meaning of the predicate which supplies the defining marks. The term to be defined, the subject-term or definiendum, is a term which threatens to give ambiguity, and calls for the remedy of definition: it therefore cannot be identical in meaning with the predicate which supplies the definition.

Similarly, to predicate genus alone, or differentia alone, of an as yet undefined concept is in no sense to repeat a part of that concept. The subject-term is here a relatively indeterminate meaning which gains through the predication of genus or of differentia some further though partial development. We conclude, then, that whatever predicable be predicated of the undefined subject-term, the predication is ampliative. When once the meaning of the term is fixed through definition, the term, *qua* defined, may, of course, become the subject of further predication. Hence, when we predicate propria of a given term, we are amplifying that meaning of the term which it has secured through its definition. But until the definition of a term is fixed, the term, *qua definiendum*, cannot be treated as already defined.

With the recognition that the Proposition is intrinsically amplia-

tive in character, the distinction between ampliative and explicative —and with it the synonymous distinction between analytic and synthetic propositions—is necessarily abandoned.

The distinction between Analytic and Synthetic has been re-interpreted by some logicians (*e.g.*, Mr. Joseph) so as to correspond to the distinction between the differentiation of S from P and the identification of S and P in the Judgment considered as an Identity-in-Difference. 'Every judgement,' writes Mr. Joseph, 'is at once analytic and synthetic; for the act of judgement at once holds different elements apart and recognizes them as elements in a single whole. As held apart, it requires an act of synthesis to see that they make one whole: as recognized to make one whole, it requires an act of analysis to find and hold them apart.'*

If we do not endorse this application of the words 'analytic' and 'synthetic,' it is partly because we require the word 'synthetic' for another office in connexion with the distinction between Affirmation and Negation, and partly because the function of holding S apart from P, so that their identity does not degenerate into a fusion of one with the other, appears to be more specifically and accurately referred to as a 'differencing' or 'differentiation' than as an analysis. We reserve the use of the term 'synthesis,' *as contradistinguished from 'analysis,'* for the purposes of Explanation and Inductive Method.†

2. *The Quality of a Categorical Proposition.*

The psychological analysis of a *complete* act of judgment, as we understand it, shows (1) that it is an assertion or statement of intended meaning, whether affirmative or negative; (2) that, *qua* assertion, it is an identity-in-difference developing its meaning according to the principle of logical Identity; (3) that it involves an accompanying attitude of belief.

Some psychologists make no distinction between Assertion and Belief. But the distinction is surely necessary. I may assert a statement as stating what I mean to say, and not at all what I believe to be true or hold to be false. Such a proposition as 'All donkeys are daffodils' may be taken in this sense as a mere statement of meaning.‡ Moreover, in so far as we are studying proposi-

* 'An Introduction to Logic,' ch. viii., p. 187. † *Vide* Chapter XXXIX.

‡ It has been urged that propositions of this kind are meaningless formulas, and therefore no propositions at all. But is this really the case? A *reductio ad absurdum* might very well conclude with the statement in question, whereas, if this statement were in reality no statement at all, the *reductio ad absurdum* would at once be vitiated. Further, the writer of a fairy-tale, or a defender of transmigration, or a believer in the absolute identity of all being might very well maintain that donkeys were daffodils *in posse*, or that, in so far as donkeys had any reality at all, they were essentially identical with daffodils, and that the apparent difference between the two species was a mere illusion. The proposition, in fact, is not necessarily self-contradictory, and only self-contradictory statements are meaningless.

tions in themselves, their logical relations with other propositions, or their logical implications, we cease to interest ourselves in their reference to reality or their significance for knowledge. The question whether we believe or disbelieve them is irrelevant to the purpose we have in considering them. Judgments, when thus abstractly studied, are asserted without being either believed or disbelieved.

Some difficulty may, perhaps, be felt in accepting Affirmation and Negation as co-ordinate forms of Assertion. A consideration of the parallel relation between belief and disbelief may help to remove it. It is a psychological commonplace that the true opposite of belief is not disbelief, but doubt. Disbelief is a form of belief; or, at any rate, belief and disbelief alike are forms of conviction. Again, the real opposite of a given extreme is not the counter-extreme, but the mean between them. If two men are fighting, they are, *qua* fighters, akin, and their common foil is the man who stands by and watches. These illustrations suggest a corresponding relation between Affirmation and Negation. They suggest that, just as belief and disbelief agree in being forms of conviction, so Affirmation and Negation agree in being forms of Assertion; and, further, that just as belief and disbelief agree in a common opposition to doubt, so the true opposite of Affirmation and Negation alike is a suspense of judgment. If there is still a difficulty felt in apprehending the view that Affirmation and Negation alike are forms of Assertion, or if it be set down as a scholastic subtlety, the reason is probably this: that there is no word in ordinary use to set off against the term ‘ synthesis ’ (with its distinctively affirmative implication) to represent the operation involved in denial. But the term ‘ dialysis,’ which means a ‘ sundering ’ or ‘ separation,’ and is a respectable dictionary word, might very well be adopted, and we should then be left free to define a negation or denial as the assertion of a dialysis, and an affirmation or ‘ position ’ as the assertion of a synthesis.*

Affirmation and Negation, then, are differentiations of the more fundamental activity of Assertion. All Judgment or Proposition is Assertion, or statement of intended meaning, and the quality-difference between Affirmation and Negation is not ultimate: they are differentiations of a common nature, the act of Assertion. Whether affirmative or negative in *quality*, propositions are assertive in *character*. Hence, though a negation must be an assertion, it cannot, *qua* negation, involve any affirmative synthesis. Pure negation is denial pure from all affirmative or synthetic intention.

* The restriction here put upon the use of the term ‘ synthesis ’ should be noted. The term is frequently used to signify that application of the principle of logical Identity which first constitutes a judgment as such. According to this use (which is that of Mr. Joseph, referred to above) every judgment is synthetic in virtue of its expressing an *identity* in difference. It seems better, however, to reserve the term ‘ identity ’ to denote this fundamental relation, and to use the term ‘ synthesis ’ in that restricted sense in which it is opposed to ‘ dialysis.’

Our subordination of the distinction between affirmative and negative quality to the common assertive character proper to all propositions does not in any way affect its genuineness. This genuineness of the qualitative distinction has, however, been disputed. Some logicians have attempted to state all propositions in affirmative form, insisting that 'S is-not P,' the typical form of the negative proposition, is logically equivalent to 'S is not-P.' The attempt, however, ignores the primary significance of negation in the act of judgment. We often require to negate simply, to give to a statement a bare, blank denial, without any positive implication in the background. If we say that a thing is not white, we merely mean that the term 'white' is not applicable to it. We just contradict quite barely the statement that the thing in question is white. We destroy the predication 'white' without making the slightest mental effort to replace it by another. Genuine logical denial can, in fact, do nothing more than deny: it fulfils itself in negating. We conclude, then, that 'S is not-P,' the meaning of which we have discussed in connexion with our treatment of negative terms, is not synonymous with 'S is-not P,' and that the two can be equated only by weakening the natural distinction between the affirmative and the negative.

3. *The Copula.*

The Copula may be briefly defined as the identity-principle* as operative within a categorical judgment. This principle is the reason itself as inspired by a logical ideal. Present in subject and predicate alike, it claims both, in their interrelatedness, as specifications of its own meaning. It is in virtue of the unity and continuity of the logical interest that our meaning developes all of a piece. Oneness and identity of logical interest means oneness and identity of the judgment, or the system of judgments, through which that interest expresses itself. We have here to do with the very nature of the logical interest itself, the fundamental and authoritative factor in all logical inquiry, and we cannot get behind it. What makes an identity of an affirmation (or a denial) is that a reasoned interest specifies, and to that extent fulfils, itself in the assertion of it. The general form in which such an interest fulfils itself is in specifying the meaning of a subject through a predicate. The copula is, strictly speaking, the judgment-activity itself in process of self-fulfilment in the form of a judgment: this active logical interest is, in fact, constitutive of the identity-in-difference which pervades and characterizes the Judgment.

* The copula should not be confused with the copula-*mark*. The copula-mark takes two typical forms: 'is' when the assertion is affirmative in character, 'is-not' when the assertion is negative. 'Is' may conveniently be called the synthetic, 'is-not' the dialytic copula-mark.

The Copula, then, is omnipresent in the Judgment. It cannot be identified with any part or aspect of it, for in that case whatever lay outside could have no part or lot in the judgment. An element that had no share in that which unifies a proposition could not be an element *of* that proposition. This point is important, and we proceed to amplify and to emphasize it.

The copula is frequently called a coupling-link between the subject and the predicate. This metaphor is misleading if it be interpreted as meaning that the terms *of a judgment* can be given independently of the relation between them. This is, in the nature of things, impossible. For the words or concepts, set down independently of the relation between them, are not 'terms in the judgment,' but terms out of the judgment. The act of synthesis or dialysis which defines the relation between S and P first brings S and P into the judgment. They are not there prior to the relating act. But a coupling-link cannot be considered as first constituting railway-carriages into railway-carriages. These remain the same before and after the coupling. It is true that the car first becomes part of a *train* by being thus coupled with another ; and if this is insisted on, the coupling metaphor might serve the purpose, though still rather lamely. The essential point to recognize, however, is that the term *of a proposition* exists only in the proposition itself as organically one with it, so that the Copula, as relating activity, cannot be identified with any single partial element in the Judgment—with a relation, for instance, that is outside the terms. It must be the activity which brings terms and relation, content and form, not *together*—for this implies a previous separate existence in mutual isolation—but *into-birth-together*.

We may illustrate this important point by what is really much more than a mere analogy. It is sometimes stated that a poet works *upon* a certain content, moulding it into poetic form ; and we are left with the impression that what is intrinsically poetic is the *form*. But the truth is that the poet works upon a certain subject-matter, which, as such, is certainly not the content of his poem. In bringing poetic unity into this subject-matter, he brings into birth, in intimate unison, content and form together. With the content comes the form, and with the form the content. The content is the subject-matter poetically *formed ;* the form is the form *of* the content. The poetic end is not to superinduce form upon content, but to transform subject-matter into a formed content.*

So with the Categorical Judgment. The matter of the judgment, that about which we judge, exists prior to the judgment. But the act of judgment consists not in superinducing a relation upon given terms, but in transforming the given matter, through the selective and unifying agency of a dominating interest, into terms-in-relation

* *Cf.* Professor A. C. Bradley, 'Poetry for Poetry's Sake.'

(the relation of subject to predicate), or into that formed content which we call a proposition.

The systematic intimacy of subject and predicate within a proposition is customarily indicated through the use of *copula-marks*—of the expressions 'is' and 'is-not.' The function of the word 'is' in the judgment 'S is P' is not that of serving as a coupling-link between subject and predicate, but that of indicating the identity-relation between the two. It tells us that the term 'P' which it precedes and introduces must be used as predicate—*i.e.*, as the predicate of the subject-term S.*

The foregoing discussion will serve to meet Dr. Sigwart's objection against a 'negative copula.' A copula, he argues, has by its very nature a synthetic function. Hence a copula that divides is a self-contradictory absurdity. 'There is no such thing,' he writes, 'as a negative, but only a negated copula.'† If the function of the copula were 'synthetic' in that sense of the word in which it is opposed to 'dialytic,' this argument would be pertinent enough. But, as we interpret the copula, its function is to operate as the identity which the assertion expresses, whether that assertion take the form of a synthesis or that of a dialysis. Concepts do not cease to be related to each other because the relation between them happens to be an opposition or a severance. We show our dependence upon society most energetically, it has been said, when we assert our independence against it. The ascetic who renounces the world has been made an ascetic by the very evil which he shuns. The young lecturer who lectures in direct opposition to the tenets he has imbibed from his late University teachers thereby proclaims the potent effect which the lectures have produced upon his mind.

The assertion of a relation between‡ S and P which the copula-mark expresses is independent of the nature of that relation. Hence, if by 'negative' copula we mean the dialytic relation between S and P, the expression is perfectly reasonable. The 'negated copula,' on the other hand, as we understand the term 'copula,' is a logical fiction. It means nothing. It indicates an operation that cannot be carried out. We cannot deny an assertion without ourselves asserting. For denial is, as we have seen, a form of assertion. Hence the negated copula is itself a copula—a self-contradictory conclusion which we could never have reached had not the original conception itself been self-contradictory and therefore meaningless.

* *Cf.* Dr. Christoph Sigwart, 'Logic,' vol. i., ch. ii., § 17, English translation by Helen Dendy (Mrs. Bosanquet), p. 94.

† *Ibid.*, ch. iv., § 20, Eng. tr., p. 122.

‡ The word 'between' must not be understood in the coupling-link sense. What the copula-mark expresses is the S and P relation, a relation which cannot be understood as distinct from the terms related.

CHAPTER XIII.

IV. (iii.) THE MEANING OF POSSIBILITY.

Real Possibility.

THE ultimate source of the idea of *possibility* is to be found in the 'I can' of the free agent—the 'I can' itself implying the 'I ought.' As personalities, we have ideals which we ought to cherish, duties which we ought to perform, spiritual imperatives which we ought to obey; and, as personalities, it is also possible for us to fulfil the essential obligations of our free-born nature. It is as an agent capable of free choice that a man is justified in saying 'I can be patient (or sincere),' 'I can obey my conscience (or my reason).'

Similarly, the ultimate source of the idea of *necessity* is to be found in the 'I cannot' of the free agent, in the compulsion which limits his freedom. 'I must' because 'I cannot help it.' It is true that we frequently use the phrase 'I must' in a more positive sense. But such usage is misleading. It is misleading to say 'I must' when what we really mean to say is 'I ought to,' or, more decisively, 'I ought to, and I will.' 'I must obey my conscience,' we say, whereas what we really mean to say is, 'I ought to obey my conscience.' So, when Regulus insists that he must return to Carthage, his true meaning is given in the words, 'I ought to return to Carthage, and I will.'

There are, however, many derivative uses of the words 'can' and 'must.' Thus, restricting our attention to the former word and its uses, we may allow that there are many aspects under which even that which is not free admits of being treated as free; and consequently a corresponding set of senses in which the word 'possible' can be used. We may conceive the universe, for instance, as containing the whole ground of everything which the future will bring forth; and we should then say: 'The universe can be what in the fullness of time it will be.' So, again, we may appropriately say, 'Water can freeze and evaporate,' for we here mentally isolate the idea of water, and, abstracting it from the conditions of actual existence, consider its changes of composition as though they emanated from itself alone, as the changeless, persistent ground of all these various transformations. Thus it comes to pass that we say 'Water can freeze and evaporate,' much as we should say 'A cat can mew and purr.' Again, the word 'can' may suitably be used to denote the power of further determination that a general idea possesses. Thus we say 'A triangle can be obtuse-angled; it can also be right-angled;' or, again, 'A horse can be black; it can also be grey;' and so forth.

The Scheme of Opposition* proper to these relations of intrinsic necessity or possibility may be set down as follows :

Assertion of Intrinsic Necessity : S must† be P.
(*E.g.*, 'A proposition must be an identity-in-difference'; *i.e.*, the nature of a proposition is such that it must be an identity-in-difference.)
Contradictory : S need not be P.
(*E.g.*, 'A proposition need not be affirmative.')
Contrary : S cannot be P.
(*E.g.*, 'A proposition cannot be self-contradictory.')
Subalternate (*Contradictory of Contrary*) *:* S can be P.
(*E.g.*, 'A proposition can be indesignate.')

Where the necessity—or possibility—is explicitly teleological, bearing on the relation between means and end, the Scheme of Opposition needs a certain readjustment, giving what we may call the Teleological Scheme :

Assertion of Teleological Necessity : If X is accepted as end, then Q must‡ be accepted as means; or : Teleologically, S must be P.
Contradictory : If X is accepted as end, then S need not be P.
Contrary : If X is accepted as end, then S must not be P.
Subalternate : If X is accepted as end, then S may§ be P.

Example of Teleological Opposition.
Assertion of Teleological necessity : If a man's aim is to keep well, he must take regular exercise.
Contradictory : If a man's aim is to keep well, he need not take regular exercise (*i.e.*, the failure to take regular exercise will not be fatal to the attaining of his end).
Contrary : If a man's aim is to keep well, he must not take regular exercise.
Subalternate : If a man's aim is to keep well, he may§ take regular exercise.
(*I.e.*, taking regular exercise will not be fatal to the attainment of his end.)

The various uses of 'possibility' we have so far discussed have pointed to a positive capacity in the subject considered, whether that subject be a personal agent, a spatial object, a proposition, means to an end, or what not. Thus, when I say 'This acorn can

* This section presupposes an acquaintance with ch. xix.
† Note that 'must' is here not exclusive of 'can.' The necessary is also the possible.
‡ Here, again, 'must' is not exclusive of 'may.'
§ The 'may' is here permissive, and by no means implies limitation of knowledge (*vide infra*).

become an oak,' or 'Water can freeze and evaporate,' or 'The aspirant after health may forgo regular exercise,' I am abstractly considering the changes which it lies in the nature of these subjects to undergo.

Possibilities of this kind are potentialities, capacities, real dispositions, and are frequently referred to as *real possibilities*. They presuppose, in each case, some positive nature of which they are the capacities or dispositions. We might refer to them as DISPOSITIONAL POSSIBILITIES, and in designating them we would propose, as far as possible, to adopt the word 'can.' To say that 'S can be P' is to say that S is potentially or dispositionally P. It is only in the case of teleological connexions that it seems necessary to substitute 'may' for 'can' in order to express this dispositional quality. But the teleological 'may,' as in the instance already cited, is closely allied to the potential 'can.' It presupposes, in particular, a positive nature (the agent inspired by an end or motive), to which it adds a qualification in the form of a permissive and purposive possibility.

Modal Possibility.

The distinctive use for which we reserve the term 'may' is that of expressing MODAL POSSIBILITY.

A *modal* possibility is a possibility of which the problematic character expresses, not a potentiality on the part of the object concerning which some predication is made, but imperfection of knowledge on the part of the predicating subject. The 'may' which expresses modal possibility is known as the PROBLEMATIC 'may.'

Let us consider the case of a particular object whose development from within is circumscribed by its environment. Here, in so far as the object contains in itself only the partial ground of its future evolution, the possibility of its reaching certain subsequent states is not only a question of time, but also of external influences beyond our power to determine or foresee. The plant-embryo in the seed *may* become a full-grown tree; the child *may* become a man. When the nature of the environment is thoroughly understood, as also the connexion between it and the object, the possibility gains a real, objective value, and is no longer modal. In that case (as when we say, 'The moon can be eclipsed'), we no longer use the problematic 'may.'

Consider, again, the specific statement 'It may freeze to-night.' In making this statement, I ground my judgment on a very partial knowledge of the conditions which occasion frost. The consequence is that I can draw no decisive conclusion, and must content myself with predicating possibility. But this possibility which I predicate is simply a confession of ignorance—*i.e.*, of inadequate knowledge.

My judgment, while it appears to be busy with the frost, treats really of my own scant knowledge of the frost-bearing conditions. Should it happen that I am conscious of none of the conditions on whose agency the event depends, 'the judgment,' as Dr. Sigwart says, 'passes into the subjective possibility of conjecture, and thus into the expression of uncertainty.'* It is in this purely subjective sphere that possibility becomes synonymous with mere conceivability or 'absence of contradiction.'

The Modal Square of Opposition.†

The attitude which contradictorily opposes the suggestion of a limitation of one's knowledge on a certain point may be said to find suitable expression in the proposition 'S is assuredly P.' The contradictory of this is 'S is-not assuredly P'; the contrary, 'S is assuredly not P'; and the subaltern (or the contradictory of the contrary) 'S is-not assuredly not P.' If we construe these propositions into equivalents expressive of the subjective attitude toward the evidence which we may conceivably adopt, we obtain the following Modal Square :

M_a : The evidence *for* the statement S is P is decisive.
M_o : The evidence *for* the statement S is P is not decisive.
M_e : The evidence *against* the statement S is P is decisive.
M_i : The evidence *against* the statement S is P is not decisive.
E.g., The evidence in support of a whale being a fish is decisive.
The evidence in support of a whale being a fish is not decisive.
The evidence against a whale being a fish is decisive.
The evidence against a whale being a fish is not decisive.

[*The So-called Inferential Possibility.*

Where, as in Formal Inference, the reference to knowledge is excluded, and with it the modal use of 'may,' the idea of a possibility ceases to have meaning. If by a logical possibility we mean an 'inferential' possibility, there can be no such thing as a logical possibility. All Inference is necessary. If the premiss is 'All S is P,' then all we can inferentially say concerning the proposition 'All P is S' is that our accepted premiss affords no ground either for accepting it or for rejecting it. But when there is no ground for inference, we cannot have an *inferential* possibility.

* 'Logic,' vol. i., English translation, p. 204. The analysis given above of the uses of the term 'possibility' owes much to Dr. Sigwart's treatment.

† This section presupposes an acquaintance with Chapter XIX.

CHAPTER XIV.

IV. (iv.) THE DISJUNCTIVE PROPOSITION.

THE essential function of the Disjunctive Proposition is to develop a given categorical basis, of a more or less general and indeterminate character, by specifying the alternative possibilities* which the predicate of the given categorical presents. Thus, the Disjunctive Proposition comes to the aid of the Categorical 'All plane rectilinear triangles are three-angled plane rectilinear figures' by adding that every three-angled plane rectilinear figure has either one obtuse angle, or one right angle, or three acute angles. The relation in which the Disjunctive Proposition stands to the Categorical is analogous to that in which Division stands to Definition (*vide* p. 41). In so far as weins ist on the mutual exclusiveness of the alternatives, the correspondence is complete.

'The disjunctive expresses the relation of possibilities to their categorical basis—ultimately to the nature of actual existence—which determines them and limits their range. There is no actual existence which has not a common nature which it shares with other actual existences, and which constitutes it a member of a class—an instance or example. But the common nature, when scrutinized, is seen to be capable of certain alternative determinations, and not of others. To enumerate a series of such determinations which omits no possible alternative is to state a disjunctive proposition. . . . Among the specific determinations which a common nature admits of, there may be many which are not actualized, and perhaps never have been nor will be actualized. Nevertheless, they possess the kind of being which we call possibility. The common nature which we are dealing with is intrinsically capable of them. Thus, a fluid is either "partially viscous or a perfect fluid." A colour is either red, green, blue, yellow, white, black, or of some intermediate quality—*e.g.*, blue-green, blue-yellow, etc. Here blue-yellow is not, as a matter of fact, an actual colour; and the actual physiological conditions of human vision seem to exclude its occurrence. None the less, we cannot say that a "blue-yellow" is intrinsically impossible, any more than a "blue-green." If we consider only the intrinsic nature of colour-mixture and of blue and of yellow, there is nothing to exclude the possibility of a blue-yellow. On the other hand, a "four-sided triangle" is an impossibility. It is not a specific determination of "triangle" as such' (Professor Stout).

The essential requisite of a disjunctive proposition, if it is to have any logical value, is that the alternatives which it enumerates shall

* The possibilities with which the disjunctive proposition is concerned are real. dispositional possibilities. They are not modal possibilities.

be *exhaustive*. If I say that men are either white-skinned or black-skinned, the statement is positively misleading, as the assumption is always tacitly made that the alternatives of a disjunction are exhaustive, and the conclusion would naturally be drawn that if a man's skin is not white, it must be black. But if I say that men are white-skinned or black-skinned, or have their skins otherwise coloured, the statement is sound enough, and is in principle exhaustive. We learn that white and black are two definite possibilities of skin-colour in the case of human beings, and the disjunction exhausts the possibilities.

For brevity's sake, we have in the foregoing assumed that the typical disjunctive proposition takes the form 'A is either B or C.' This, however, is not the case. No doubt the *meaning* of this formula is genuinely disjunctive, and the proposition may be expressed in the genuine disjunctive form 'Either A is B or A is C'; but the form as it stands is categorical, A being the subject-term, and 'either B or C' the predicate-term. No proposition is truly disjunctive that cannot be appropriately expressed in the form 'Either P or Q,' where P and Q are statements. Thus, the proposition 'All A's are either B or C' is disjunctive only because it may be read in the sense 'Either an A is B or it is C.' The proposition 'No A is either B or C' is equivalent to 'A is neither B nor C'—*i.e.*, to 'A is not B and A is not C'—a compound categorical proposition. With regard to the particular propositions 'Some A's are either B or C' and 'Some A's are neither B nor C,' it seems difficult to consider them in any other light than that of categoricals with 'either B or C' and 'neither B nor C' as their respective predicate-terms.

Of the many problems connected with the import of the Disjunctive Proposition none has excited so much interest as the question whether the alternatives of a disjunction should be treated as mutually exclusive or not. Does the expression 'Either . . . or' include or exclude the possibility of both alternatives being true? When we say 'Either this man is a fool or he is a knave,' do we imply that he may be both, or that he must be one and not the other? In a word, is 'either . . . or' to imply 'not both' or 'it may be both'?*

Before we pledge ourselves to either of these uses to the exclusion of the other, it would be well to ask whether the option is a forced one. Is it necessary, is it even relevant, to decide either in one way or the other?

* The distinction between these two alternative uses may perhaps be expressed more clearly as that between 'assuredly (or knowedly) not both' and 'not assuredly both.' The truth-view of import is here presupposed. From the standpoint of the mere statement-view of import the distinction would be best expressed as that between 'statedly not both' and 'not statedly both' (*cf.* pp. 148, 157). This interpretation of 'Either . . . or' is exactly parallel to the interpretation of 'some' adopted in the case of the particular categorical. In the non-exclusive disjunctive 'Either P or Q,' '*Both P and Q*' is *an unstated possibility* (*vide* pp. 156, 157).

This query does not imply any dislike of giving a fixed meaning to the expression 'either . . . or.' In the interests of the Principle of Non-Ambiguity, it is essential that we should know what we mean when we use the expression, *so far as such knowledge is relevant* to the logical purpose we have in using the disjunctive form. But, in ordinary cases, the only meaning which it is relevant to give to 'either . . . or' is that of 'not neither.' This meaning satisfies the requirement of exhaustiveness, and it enables us to waive altogether the issue as to whether 'either . . . or' is to imply 'not both' or 'it may be both.'

Let us consider the types of inference that we are naturally inclined to draw from the disjunctive proposition 'Either P or Q.' They are two in number: (1) The hypothetical proposition—'If not P, then Q'; and (2) the hypothetical proposition—'If P, then not Q.' The first of these is the inference from the exhaustiveness of the disjunction; the second is the inference from the mutual exclusiveness of the alternatives. If there are more than two alternatives, as in the proposition 'Either P or Q or R,' the two inferences will take the forms: (1) If not P, then either Q or R; and (2) if P, then neither Q nor R.

Now, in so far as it is the first of these two inferences that alone concerns our logical purpose in the framing of a disjunction, the question as to whether we are to side with the exclusivists or the non-exclusivists is wholly irrelevant. We are 'exhaustivists'—that is enough. 'Either . . . or' means for us 'not neither,' and nothing further.

In an able article in *Mind*, October, 1903, Mr. G. R. T. Ross has endeavoured to show 'that the function of the disjunctive judgment both in science and in practical reasonings is to be exhaustive and not necessarily exclusive.' The contention appears, as a *primary* contention, at any rate, to be essentially sound. When we develop an indeterminate categorical basis disjunctively, our primary aim is attained if we can substitute for the comparatively indeterminate categorical statement a list of specific possibilities which collectively cover precisely the same range as the vaguer generality which they disjunctively specify. For, since the same range is covered in both cases, the choice between a number of specific possibilities is a fresh advantage that the disjunctive offers over and above those offered by the indeterminate categorical. This particularization without loss of scope is in itself something to the good. But once we have the specific possibilities *exhaustively* before us, the question as to whether they mutually exclude each other may remain a matter of complete indifference to us.

Let us suppose that the assertion that X is an objectionable person is adequately represented by the disjunctive statement that 'Either X is a knave or he is a fool.' Without inquiring whether X's knavery is or is not compatible with his foolishness, we can infer at

once, from this statement, that if X is not a knave, he must be a fool, and that if he is not a fool, he must be a knave, and thereby emphasize in a fresh and specific way our original contention that X is an objectionable character. The inference that 'If X is a knave, he is not a fool' would be beside the point. It would not emphasize his objectionableness, but qualify and limit it.

Suppose, again, that X is in a perilous situation at the edge of a chasm, and that the categorical requirement of immediate action is presented in the disjunctive form: 'Either jump or starve.' Here the essential inference is that 'If X does not jump, he will starve,' for it is this consideration that supplies the incentive to action. Little can be gained by inferring that 'If X does jump, he will not starve,' and that 'If X starves, he will not have jumped.'

I quote from the article by Mr. Ross the following illustrations in further support of the contention that, under ordinary circumstances, an exhaustivist need not pledge himself either to the exclusivist or to the non-exclusivist theory.

1. 'Planetary orbits* fall either wholly inside or wholly outside the Earth's orbit.'

Here the disjunction admits of being read exclusively. We can infer, if we like, that 'Jupiter's orbit, lying without that of the Earth, cannot lie wholly nearer to the sun than it.' But this is futile. The real force of the disjunction lies, as Mr. Ross points out, in its exhaustiveness, in the implied denial that there are any planets with orbits intersecting that of the Earth. It lies in the possibility of the inference that if a planetary orbit does not fall wholly inside the Earth's orbit, then it must lie wholly outside it.

2. 'Planets whose orbits lie between the Earth and the Sun are, when visible, to be seen either in the morning or in the evening.'

Here we have a statement which we may very profitably utilize without possessing any proof that Venus, for instance, when visible in the evenings, must in the morning rise after the Sun, and so be lost in his light. The force of the disjunction lies, again, in its exhaustiveness, in the certainty which it professes to give us that planets of the kind specified are never to be seen during the middle of the night. It assures us that if we see a starry object of peculiar brilliance at midnight, though it may be Jupiter, it cannot be Venus.

We have so far been considering the Disjunctive Proposition from a point of view that compels no reference to the Exclusivist controversy. But a point is necessarily reached where a decision on this question becomes imperative. The point is reached so soon as the distinction of the alternatives from each other becomes a matter of logical interest. It may become important to decide whether an

* In this statement the Earth is not included among the planets.

objectionable person is a pure knave or a pure fool, the knavery being unadulterated with foolishness, and *vice versa*, or whether he is at once a knave and a fool. In that case, when we make the statement that 'Either X is a knave or he is a fool,' we are directly interested in knowing whether the alternatives are intended or are not intended to be mutually exclusive.

We must distinguish here between two quite different interests—the interest in non-ambiguity and the interest in scientific precision.

1. The statement in question may not be our own, but may be made by some other person, and we may be interested in understanding what that same person precisely means by his statement. In this case, however, it does not lie with us to decide on the sense to be put upon the 'either . . . or.' We must leave that to the 'other person.' The most we can do in defence of logical interests is to insist that he shall make his meaning explicit, and that a rational convention shall be reached which, in respect of the assertion in question, shall secure complete freedom from ambiguity. Thus, we might suggest that, if the exclusive reading be intended, the statement should take the form 'Either he is a non-foolish knave or a non-knavish fool,' or perhaps, more simply, 'Either he is a fool *or else* he is a knave'; the ordinary form, 'Either he is a knave or he is a fool' being reserved for the non-exclusive reading.

If our interlocutor is stating his disjunctive proposition in symbolic form, it seems simpler to reserve the ordinary form 'Either P or Q' for the non-exclusive reading. The attempt to expand this into 'Either P or Q or PQ' (the three alternatives being taken as mutually exclusive) results in a logical disaster or else in a confusion of symbols. For 'P' here means 'P and not Q,' and 'Q' means 'Q and not P.' Hence PQ should mean 'P and not Q combined with Q and not P,' which leaves little distance between PQ and logical nonentity. In symbolic language, then, 'Either P or Q' represents most suitably the non-exclusive reading, and 'Either $P\bar{Q}$ or $Q\bar{P}$'—or, more simply, 'Either P *or else* Q'—the exclusive reading. The form 'Either P only or Q only' is obviously unsatisfactory as an interpretation of the exclusive meaning, for 'P only' implies a much greater restriction than 'P but not Q.' There is, indeed, another alternative. The exclusive form may be uniformly adopted. In this case the proposition 'Either P or Q' would be transformed into 'Either $P\bar{Q}$ or $Q\bar{P}$ or PQ' when 'Either . . . or' implies 'it may be both,' and into 'Either $P\bar{Q}$ or $Q\bar{P}$,' when 'Either . . . or' implies 'not both.' But it is very doubtful whether this gain in uniformity would sufficiently compensate for the loss of the simple form 'Either P or Q.'

2. We come now to the second of the two interests in connexion with which a decision on the Exclusivist question becomes essential. The *distinction* of alternative possibilities from each other may have

a real value for Science, and the interest in exclusiveness may ally itself with the interest in a logical ideal of disjunction. Assuming that this distinction of alternative possibilities is desirable—as it is, for instance, in the classification of species under a genus—we have to ask what the logical ideal of disjunction demands of the disjunctive proposition in view of this requirement.

The natural answer would seem to speak wholly in favour of the Exclusivists. For surely it is only when the disjunctive proposition is read exclusively that we obtain this desired distinctness between the various alternatives.

But in this view there is involved an assumption which we must try to make clear. It is this : We are assuming that the way in which the disjunctive proposition can always best further the logical idea of precision is by being itself ideally precise. But this is a fundamental misconception. In the service of logical ideals the various forms of judgment must co-operate. In some cases the ideal—that of precise characterization, for instance—will be reached by means of a categorical proposition. As an example of this, we may cite the record of some delicate scientific observation. In other cases, as in the development of a supposition into its consequences, it is the hypothetical proposition that must embody the logical ideal—in this case that of necessary connexion between the parts of the proposition. In other circumstances, again, as in scientific Classification, the ideal, that of the mutual exclusiveness of a number of co-ordinate possibilities, requires for its embodiment a disjunctive proposition.

Now, where the function of embodying the logical ideal falls to the lot of the disjunctive proposition, there can be no doubt that the alternative possibilities which it enumerates must be mutually exclusive as well as exhaustive. Here the *function* of the disjunctive logically requires that its form shall be perfect. Hence, when scientific results are tabulated in disjunctive form, the disjunctive should be of the exclusive type, or should approximate as closely as possible to that type. In Mathematics these perfect disjunctions are always obtainable, but in the more concrete sciences this is not the case. In that classification of species in which the species are defined 'by type' (*vide* p. 62) the boundaries between two co-ordinate species will often be somewhat uncertain, though the various types, as centrally defined, are distinct and mutually exclusive.

But where the function of embodying the logical ideal falls, shall we say, to the lot of the categorical proposition, and the disjunctive proposition fulfils its service as a mere means towards ensuring to the categorical the maximum precision of statement, that service may frequently be best fulfilled when the disjunctive is non-exclusive. The ideal functioning of the disjunctive requires here the imperfection of its form. Suppose that, starting from a given categorical basis, we are able to state our alternatives exhaustively

in the form 'Either P or Q or R.' A methodical scientific inquiry may cancel P as a possibility which in the given circumstances cannot be actualized, and we are left with the conclusion 'Either Q or R.' The possibility Q, we suppose, is similarly cancelled, and we are left with the categorical assertion 'R.' In a case of this kind the imperfect, non-exclusive type of disjunction is the most serviceable, and for that reason the most ideal. Where our aim is to reach the truth of one alternative by the elimination of the others, it is otiose to insist on the various alternatives being *ab initio* mutually exclusive. The successive cancelling of P and Q is a process that is quite independent of the question of exclusiveness or non-exclusiveness. Suppose that the disjunction is given in exclusive form. We are then told that either $P\bar{Q}\bar{R}$ is true, or $Q\bar{P}\bar{R}$ is true, or $R\bar{P}\bar{Q}$ is true. P and Q are cancelled, and we are left with $R\bar{P}\bar{Q}$. But this is really the same result as that which we reached above, when we started with the imperfect, non-exclusive disjunctive 'Either P or Q or R.' For the result R there obtained might equally well have been expressed '$R\bar{P}\bar{Q}$,' since it was certainly exclusive of P and Q.

The inconvenience of refusing to utilize a disjunctive proposition at all except when expressed in its most perfect form may best be brought out by a comparison which is much more than a mere analogy. It would be like refusing to use a hypothetical proposition until it had been rendered so precise in both its parts that not only should the affirmation of the antecedent involve that of the consequent, but also the affirmation of the consequent should involve that of the antecedent.

We conclude, then, that the disjunctive, in the service of the categorical, may profitably be left in non-exclusive form, and that this imperfect, non-exclusive form is the form required in the interests of Scientific *Explanation.* Formal Classification, on the other hand, and the laying out of alternative possibilities in the mathematical sciences, require the service of the disjunctive judgment in its perfect and exclusive form.

A disjunctive proposition, then, may be defined as a statement to the effect that, of a closed number of alternative possibilities, one is taken to be actualized. The one obligatory *rule* of disjunction—Rule I.—is that the alternatives shall exhaust the possibilities. Where it falls to the lot of the disjunctive to uphold the ideal of scientific precision, we have further to observe Rule II., that the alternatives shall be reciprocally exclusive. In mathematical inquiry, where scientific precision is as imperative at the beginning of the inquiry as it is at its close, both rules must necessarily hold good for all disjunctive propositions.

Example.—Criticize the following disjunction :

> 'Either triangles are equilateral, or they are isosceles, or they are right-angled.'

This breaks Rule II., for an isosceles triangle may be right-angled. Whether it can be equilateral depends on the precise definition of the isosceles triangle.

It also breaks Rule I. There are triangles which are neither equilateral, isosceles, nor right-angled—namely, the scalene triangles which are not right-angled.

CHAPTER XV.

IV. (v.) THE HYPOTHETICAL PROPOSITION.

THE Categorical Proposition is a proposition which purports to be a statement of fact. The fact to which the statement refers need not be a 'real' fact, or fact of Nature, a fact under Causal Law. It may be a 'formal' fact, a fact that can have actuality only in reference to a certain limited universe of discourse.

The Hypothetical Proposition, on the other hand, may be defined as the statement of a connexion between two possibilities. It contains two clauses, of which the first is called the Antecedent, and the second the Consequent. The Disjunctive Proposition also, as we have seen, states a connexion between possibilities, but in that case the possibilities are regarded as *alternative* possibilities.

The connexion between the Hypothetical Proposition and the Disjunctive is, from the point of view of the logical development of thought, a very close one. The Hypothetical Proposition, as we have already seen (*vide* p. 113), takes one of the possibilities which the Disjunctive Proposition specifies, and develops by connecting it with another possibility. Thus the hypothetical selects for its antecedent one of a number of possibilities disjunctively presented. The consequent also may be regarded as having been drawn from the alternatives of another disjunctive series. Thus both the possibilities with which the Hypothetical Proposition is concerned are of the *disjunctive* type, and therefore real, and not modal, possibilities.

Suppose that we have before us the disjunctive proposition 'Either a triangle is obtuse-angled, or it is right-angled, or it has three acute angles.' Selecting one of these alternative possibilities, we say 'If a triangle is right-angled, a semicircle may be circumscribed to it having its hypotenuse as diameter.'

As the Hypothetical Proposition is concerned with possibilities, and the Categorical with actualities, or with what purport to be actualities, whether formal or real, it is logically impossible to express a hypothetical proposition as a genuine categorical, though it may be equivalently expressed in categorical form. Consider

the statement: 'If anyone trespasses on this property, he will be prosecuted.' We may transvert this into categorical form, but we cannot give it a categorical meaning. We may state it in the form: 'All trespassers will be prosecuted.' But this is not a proposition which purports to be a statement of fact. It does not imply that anyone has trespassed in the past, nor even, indeed, that anyone will do so in the future; it does not imply that any prosecution has taken place, or is certain to take place. On the other hand, the statement: 'All those trespassers are being prosecuted' is a categorical proposition. It claims to state an actual fact, and it cannot be equivalently expressed as a genuine hypothetical.

We conclude, then, that, since it is a fundamental requisite of logical thinking to be guided by meaning and not by form, and since the *meaning* of a hypothetical statement is entirely different from the *meaning* of a categorical statement, the two types of statement are irreducible the one to the other. Further, since hypothetical propositions are frequently given in categorical form, and categorical propositions in hypothetical form, it is essential, when reducing a proposition to strict logical shape, to look to the meaning and adjust the form accordingly.

Consider, for instance, the proposition:

> 'If air is liquefied, it is in that state dangerous to handle.'

This is a pseudo-hypothetical. Its meaning is precisely equivalent to that of the genuine categorical 'Liquefied air is dangerous to handle.' On the other hand, the proposition 'If this fluid is liquefied air, it is dangerous to handle' is a true hypothetical. It cannot be reduced to a genuinely categorical equivalent.

In speaking of the Hypothetical Proposition as a connexion of possibilities we may be using the word 'connexion' in one or the other of two senses: either (1) as indicating an assertorial—*i.e.*, an assertorially intended connexion; or (2) as indicating an apodeictic —*i.e.*, an apodeictically intended connexion. The connexion is assertorial so far as it is merely asserted and nothing is implied with regard to its nature. It is apodeictic when it is intended to be a logically necessary connexion, a connexion that cannot be denied without either rendering Nature unintelligible or our own thought inconsistent.

As instances of assertorial connexion we may cite the following propositions:

> 'If you go out without an umbrella to-day, you will come home wet.'
>
> 'If sun-spots are numerous, magnetic storms on this planet will be correspondingly numerous.'

The following are instances of apodeictic connexion :

1. 'If there is a way up, (it logically follows that) there must be a way down.'
2. 'If all men are mortals, (it logically follows that) some mortals are men.'
3. 'If all men are mortals, (it logically follows that) all mortals are men.'
4. 'If you accept the statement that all persons are selfish, you are logically compelled to admit that you are yourself selfish.'

In the first and second of these last examples we see that denial of the validity of the connexion would render Nature unintelligible. In the third example the apodeictic intention has miscarried. In the fourth, denial of the validity of the connexion would render thought inconsistent.

When the connexion of possibilities depends on the human will as a synthetic principle, and the reference to Reality is still restrictedly conceived as either 'formal' or 'real,' the free connexion which the statement implies must be interpreted as equivalent to a material connexion in this essential respect—that it can be justified through observation or scientific investigation. So interpreted, the statement ranks as an assertorial hypothetical. Let us from this point of view consider the proposition : 'If anyone trespasses on this property, he will be prosecuted.' Here the prosecution must not be supposed to depend on the caprice of the owner. It must depend on such facts as the apparatus of the law *re* trespassing and the policing of the property, which can be investigated as actual facts can be investigated. Otherwise the validity of the proposition does not *admit* of being scientifically tested.

We have, then, two main types of Hypothetical proposition—the Assertorial (formal or real) and the Apodeictic. There are also two main varieties of the Apodeictic, respectively regulated by the Law of Non-Contradiction and the Law of Excluded Middle. The first of these two varieties may be called the Formal Apodeictic Hypothetical, the second the Material Apodeictic Hypothetical. The three types of Hypothetical Proposition may suitably be symbolized as follows :

Assertorial Hypothetical.	If P, then ascertainably* Q.
Apodeictic Hypothetical.	If P is accepted, then by implication Q must be accepted (Formal). If P is true, then by implication Q is true (Material).

* The ascertainability of Q is, of course, only a claim made by the judger, and the form of verification may vary with the Universe of Discourse.

Illustrations :

If water is heated at standard pressure, it will boil at 100° C. (real assertorial).

If Black mates White in three moves, Black will have won the tournament (formal assertorial).

If the moon is made of green cheese, (it logically follows that) it must at least be subject to the law of gravitation (material apodeictic).

If the moon is made of green cheese, (it logically follows that) some heavenly bodies are not incorruptible (material apodeictic).

If the statement that the moon is made of green cheese is accepted, (it logically follows that) we must accept the further statement that all bodies not made of green cheese are-not the moon (Formal apodeictic).

If the statements that all green cheese is corruptible and that the moon is made of green cheese are accepted, (it is logically necessary that) the further statement that some heavenly bodies are not incorruptible must also be accepted (Formal apodeictic).

Should the Hypothetical be presented in the general form 'If P, then Q,' or in the form 'If P is true, then Q is true,' the more specific nature of the given hypothetical will depend on the meaning of the connecting-word 'then.' If 'then' means 'in that case, as a matter of ascertainable fact,' the proposition is assertorial; if 'then' means 'it follows with logical necessity that,' the proposition is an apodeictic hypothetical of one kind or the other.

With this distinction between assertorial and apodeictic hypotheticals before us, we are able to give more precise expression to the distinction between the categorical and hypothetical forms of statement. A categorical proposition must admit of being presented, whether in a formal or in a real sense, as true or false. The apodeictic hypothetical can only be valid or invalid. It can be 'validated' as a sound inference, or 'invalidated' as an unsound or illegitimate inference. An assertorial hypothetical, on the other hand, may either be *justified* as a correct prediction or be *discredited* as an incorrect prediction. Moreover, it is only categorical statements that can be *verified.* When we verify the truth of the alleged connexion between antecedent and consequent, we are verifying the categorical proposition 'Q is connected with P.' Similarly, as we shall see, when we come to Induction, to verify a hypothesis is to verify not a hypothetical proposition, but its (categorical) antecedent.

The logical connexion of possibilities asserted by the Apodeictic Hypothetical, when not invalid, and therefore, in first or last resort, meaningless, may be either conditionally or unconditionally valid.

It is unconditionally valid when the antecedent cannot be affirmed without necessarily implying the consequent. Thus the propositions :

> 'If all men are mortal, then some mortals are men' (material),

and

> 'If it be granted that all men are mortal, then it must be granted that some mortals are men' (Formal)

are unconditionally valid hypotheticals. But the propositions :

> 'If all men are mortal, then Socrates is mortal' (material),

and

> 'If it be granted that all men are mortal, then it must be granted that Socrates is mortal' (Formal)

are conditionally valid hypotheticals. They are conditionally valid because in each case the consequent can be inferred from the antecedent only in virtue of a *further* assumption—namely, the assumption that Socrates is a man, or is granted to be a man. The strictly logical forms of these propositions would be :

> 'If all men are mortal, and Socrates is a man, then Socrates is mortal' (material),

and

> 'If it be granted that all men are mortal, and that Socrates is a man, then it must also be granted that Socrates is mortal' (Formal).

All hypothetical validity-connexions, in fact, when expressed in strict logical form, are of the unconditionally valid type. The hypothetical assertion cannot justify itself except by, as it were, elongating into an explicit logical inference. We have here the point of departure for the study of Inference.

V.

THE FORMAL TREATMENT OF THE LOGICAL PROPOSITION.

CHAPTER XVI.

V. (i.) TRANSITION TO THE FORMAL TREATMENT OF LOGIC.

THE essential significance of the connexion between Inference and the apodeictic hypothetical form of statement is concentrated in the meaning of the word 'if.' Whether we say 'If P is true, then Q is true,' or 'If P is accepted, then Q must be accepted,' 'if' means no more than 'assuming that.' If the word is understood as equivalent to 'given that,' then by 'given' we must mean 'given as an assumption.' 'If' therefore plays the important part of introducing truth and falsity relations in a purely hypothetical way. When a person states an inference in the form 'If P, then Q,' no call is made on him to substantiate the truth of P. He is not expected even to have an opinion as to the truth or untruth of P. When respectively introduced in the form of antecedent and consequent of a hypothetical proposition, the premisses and conclusion of an argument are shielded from all direct contact with the world of Knowledge and of Reality. The shelter of the word 'if' completely protects them from the fortunes and perplexities of the truth-interest.

Now it is precisely in this abstract aspect that the problem of Inference can best be studied. In order to understand the object of our study we must isolate it, after the fashion of all successful experimentation; and in the present instance the hypothetical form of statement is the isolating apparatus. This provisional isolation from the larger interests of Truth has very great advantages. It enables us, more particularly, to present our object in such artificially devised forms as will best serve our purpose, and thus makes possible a degree of accuracy and precision which would not be possible were the object studied in the vast, uncontrolled context of its natural surroundings.

We propose, then, to study Inference as Inference, to substitute a pure validity-interest for the more comprehensive truth-interest, and—as an important provisional step—to adapt the statement-import of the proposition in such wise as to facilitate still further our study of the conditions of valid Inference. Moreover, of the two types of apodeictic hypothetical we take as our model the Formal type, of which the purport is to uphold the necessity for *consistent statement* in the name of the Formal principles of Identity

and Non-contradiction. Adopting, as we propose to do, the *statement-import* of the Proposition, the material type 'If P is true, Q is implicitly true,' with its ultimate appeal to the Law of Material Identity and the Law of Excluded Middle (*vide* p. 99), would be a wholly incongruous model.

The study of the Inference problem under the restrictions above set forth is the central, and perhaps the exclusive, topic of a 'Formal' Logic. Where Proposition and Inference are studied not only as more or less complicated statements of meaning, but also on the basis of an artificial modification of statement-import, the treatment is at once abstract and conventional, and therefore Formal in a very genuine sense of the word; and what we have here called a Formal treatment of Logic is usually more briefly referred to as Formal Logic. It is with this Formal aspect of Logic that we shall now be exclusively concerned until we pass from the problem of Inference to the wider and deeper problem of Scientific Explanation.

CHAPTER XVII.

V. (ii.) THE FORMAL IMPORT OF THE CATEGORICAL PROPOSITION.

THE import, or structural significance, of a proposition depends on the character of the identity relation which it affirms. Identity determines Import: as the identity is conceived, so will the import be conceived. Where this identity is conceived in the form which, being the simplest and most manageable, is therefore, regarded from the point of view of the Formal treatment of Logic, the most relevant, we have what is known as the Formal import of a proposition.

The most simple interpretation of the meaning of identity is that of numerical coincidence. When I say 'All lions are carnivores,' I am considering lions and carnivores alike simply as countable objects. *Qua* countable objects they are identical, and the difference is a mere difference of quantity: the lions as countable objects are numerically coincident with the same number of carnivores. Each of all the lions is a carnivore.

The form of propositional import which interprets the Proposition in this simple way, and reads both subject and predicate terms in extension, may suitably be called the Extensive Import of a proposition. The extension of a term, as we have seen, is not to be confused with its conno-denotation or *intension*, the joint product of those processes of Definition and Division through which the meaning of a term is developed. The extension includes the objects to which the meaning of the term *applies*. But in *referring* to the extension of a term we need not refer to the whole

of it : the reference may be either partial or total. Hence a term read 'extensively' is a term considered from the point of view of its extent, but dependent on a defining quantity-mark (or its logical equivalent) to make clear whether this reference to extension is partial or total.

The 'extensive' interpretation or reading of a proposition is customarily called its 'denotative' reading. But this usage involves a confusion between extension and denotation, and must carefully be avoided by those who hold, as we do, that the meanings of these words should be differentiated. The proposition 'All men are mortal,' when taken as equivalent to the true extensive meaning, 'The objects *indicated* by the term "man" are numerically coincident with objects *indicated* by the term "mortal,"' is commonly said to have its terms read in denotation, and the word 'denoted' is substituted for 'indicated.' The same misuse (as we hold it to be) of the word 'denoted' occurs in the enunciation of what is called the predicative view of Import. Here the subject-term is said to be read in denotation, the predicate-term in connotation, and the proposition 'All men are mortal' is then rendered as follows : 'The objects denoted by "man" possess the attributes connoted by "mortal."' We should prefer to say that in this case the subject-term is read in extension and the predicate-term in intension.*

We are not, however, directly concerned with the various ways of interpreting a proposition, but rather with explaining and developing that one form of propositional import which we have selected as most adequately meeting the requirements of a Formal logical treatment.

The extensive proposition is most conveniently stated in one of four ultimate forms, which are traditionally known as the A, E, I, and O propositions. The scheme is as follows :

†Universal Affirmative : A : 'All S's are P's,' symbolized by SaP.
Particular Affirmative : I : 'Some S's are P's,' symbolized by SiP.
} (*AffI*rmo.)

†Universal Negative : E : 'All S's are-not P's,' symbolized by SeP.
Particular Negative : O : 'Some S's are-not P's,' symbolized by SoP.
} (n*E*g*O*).

* A third variety in the rendering of a proposition may here conveniently be noticed. It is called the *attributive* view. On this view both terms are read in connotation. Thus, the proposition 'All men are mortal' would run as follows : 'The attributes connoted by the term "man" are accompanied by the attributes connoted by the term "mortal"; ' or else 'The attributes connoted by the term "man" are accompanied by the attribute "mortal."'

† A distinction is commonly drawn between the generic universal judgment and the 'general' or enumerative universal judgment. The A and the E pro-

Illustrations :

A : All dukes are members of the House of Lords.
I : Some lords are members of the House of Commons.
E : All dukes are-not members of the House of Commons.
O : Some lords are-not members of the House of Commons.

The fact that according to this scheme there are four propositions, and only four, follows from the nature of a proposition as Formally conceived. A proposition must be either affirmative or not; and in either case its subject-term must either be used in its whole extent or not so used.* Moreover, the choice of the two fundamenta of Quality and Quantity, upon which this fourfold division depends, is necessitated by the adoption of the extensive view of Import. The adoption of this specific view makes it imperative to accept the basis of Quantity or extensive reference as one fundamentum; and the fact that the distinction in quality is a distinction which is logically prior to any specification of identity-import whatsoever obliges us to include the quality-basis as a second fundamentum.

On the Distribution of Terms in a Proposition.

In a previous paragraph we pointed out the fact that our reference to the extent of a term may be either *total* or *partial*, or, to use the more technical language of Formal Logic, either universal or particular. We have now to give to this distinction the preciser form required by the abstract validity-interest which is at present dominating our whole inquiry. From this restricted standpoint our sole concern is with unambiguous statements of meaning, and inquiry into the implications of these same statements. It is with the aim of clearly expressing this limitation of standpoint that, when speaking of the *extension of terms*, we shall use the words 'distributed' and 'undistributed,' as defined below, in the place of the words 'universal' and 'particular.'

A term is said to be distributed (within a proposition) when it is used in its whole extent—that is, when there is either an explicitly stated or a logically implied reference to all the individuals contained in the class for which it stands.

A term is said to be undistributed when it is *not* used in its whole extent—that is, when no reference to the whole extent of the class is either explicitly stated or logically implied.

Let us consider A, E, I, O from this point of view.

---

positions are, in our view, enumerative universals. The difference between 'generic' and 'enumerative' corresponds, in fact, to the distinction between 'intensive' and 'extensive.'

* For further substantiation, see the discussion on the Quantified Predicate, pp. 159-161.

i. *Distribution of S :*

A, I, and O present no difficulty. S is distributed in A, undistributed in I and O.

With regard to E a difficulty may present itself, arising from the familiar, though misleading, form 'No S is P.' In objection to the statement that S is distributed in the E proposition it may be urged that, since the proposition states that no S's are identical with P's, therefore no individuals belonging to the class S are being referred to. Now, it is quite true that no such individual objects are being referred to *as being identical with P's*. But 'No S is P' is not an affirmative proposition with a subject of zero extent. It is a *negative* proposition. It is therefore equivalent to 'All the S's are-not P's.' Thus E, the universal negative proposition, distributes its subject. It is obvious that, so long as the E proposition is consistently presented in the stricter form 'All S's are-not P's,' the misapprehension cannot arise.

ii. *Distribution of P :*

A. Does the statement 'All S's are P's' imply the statement 'All S's are *all* P's'?*

Obviously it does not. For the statement that all the S's are severally identical with the same number of P's does not imply that the two classes S and P are coextensive, coinciding point for point. What the given statement necessitates is simply the identification of each of all the S's with each of some of the P's. It is equivalent to the statement that 'All S's are some P's.'

Therefore *P is undistributed.*

E. Does the statement 'All S's are-not P's' necessarily imply the statement 'All S's are-not *any* P's'? It must be so. If it did not imply this, its acceptance would not preclude the acceptance of the statement 'Some P's are S's,' which, if accepted, would necessitate the acceptance of 'Some S's are P's.' But if 'All S's are-not P's' does not preclude the acceptance of 'Some S's are P's' it is meaningless.†

Therefore *P is distributed.*

I. Does the statement 'Some S's are P's' imply the statement 'Some S's are *all* P's'?

No, certainly not. The assertion 'Some cats are black objects' does not imply that, when each of the 'some cats' referred to has been identified with a black object, there are no black objects remaining over. It does not imply that the 'some cats' are numerous enough to account for *all* the individuals contained in the class Black object.

* On the quantified predicate, see p. 159. † *Cf.* pp. 98, 104.

Therefore *P is undistributed.*

O. Does the statement 'Some S's are-not P's' imply the statement 'Some S's are-not *any* P's'?

Yes, for if not, its acceptance would not preclude the acceptance of the statement that Some P's are identical with these same 'some S's' of which we are stating that they are not identical with P's. But if we were to state that P's are identical with them, we should also be stating that they are identical with P's, which is contrary to the given statement. Now an accepted statement which does not preclude the acceptance of a statement inconsistent with itself is self-contradictory, and therefore meaningless.

Therefore *P is distributed.*

From the consideration of E and O we see that negative propositions, as such, must have their predicates distributed.

We have, then, the four following rules for the distribution of terms :

1. All universal propositions distribute their subjects.
2. No particular propositions distribute their subjects.
3. All negative propositions distribute their predicates.
4. No affirmative propositions distribute their predicates.

Rule No. 3 in particular is worth remembering.

An A proposition, then, is technically defined as a logical proposition having its subject distributed and its predicate undistributed; and E, I, and O are similarly definable. These definitions are fundamental in a Formal treatment of Logic, and it is important, when thinking of these propositions, to think of them in this way.

The Diagrammatic Representation of A, E, I, O.

'Diagrams,' as Professor Welton says, 'are intended to make obvious at a glance the relations between the terms expressed in a proposition.'* The relations thus diagrammatically expressed are necessarily extensity-relations, so that it is only when the extensive view of Import is adopted, and both subject and predicate terms are read in extension, that we can express diagrammatically the relations which we state to exist between the two classes S and P. Each term is treated as a class-term and diagrammatically represented by a circle. The relation stated to hold between the two terms is thus represented diagrammatically by a mutual coincidence or non-coincidence, partial or total, of two circles.

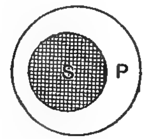
We proceed to point out the precise diagrammatic equivalents of the A, E, I, and O propositions.

In the following diagrams the shaded part of a circle stands in

* 'A Manual of Logic,' vol. i., ch. iii., p. 215.

each case for that part of the class-extension to which reference is made either explicitly or by logical implication, so that distributed terms are represented by wholly shaded circles, undistributed by partially shaded circles. The horizontal shading in each case belongs to S, the perpendicular to P.

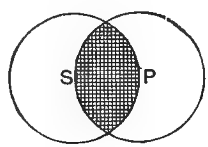
The A proposition must be represented thus :



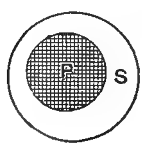
It is, indeed, frequently stated that A requires two diagrams to represent it adequately—the one already given, and a second representing the coextensiveness of S and P :

But since, when asserting the A proposition, we do not state (or imply) that we are referring to the whole extension of P, we are not at liberty to make the statement diagrammatically. We cannot logically draw what it is illogical to state.

The I proposition gives :

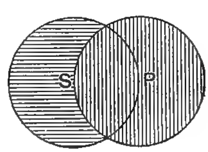


The alternative

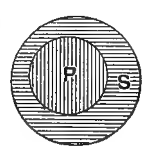


is ruled out, since we do *not* state that some S's are *all* the P's, and, as we do not state this, we are not at liberty to indicate it diagrammatically.

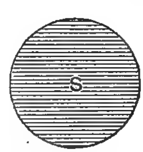
The O proposition gives :

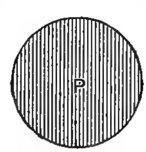


or



Finally, the E proposition gives :

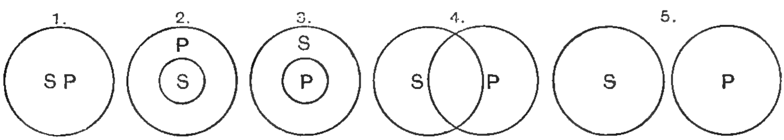




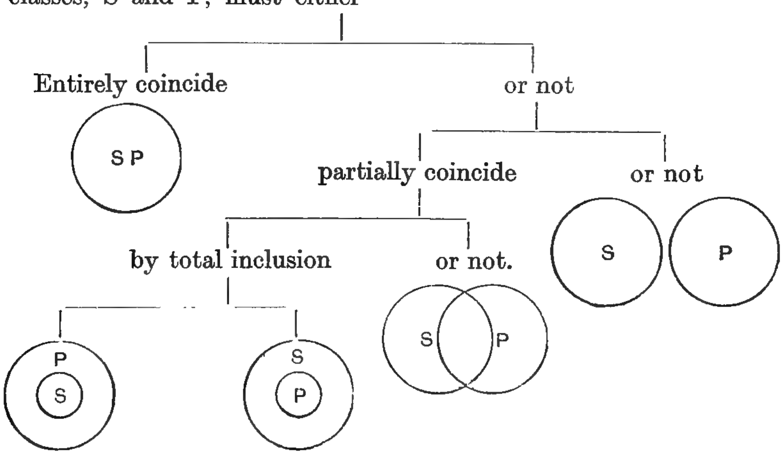
It follows from the above that an A or I proposition must *invariably* be taken as possessing an undistributed predicate, even though we happen to *know* that the statement made by the proposition would still be true though taken to involve a reference to *all* the individuals included in the extension of the predicate term. Thus the predicate of the proposition 'All equilateral triangles are equiangular triangles' is undistributed, and so is the predicate of the proposition 'Some animals are horses.' If our intention is to distribute both subject and predicate—if we wish, for instance, to assert that all the objects contained in the class Equilateral triangle are severally identical with all those contained in the class Equiangular triangle, we must make use of two propositions, and say 'All equilateral triangles are equiangular triangles,' *and* 'All equiangular triangles are equilateral triangles.'

It is most important, in connexion with logical diagrams, not to confuse the diagrammatic representation of *statements* concerning the relations of classes to each other with the representation, in diagrammatic form, of the possible class-relations themselves.

Thus, as Dr. Venn and Professor Welton insist, there are only five ways in which two classes can partially or wholly coincide with one another—namely, those represented by the five diagrams :



For, as Mr. Stock points out ('Logic,' 2nd edit., p. 85), two classes, S and P, must either—

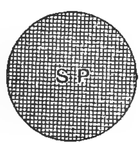


But this does not justify us in concluding that, on the extensive view of import, there are five, and only five, elementary *propositions* :

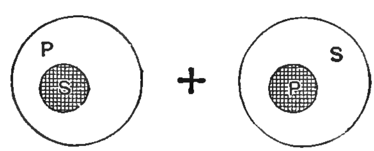
1. S coincides with P :
2. S is wholly included in P :
3. S wholly includes P :
4. S partially includes and partially excludes P :
5. S entirely excludes P :

Qua statement of meaning, the first of these is, in fact, not an elementary type. It is equivalent to the second and the third conjointly.

More generally we may say that, whilst we admit that classes can be related to each other (in respect of mutual coincidence and non-coincidence) in five ways and five only, we do not hold that there are only five ways in which such relations of classes can be stated. We may, as we have just seen, reduce them to four ; or we may extend them, though perhaps without sufficient reason,* to eight. And it is particularly to be noted that in Formal Logic we are concerned not with the relations of classes, but with the statement of their relations. Logical diagrams should represent statements only, sc that two ways of saying the same thing may appropriately be represented by different diagrams. Thus the proposition 'All the S's are all the P's' is represented by the

diagram 

which directly expresses the coextensiveness of the classes S and P. But when our statement takes the compound form 'All S's are P's and all P's are S's,' the corresponding

diagrammatic rendering will be : 

'All' and 'Some.'

The meanings which we ascribe to the words 'universal' and 'particular' as applied to propositions, and to 'distributed' and 'undistributed' as applied to terms, need to be further defined by a discussion of the precise meanings of the two words 'All' and

* *Vide* p. 159.

'Some' as used to introduce, respectively, the universal and the particular proposition.

The extension of a term, as we have seen, must be *conjunctively* interpreted (the intension being *disjunctively* interpreted). Thus, the extension of 'Man' is 'this man *and* that man *and* the other,' etc. 'All' and 'Some,' then, in Formal Logic are used conjunctively. This conjunctive use must not be identified with the *collective* use of these words. In ordinary discourse 'All' and 'Some' *may* be used collectively. In speaking of 'all the men' we may be referring to them as a single group, as in the statement 'All the men in the room were but a tithe of those who were invited'; and 'Some' is collectively used in such statements as 'Some days of rest are all that he needs,' 'Some coppers will make these children happy.' But the two words may also be distributively understood, and this is the case in their conjunctive use. The expression 'A *and* B *and* C,' when conjunctively understood, means 'A, *also* B, *also* C,' and in this case whatever is predicated of 'A and B and C' is predicated of each individually. This is the case when All and Some are used to introduce the universal and particular propositions respectively.

1. *On the Logical Meaning of 'All.'*—We have seen that in ordinary discourse the word 'All' (as also the word 'Some') is ambiguous. It bears two distinct meanings of which one only is accepted as appropriate to the purposes of Formal Logic. Thus, when I say 'All these articles cost sixpence,' I may mean either that I paid sixpence for each article, or that I gave sixpence for the lot. In the former case, 'All' is the universal quantity-mark. Both it and the statement that it introduces are distributive in meaning. The sentence is an A proposition in strict logical form. In the latter case, the meaning is collective, and 'All' is not a quantity-mark in the strictly logical sense. For the purposes of Formal Logic the statement needs to be reduced to strict logical form, and will then appear as a Singular Proposition: 'This collection of articles is a lot that costs sixpence.'

The simple test for discriminating between the distributive and the collective 'all' is to substitute 'each of' for 'all,' and see whether the sense of the statement is thereby affected. If the substitution does not alter the sense, the proposition is distributive; if it does, we are dealing with a collective statement which is not in strict logical form. For instance, in the sentence 'All these trees here hide the view' we cannot read 'each of' for 'all' without destroying the sense. Therefore the statement is collective. It is a Singular Proposition in disguise.

2. *On the Logical Meaning of 'Some,' and the Import of the Particular Proposition qua Particular.*—The distinction between the universal and the particular proposition coincides, as we have seen, with the distinction between a proposition with a

distributed subject and a proposition with an undistributed subject. The logical meaning which 'Some' bears in the particular propositions I and O must, therefore, be that which appropriately expresses the characteristic of undistributedness. As we have seen, a term is undistributed when there is no reference, either expressed or implied, to the whole extent of the class for which it stands. This definition of an undistributed term is not arbitrary. It is the meaning dictated by the true interests of Inference. A term is of interest for purposes of inference just in so far as we definitely state or else logically imply what we take its extensity-reference to be. The primary essential here is to be able, in any given case, to distinguish with certainty between what is stated and what is not, and it is this very distinction which the use of the words 'distributed' and 'undistributed,' as above defined, enables us to express.

The true logical meaning of 'Some' in its relation to 'All,' a meaning which we may call the *undistributed* meaning of the word, is therefore correctly expressed in the phrase 'not statedly all.' As regards the inferior limit, the appropriate phrase is 'one at least.' If we are told that Socrates was wise, and that he was a citizen of Athens, we are entitled to conclude that at least one Athenian citizen was wise. But this is essentially a proposition with undistributed subject, for we do not state that all Athenian citizens were wise. Hence, in order to include such cases as this, 'Some' must be able to stand for 'one at least,' and we see that 'one at least, but not statedly all' is the proper logical equivalent of the word 'Some.'

From this strictly logical meaning of 'Some' we must carefully distinguish other meanings that the word is capable of bearing :

(i.) There is the *popular* use of 'Some' in the sense of 'a few at any rate, yet considerably less than all.' The logical disadvantage of this use of the word is that, if the particular quantity-mark is thus interpreted, there are many propositions which can then be neither particular nor universal. Thus, 'One S at least is a P,' and 'Very nearly all the S's are P's' would not be particular propositions if this popular meaning of 'Some' were adopted, and they certainly are not universal. This, of course, presents no difficulty to the practical consciousness. If it wishes to talk about 'one at least,' it says 'one at least'; if it desires to refer to 'nearly all,' it says 'nearly all.' It is only when we wish to generalize and to regulate that we aim at minimizing distinctions and making them strictly relevant to our purpose—to the interests, for instance, of logical inference.

(ii.) Secondly, there is the *exclusive* use of 'Some.' That which we have called the strictly logical meaning of 'Some' is open, in the opinion of a certain school, to a fundamental objection. It shares with other interpretations the defect of not allowing our

statements to be sufficiently explicit. A man may reasonably be challenged to explain why he prefers to point out what he does not state instead of stating clearly and exclusively what he does intend to say. Granting that a proposition, as Formally used, expresses nothing beyond a statement of meaning, would it not be better to render 'Some S's are P's' as follows: 'What I mean to state is that one S at least is a P, but that not all of the S's are P's.' Here there is no reticence, no inscrutable reference to what is *not* stated. Why not define 'Some' as exclusive of 'All,' and 'All' as exclusive of 'Some'?

This proposed interpretation of 'Some' is known as the 'exclusive' meaning. 'Some' is here taken as equivalent to 'one at least, but *not* all,' or, if we prefer to keep more explicitly to the statement-view of Import, 'one at least, but statedly not all.'

The essential objection to this exclusive interpretation of 'Some' is that it reduces the distinction between the I and O propositions to a mere difference of emphasis. If we state that one S is a P, but that not all S's are P's, we are also stating that some S's are-not P's. The fourfold scheme reduces to a threefold scheme, including three types of proposition—A, E, and Φ. This would confuse the whole scheme of logical opposition (*vide* Chapter XIX.).

There is also the further objection that this use of 'Some' cannot cover all cases. If I say 'Some cats are fond of fish,' I do not, as a rule, wish to imply that some cats are-not fond of fish. What the statement *does* invariably imply is that one cat at least is fond of fish. As to the question whether all cats are fond of fish no statement is made.

(iii.) There is, further, an indefinite or semi-indefinite use of the word still suggested as the correct use in many logical treatises. According to this interpretation 'Some' is defined as meaning 'one at least, possibly all,' or 'one at least, it may be all.' 'Some S's are P' could then be paraphrased thus: 'I state the predicate P of at least one S, but I do not state of how many S's it holds good: *it may hold good of all of them.*' Thus, when I say 'I saw some of your friends at the gathering yesterday,' I may mean to include the possibility of my having seen *all* the friends of the person to whom I am speaking.

This interpretation of 'some' has a plausible appearance. But this plausibility is really derived from a confusion of the positive expression 'it may be all' with the negative limitation 'not statedly all'; and it shares with other renderings the inconvenience of leaving certain types of proposition not accounted for either as particular or as universal. Thus, 'Some human beings are children' cannot be interpreted as meaning 'Some human beings, possibly all, are children.' 'Some . . . it may be all' is incompatible with 'Some . . . but not all.' On the other hand, 'Some . . . but not statedly all' *is* compatible with 'Some . . . but not all.'

If I say 'I don't state that all human beings are children,' it is open to some one else to complete my statement by saying: 'But I do state that some human beings are not children.' The rendering 'one at least . . . it may be all' lacks the comprehensiveness of the rendering 'one at least . . . but not statedly all.'

But there are other objections of a more fundamental kind. The reading 'possibly all' does not truly interpret the meaning of the undistributed term. By unduly emphasizing the indefinite import of the word 'some' it fails to grasp the true logical character of the particular proposition. It is quite true that 'indefinite' is a better label for the I and O propositions than is the word 'particular.' For 'particular,' as popularly used, implies a definiteness of a positive kind, as when we speak of 'this particular individual.' So, again, we talk of reasoning from particulars to particulars, when we mean that in each case we argue from one given instance to another. But we really need a still better label which should convey the negatively definite idea that the proposition makes no universal statement. For there is certainly no logical value in 'indefiniteness.' Merely to suggest the possibility that all of the S's are P's is to make a statement that has no inferential value whatsoever. There is no logical vitality in a mere possibility.

The reasons given above are perhaps sufficiently decisive against the use of 'Some' in its indefinite sense. But the crucial objection arises out of the implications of the word 'possibly.' The phrase 'Some, possibly all' may indicate either a limitation of subjective certainty, in which case it means 'One at least, all for aught I know,' or else a more definite limitation of knowledge, its meaning then being 'One at least, not assuredly all.'* But both these readings of the word 'Some' are entirely inadmissible on the view we have adopted as to the import not only of the undistributed term, but of a Formal Logic generally. It cannot be too emphatically stated that in Formal Logic—*i.e.*, in a Formal treatment of Logic—we are not directly concerned with any questions of truth or knowledge or ignorance, or even of opinion, or with the limitations of any of these. No doubt a more intimate aim of Logic is to analyse the scientific reasoning which is concerned with knowledge and with truth; but Formal Logic is a propædeutic which is abstractly concerned with consistency of reasoning without any reference to the truth or the falsehood of the accepted premisses, or to the knowledge or the ignorance of the reasoner. In Formal Logic we are concerned not with what is, or with what is known, but with what is *stated* either explicitly or by logical implication. Our business is to develop not knowledge or opinion, but *significant statement*. Doubtless the Formal logician may have to deal with

* The word 'knowedly' or 'knownly' would perhaps have expressed the meaning more directly than the word 'assuredly,' but might not have sounded so well.

statements about truth, as also with statements concerning knowledge, certainty, and possibility; but with the *truth of statements* or with the knowledge, ignorance, certainty, or possibility from which those statements spring he has nothing at all to do. He develops the meaning of statements, and notes their limitations of meaning. His business is not with what is known or not known, but simply with what is stated. Thus, for him the particular as opposed to the universal quantity-mark indicates never the *statement of a limitation*, but always the *limitation of a statement.*

Justification of the 'Extensive' View of Import.

Mr. Joseph has brought against the conception of extensive import the crucial objection that 'we cannot predicate of the extension of one term the extension of another.'* But what does this precisely mean? It is true that we cannot predicate of all the S's, taken distributively, that they are all the P's, or some of them, if the word 'all' or 'some' in the predicate is used collectively. We cannot say 'All S's (distr.) are all, or some, P's (coll.).' Nor can we say 'All S's (distr.) are all, or some, P's (distr.)' if by this is meant 'Every S is every P (or each of some P's).' But we *can* say 'Every S is a P,' and the predicate of this proposition may be regarded as a genuinely 'undistributed' term. The meaning is 'S_1 and S_2 and S_3 . . .—each of all the S's—is a P.' Evidently the P's must be at least as numerous as the S's, but whether they outnumber them or not we do not state. If we wish to state that they do, we may do so unambiguously by means of the compound proposition 'Every S is a P, and each of some P's is-not any S.' And if we wish to state that the two groups exactly coincide point for point, we can say 'Every S is a P, and every P is an S.' When we employ the simple form 'Every S is a P' we are not identifying each of all the P's with an S. The reference, therefore, is neither statedly nor implicitly to each of all the P's, and hence the predicate is undistributed. Thus we see that the mark 'a' in 'Every S is a P' may logically represent the undistributedness of the predicate-term. It may do so with equal appropriateness in the particular proposition 'Each of some S's is a P.' Here again the proposition does not imply that each of all the P's is an S. If we desire to distribute the predicate term, we can use the compound proposition 'Each of some S's is a P, and each of all P's is an S.' In the case of negative propositions, for E we have 'Every S is-not any P,' and O is equivalent to 'Each of some S's is-not any P.' Here the 'any' is a mark of distributedness. We state that no one of the S's referred to is any single one of all the P's.

It follows from the above that there is a sense in which we can

* 'An Introduction to Logic,' ch. ix., p. 202.

perfectly well 'predicate of the extension of one term the extension of another.' What we have already said may perhaps be sufficient to establish the point. But it may be useful, further, to draw attention to the ambiguity of the verb 'to predicate.' It is possible to use this verb in such a way as to presuppose the predicative view of import, and so exclude *ab initio* the possibility of adopting an extensive reading. In the proposition 'All S's are P's' the predicate is not predicated of the subject as its *attribute.* P cannot be a differentia or a proprium of S. But it may still be a predicable, for it may stand to S as genus to species, or as a class-term in one division may stand to a class-term in another division. What is predicated of the extension-of-S is that it is related in the way of at least partial coincidence to the extension of a certain correlate P. Thus P is predicated of S, not as its attribute, but as its extensional correlate.

Our conclusion, then, is that we are justified in retaining the fourfold scheme in its extensional form, and that we may posit the following equivalences :*

All S's are P's ≡ Each of all S's is a P.
Some S's are P's ≡ Each of some S's is a P.
All S's are-not P's ≡ Each of all S's is-not any P.
Some S's are-not P's ≡ Each of some S's is-not any P.

The Quantification of the Predicate.

Sir William Hamilton proposed to develop the fourfold scheme of Categorical Propositions by adding the quantity-mark 'all' or 'some' to the predicate of A and I, and 'any' or 'some' to that of E and O, thus obtaining an eight-fold scheme.

From A	All S is all P..	U.
	All S is some P	A.
From I	Some S is all P	Y.
	Some S is some P	I.
From E	No S is any P	E.
	No S is some P	η.
From O	Some S is not any P	O.
	Some S is not some P	ω.

The interpretation and discussion of this scheme still occupies a considerable place in modern treatises on Logic. The reader will find excellent critical appreciations in Professor Welton's 'Manual of Logic' (vol. i., bk. ii., ch. ii., pp. 200 *ff.*) and Mr. Joseph's 'Introduction to Logic' (ch. ix., pp. 198, *ff.*). The ambiguities of the

* For the relation of this 'coincidence' or 'identity' form of extensional import to the class-inclusion view, *vide* p. 239.

words 'All' and 'Some' and the confusion between the distributive and collective uses of these marks supply ample opportunity for criticism and reconstruction.

Into the complexities of this discussion we do not propose to enter. We content ourselves with connecting the doctrine of a quantified predicate with the fourfold scheme as we have adopted it, and considering the logical significance and importance of the eightfold scheme from this single point of view.

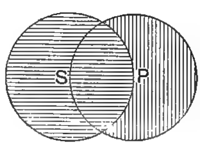
In the first place, the conception of a quantified predicate appears to us to be perfectly reasonable. The quantified predicate is indeed already present in the fourfold scheme under the guise of the distributed and undistributed predicate-term. Hence, assuming as we do the undistributed meaning of the word 'Some,' and reading both subject and predicate terms in extension, we hold that there is obviously no difference between the A, I, E, and O of the quantified scheme and the corresponding propositions of the fourfold scheme.

Our sole criticism of the Hamiltonian scheme *when interpreted in this way* is that the four additional propositions, U, Y, η, ω are superfluous.

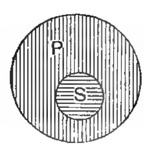
The U proposition, as we have already seen, is equivalent to the compound proposition 'All S is P and All P is S,' of which the elements are A propositions.

Again, the Y proposition is equivalent to the A proposition 'All P is S.'

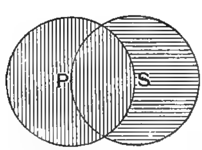
The η proposition may be disposed of in a similar way. We may diagrammatically represent the η proposition—'No S is some P'—as follows:



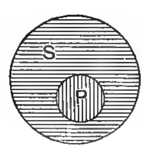
or



But if we exchange the S and P in these diagrams, we obtain the diagrams of the O proposition:



or



'No S is some P' is, in fact, identical with 'Some P is no S.'

As for the ω proposition, it cannot be denied *whatever* we intend to state. Even if we intend to state that the extension of the

class S exactly coincides with that of P, still we evidently cannot deny the statement that a certain number of S's are not coincident with a certain other number of P's.

Thus, of the four propositions U, Y, η, ω,

U is reducible to two A propositions,
Y is reducible to an A proposition,
η is reducible to O,
ω is truistic, and therefore useless.*

CHAPTER XVIII

V. (iii.) THE REDUCTION OF CATEGORICAL PROPOSITIONS TO STRICT LOGICAL FORM.

IN reducing a proposition to strict logical form, our first care must be to interpret the given statement as an 'extensive' proposition. Thus, if 'S is P' is naturally read on the predicative view, to the effect that the objects indicated by S possess the attribute P, it must be reinterpreted, so as to read as follows: 'The objects indicated by S are objects which possess the attribute P.' In the case of abstract propositions (statements, that is, with an abstract term as subject) the object indicated by the subject-term will be abstract, and the correct interpretation will be somewhat as follows: 'The abstract object indicated by the term S is identified with an abstract object indicated by the term P.' Thus, 'Mercy is twice blest' may be interpreted as 'Mercy is a twice-blest virtue.' It will then rank as a universal affirmative with distributed subject and undistributed predicate.

The singular proposition need cause no difficulty from the point of view we are here considering. For, as we have seen, it is but the limiting form of the universal proposition. 'This S is P' means 'All objects indicated by the term "this S" are objects indicated by the term P.' As a matter of fact, 'this S,' like a proper name, restricts the extension to one object, so that, while the proposition is singular, its subject-term is distributed. The singular proposition ranks, therefore, as a universal. It could not rank as a particular proposition for another reason. For 'some' means 'one at least,' and 'one at least' is not the same as 'one.' Hence 'This S is P' could not be brought under the form 'Some S's are P's,' for

* In the one case in which it might seem possible not to accept ω (namely, where S and P extensionally coincide, and both refer to one and the same individual) the use of the proposition would be inappropriate. If we wished to state such coincidence we should employ a singular proposition.

the latter form implies that one S *at least* is P, and this rendering goes beyond the meaning of the singular proposition.

Once a proposition is understood extensively, the main rules for reduction to strict logical form may be briefly formulated.

Putting a categorical proposition into strict logical form means, we may say, expressing it in one of the four typical forms, A, E, I, O. This involves :

1. Finding which is the true logical subject and which the true logical predicate.
2. Giving the subject its correct quantity-mark, either 'All' or 'Some.'
3. Giving the proposition its correct quality-mark, either 'is' or 'is-not.'

The second of these requisites, however, is subject to an important modification. We have seen that, for the purposes of a Formal treatment, the Singular Proposition ranks as a universal. Consequently we have a 'singular' form of the A proposition, the form typically represented by 'This S is a P.'

We have, then, as the two recognized forms of the A proposition—

1. 'All the S's are P's,' where 'All' is understood distributively.
2. 'S (singular) is a P.'

A difficulty frequently arises from the fact that propositions collective in meaning are presented in the ordinary distributive form. Thus, a sentence given to us in the form 'All the S's are P's' may be incorrectly expressed. A proposition, we know, is distributive if, on putting 'each of' in the place of 'all,' we find that the sense remains unaffected. The distributive use of 'all' being accepted as its correct Formal use, the word 'all' should be retained, in the logically stated proposition, whenever this substitution does not alter the sense. But if the substitution changes the sense, then 'all' is used collectively, and therefore must not be retained in the proposition when this is stated in logical form. In its place we must use a collective expression with a singular import,* so that the elaborated proposition will take the form 'S (singular) is a P' instead of 'All the S's are P's'; the essential defect of the latter expression in such a case being that it has

* Collective terms, like 'family' or 'regiment,' which refer to a collection of objects *qua* aggregate, may be either singular or general. Such terms as 'This regiment' or 'The Light Brigade' are singular collectives. 'Regiment' is a general collective. Collective terms, whether singular or general, are always used collectively with regard to the individuals of the group or kind of group specified. But general collectives are used disjunctively with regard to the various kinds or classes which constitute their denotation. 'Family' indicates the members collectively, but it denotes the various kinds of family disjunctively. A family is large or small, rich or poor. The term 'family' may correctly be predicated of *each* of these types taken apart from the rest.

distributive form but collective meaning. This distinction applies also to some propositions which are apparently particular. 'Some' may be used in a collective sense, as meaning, for instance, 'a handful of'; and, when so used, it should not appear as mark of quantity in the proposition logically expressed, but should be superseded by a collective expression. Thus,

'All these weeds choke the flower-bed'

should be transformed into

'This mass of weeds is a mass that chokes the flower-bed.'

So also 'Some soothing words appeased him' is only apparently an I proposition, for the 'some' is here used collectively. Reduced to proper logical form, the proposition would run :

'A string of soothing words is a thing that appeased him.'

Further difficulty is caused by propositions which refer to a single individual, but do not specify that individual, and therefore are not singular in the sense required to justify the use of the singular form. Examples of such propositions are :

'A friend of mine has gone abroad.'
'A ship was on fire.'
'An earthquake had occurred in Jamaica.'

A legitimate method of reducing sentences of this type would be to utilize x, the usual symbol for the unknown quantity, as follows :

'The friend whom I call X is a person who has gone abroad.'
'The ship X is a ship that was on fire.'
'The earthquake specified as X is an event that had occurred in Jamaica.'

In arguments this substitution would perhaps be convenient.

The third of the requisites of strict reduction also needs some words of comment. The quality-mark may take any one of several forms. If affirmative, it may be 'am,' 'art,' 'is,' or 'are.' If negative, 'am-not,' 'art-not,' 'is-not,' or 'are-not.' Thus, 'I am a man,' 'Thou art a woman,' 'We are human beings,' may be regarded as reduced propositions. They are all universal affirmatives.

It is important that the quality-mark should not be confused with the tense-mark. The quality-mark is the copula-mark, and has a strictly logical significance. Distinctions between present, past, and future belong to the predicate. Thus 'were' and 'were not,' 'will be' and 'will not be' are not strictly permissible as copula-marks.

Example.—'The *Drake* was in harbour. She is a splendid vessel, and will be the Admiral's flagship.'

Strict Logical Form : 'The *Drake* is a vessel which at the time we are speaking about was in harbour. The *Drake* is a splendid vessel. The *Drake* is a vessel destined to be the Admiral's flagship.'

Where the argument presupposes throughout either the past or the future tense, so that there is no danger of confusion, disregard of these logical requirements is apt to pass without protest, even from the flintiest logician. Logic owes much to the genius of language, and may waive its claims upon *form* not only graciously, but without inconsistency, so long as the yielding does not involve or lead up to any ambiguity or contradiction.

On the Reduction of Certain Ambiguous Expressions.

Any is an ambiguous expression. The following are characteristic meanings :

Affirmatively 'Any' = 'every.'

'Any man will tell you that' reduces, therefore, to 'All men are persons competent to tell you that.'

Negatively 'Not any' = 'no'; or else, 'not any' = 'not every' = 'some . . . not.'

Thus

'I have not any money' reduces to 'I am a person with no money.'

And

'Any excuse will not suffice' reduces to 'Not every excuse will suffice'—*i.e.*, 'Some excuses are-not adequate excuses.'

N.B.—In questions, 'Any' = 'Some.'

E.g., 'Have you any coppers about you ?'

'A few are' = 'Some are.'

'A few are not' = 'Some are-not.'

Thus 'A few were present' reduces to 'Some persons are persons who were present.'

'All . . . are not' is ambiguous. It may mean either 'All . . . are-not' or 'Some . . . are-not.' The latter gives the natural meaning ; for 'All . . . are not' = 'Not all . . . are' = 'Some . . . are-not.' It is with the express object of avoiding this ambiguity that we have used the hyphen in the case of the E proposition, 'All S's are-not P's.'

E.g., 'All men are not honest who say that they are.'
= 'Not all men are honest who say that they are.'
= 'Some men who say they are honest are-not honest men.'

So again, 'Every pun is not a joke' reduces to 'Some puns are-not jokes.'

In many cases the quantity-mark is lacking altogether where it is needed, and we have then to insert it. Propositions of this kind are known as Indesignate or Pre-indesignate Propositions; and in reducing them we may guide ourselves by the following rules:

1. An Indesignate Proposition is universal if, when it is read according to the predicative view, the predicate is found to be furnishing some element of the connotation, or else a property implied in the definition of the subject; for then P belongs to S as such.*

Examples:

'Angles in a semicircle are right angles.'

'Cows are ruminants.' (The connotation of 'cow' is here assumed to include, explicitly or implicitly, the mark 'ruminant.')†

2. If P is a problematic property of S, the proposition is obviously particular.

Examples:

'Frenchmen are vivacious.'

'Italians are musical.'

Reduced to logical form, these will be:

'Some Frenchmen are vivacious individuals.'

'Some Italians are musical individuals.'

Here the 'Some' is, of course, equal to 'most'; otherwise the indesignate form would be positively misleading. Still, 'most' is a mark of particularity.

3. If P is a characteristic property, the proposition is most naturally treated as universal. There is no logical necessity for our supposing that all ruminants have divided feet; but, so far as our experience goes, the chewing of the cud is invariably accompanied by the divided foot. Hence the proposition

'Ruminants have divided feet'

would appropriately reduce into

'All ruminants are creatures with divided feet.'

* *Cf.* Welton, 'A Manual of Logic,' vol. i., pp. 169-171. Mr. Joseph uses the term '*indefinite* judgement' ('An Introduction to Logic,' ch. viii., p. 156).

† The connotation of the subject-term in this proposition is, of course, not the meaning which it actually bears in the proposition. This meaning is a relatively indeterminate meaning which the predication of the term 'ruminant' serves to render more determinate (*vide* p. 121).

Exclusive and Exceptive Propositions.

An *exclusive* proposition denies a predicate of all save the members of a specified class.

Example.—'None but the free can obey.'
= 'All not-free persons are-not persons who can obey.'

Thus the logical equivalent of the exclusive is an E proposition with a negative term as subject.

An *exceptive* proposition affirms the predicate of the members of a whole class with the exception of those constituting a certain part of it.

Example : 'All is lost but honour'
= 'All possessions other than honour are lost possessions'—*i.e.*, an A proposition with a negative term as subject.

With regard to the reduction of exclusive propositions to strict logical form, we may provisionally adopt the following rule :

Express 'Only S's are P's' or 'S's alone are P's'
as E : 'All not-S's are-not P's' (preferably),
or as A : 'All P's are S's.'

Example.—'Only drakes are curly-tailed'
= 'All not-drakes are-not curly-tailed creatures,' E.
or 'All curly-tailed creatures are drakes.' A.

It is important to note that the statement 'Only S's are P's' does not imply that 'all S's are P's.' Granted that 'only heroes are wearers of the Victoria Cross,' it does not follow that all heroes wear it.

The particular proposition in which 'Some' is used in its exclusive sense is not strictly exclusive according to the definition given above, and its reduction is different from that of the exclusive universal to which the given definition alone, in strictness, applies. It resolves itself, in fact, into two independent propositions. Take, for instance, the proposition :

'Some only who promise keep their word.'

Here the logical function of the word 'only' precisely coincides with that of the expression 'not all'—*i.e.*, 'some . . . not.' The proposition therefore means 'Some who promise keep their word, but some do not'—*i.e.* :

$\left\{\begin{array}{l}\text{1. 'Some makers of promises are breakers of promises.' I.}\\ \text{2. 'Some makers of promises are-not breakers of promises.' O.}\end{array}\right.$

Propositions which are thus analysable into two or more independent propositions are technically called Exponible Propositions.

With regard to exceptive propositions, some difficulty is presented by those in which the excepted class is numerically characterized. Such Numerical Exceptives may be divided into two classes:

1. The excepted part may be individually indefinite.

Examples :

'All except a few were lost.'
= 'The proportion of persons lost to those not lost is a very large proportion.' A.

Again :

'All except seven were saved'
= 'The number of persons who were not saved is seven.' A.

2. The excepted part may be individually definite.

Example.—'All except A and B were lost.'

This should be treated as an A proposition, since in an argument it would have the force of a universal statement. When it is further stated that C was on board, we can definitely infer that he was lost. As regards logical form, the simplest rendering is reduction to the A proposition :

'All persons other than A and B were lost.'

Propositions in which an exceptive or exclusive phrase modifies some part of the *predicate* are not exceptive or exclusive propositions, and in reducing them there is no necessity for changing the phrase in question.

Examples :

'Except when they are naughty, children are invariably good.'
= 'All children are beings who are invariably good except when naughty.' A.
'Children are only good when they are asleep.'
= 'All children are beings that are good only when asleep.' A.
'It concerns no one but myself.'
= 'This is a matter that concerns no one but myself.' A.
'No human beings experience nothing but trouble.'
= 'All human beings are-not beings that experience nothing but trouble.' E.

On the Reduction of Non-propositional Sentences to Propositional and Strictly Logical Form.

In so far as optatives, interrogatives, etc. imply assertions, this implication can be brought out, and the sentence thus transformed into a sentence indicative, or proposition.

Optative.—'Would I were a cassowary!' = 'To be a cassowary is an object of my desire.'

Imperative.—'Beware the Jabberwock!' = 'The Jabberwock is a creature to be avoided.'

(Rhetorical) *Question.*—'Is thy servant a dog?' = 'Thy servant is-not a dog.'
'Am not I thine ass?' = 'I am thine ass.'

Interrogatives with indicative implications have negative force if they are in positive form, positive force if in negative form.

Exclamation.—'Just the place for a Snark!' = 'This place is a promising spot for snarks.' To the Bellman's exclamation the crew might have replied: 'We don't believe it!' In that case they would have taken the remark as a statement that had no sufficient ground. This would have stamped it as a Proposition.

On the Reduction of Given Sentences to Strict Logical Form.

Practical Hints on what is implied by 'strict logical form':

(*a*) As regards quantity and quality.
Use 'all,' and not 'every,' 'each' or 'any.'
Use 'all . . . are-not' for the negative universal, and not 'no . . . are' or 'all . . . are not.'
Use 'some,' and not 'several' or 'many,' etc.

(*b*) As regards the constitution of subject and predicate terms.
These should be expressions that have a meaning in themselves, and they should also admit naturally of being qualified by 'all' or 'some' (except in the case of Singular Propositions)—*e.g.*, all 'glittering things,' not all 'that glitters.'

Examples.—Express each of the following sentences in such (strict) logical form as reproduces most nearly the natural meaning of the sentence:

1. Not all who are called are chosen.
= Some persons called are-not persons chosen. O.
2. Not all your efforts can save him.
= The sum-total of your efforts is-not a force that can save him. E.
3. All kings are not wise.
= Some kings are-not wise persons. O.
4. Every bullet does not kill.
= Some bullets are-not fatal missiles. O.

5. Few men succeed in life.
   = The number of men who succeed in life is-not a large number.* E.
6. All my guesses but two were correct.
   = The number of guesses which I correctly made is the total number diminished by two. A.
7. Great is Diana of the Ephesians.
   = Diana of the Ephesians is a great goddess. A.
8. An honest man's the noblest work of God.
   = Honest manhood is the noblest work of God. A.
9. He envies others' wealth who has none himself.
   = All non-wealthy persons are persons envious of others' wealth. A.
10. Only doctors understand this subject.
    = All non-doctors are-not persons able to understand this subject. E.
11. The more, the merrier.
    = All new-comers are mirth-increasers. A.
12. He has no home but Athens.
    = All places outside of Athens are-not homes for him. E.
13. A few Greeks vanquished the vast army of Darius.
    = A mere handful of Greeks is the force that vanquished the vast army of Darius. A.
14. A little knowledge is a dangerous thing.
    = A smattering of knowledge is a dangerous thing. A.
15. The Romans conquered the Carthaginians.
    = The Roman power is a power that conquered the Carthaginians. A.
16. The angles of a triangle are equivalent to two right angles.
    = The sum of the angles of a triangle is a quantum equal to two right angles. A.
17. Two blacks won't make a white.
    = All possible combinations of two blacks are-not combinations producing whiteness. E.
18. My friend plays golf.
    = My friend is a golfer. A.
19. Not all the gallant efforts of the veterans availed anything.
    = The gallant fighting of the veterans is-not fighting that availed anything. E.
20. Few dogs are not fond of fetching sticks.
    (Force Affirmative)
    = All dogs not fond of fetching sticks are exceptional dogs. A.

* As the original proposition is negative in meaning, the logical form must be sympathetically negative.

21. A few dogs are fond of cats.
= Some dogs are fond of cats. I.
22. Only a few politicians are statesmen.
(Exponible)
= Some politicians are statesmen and some politicians are-not statesmen. Φ.
23. Only ignorant persons hold such opinions.
(Exclusive)
= All non-ignorant persons are-not persons holding such opinions. E.
Or—
= All persons holding such opinions are ignorant persons. A.
24. Some men are not incapable of telling falsehoods.
(Force Affirmative)
= Some persons are persons quite capable of telling falsehoods. I.
25. Scotchmen are level-headed.
Is 'level-headed' part of the connotation of 'Scotchman'? Obviously not, unless by 'Scotchman' we mean 'typical Scotchman.' It appears to be a problematic property. We therefore have as the strict logical form :
= Some Scotchmen are level-headed persons. I.
26. To be or not to be, that is the question.
= Whether life is worth living or not is the question I must answer. A.
27. Scarcely anyone got through.
= The number of persons who passed is a very small number. A.
28. Men are not what they were.
= The manhood of to-day is-not manhood as it used to be. E.
29. The side and diagonal of a square are incommensurable.
= The ratio of the side of a square to its diagonal is an incommensurable ratio. A.
30. The only interested persons are candidates and examiners.
= All interested persons are either candidates or examiners. A.
31. Am I my brother's keeper ?
(Rhetorical question implying negative statement. The logical subject here is 'The guardianship of my brother.')
= Looking after my brother is-not my business. E.

32. Fain would I climb, but that I fear to fall.
= My wish to climb is a wish checked only by my fear of falling. A.
(N.B.—'But' has the force of 'only.')
Other alternative renderings are:
= All my impulses to climb are impulses inhibited by the single fear of falling. A.
Or—
= My fear of falling is the only thing which prevents me from climbing. A.

33. All the travellers were not provided with passports.
= Some travellers in the company specified are-not travellers who were provided with passports. O.

34. All but Noah and his family were drowned.
= All persons of Noah's day who were not of his household are persons who were drowned. A.

35. Afflictions are often salutary.
= Some visitations of affliction are salutary experiences. I.

36. All these claims upon my time overpower me.
= This multitude of claims upon my time is a burden that overwhelms me. A

CHAPTER XIX.

V. (iv.) THE OPPOSITION OF PROPOSITIONS.

IT is customary to say that two propositions are logically *opposed* when, having the same subject and predicate, they differ in quantity, in quality, or in both quantity and quality. From this point of view the relation between *any* two of the propositions A, E, I, and O is treated as an Opposition.

The definition, though convenient, is superficial and arbitrary, and it necessitates using the term 'Opposition' in an entirely technical sense, for the relation between 'All men are mortals' and 'Some men are mortals' is, according to the definition given above, an opposition.

A much sounder method is to guide ourselves by principle, and to hold that propositions are in opposition only when they violate the requirement of non-contradiction. Where this requirement is respected, as in the so-called 'subcontraries' and 'subalterns,' and the relation ceases to be one of opposition, we may suitably speak of Subcontrariety and Subalternation, but not of Sub-

contrary Opposition or of Subaltern Opposition.* The only two forms of genuine opposition between propositions are known as Contradictory Opposition and Contrary Opposition respectively. They are relations between genuine opposites, because in each case we have a pair of propositions juxtaposed which cannot logically be entertained together. If one of them is accepted, the other must be rejected.

We proceed now to the more minute consideration of the four propositions, A, E, I, O, in respect of those relations between them which are customarily known as Oppositions. The relations we have to deal with are the following:

(*a*) *Contradictory Opposition:*
A. All S's are P's.)(† O. Some S's are-not P's.
E. All S's are-not P's.)(I. Some S's are P's.

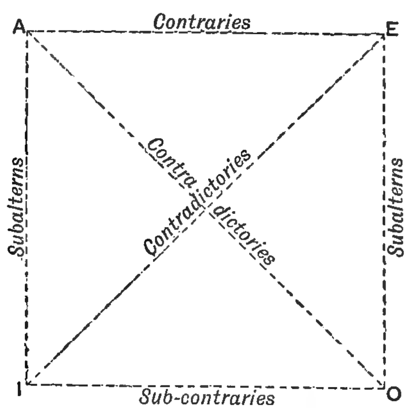
(*b*) *Subcontrariety:*
I. Some S's are P's.)(O. Some S's are-not P's.

(*c*) *Contrary Opposition:*
A. All S's are P's.)(E. All S's are-not P's.

(*d*) *Subalternation:*
A. All S's are P's.)(I. Some S's are P's.
E. All S's are-not P's.)(O. Some S's are-not P's.

These last two pairs are known as Subalterns. A is usually called the Subalternans of I, and I the subalternate of A. Similarly, E is the subalternans of O, and O the subalternate of E.

The above-named relations may be diagrammatically represented in what is known as the Square of Opposition:



* Mr. Joseph ('An Introduction to Logic,' chap. ix., p. 207, footnote) draws attention to Aristotle's own statement on this point: 'For *some are* is only verbally opposed to *some are not*' (Anal. Pri. B., xv. 63*b*, 27). At the same time Mr. Joseph holds that if subcontraries 'are not opposed, they are anyhow contrasted, and that may justify their continued inclusion' among 'forms of opposition.'

† We adopt the grammatical sign)(as signifying any kind of so-called Opposition.

Contradictory Opposition.

The Rules or Laws of Contradictory Opposition are identical with the formulæ already given (p. 98) for the Principle of Non-Contradiction :

Rule 1. Contradictories cannot both be accepted.
Rule 2. Contradictories cannot both be rejected.

These rules cannot be proved by means of principles more ultimate than themselves. But they admit of the most cogent proof possible, in another sense of the term 'proof.' If the proof of a law lies in its indispensableness, then the Rules of Contradictory Opposition may be proved to the hilt. For if they do not hold good, there is no such thing as consistency, and it becomes unreasonable to think at all. Moreover, the very attempt to deny these laws refutes itself. The statement 'Contradictories can be accepted together' implies that the contradictory of this very statement may itself be accepted. Finally, as we have already seen, the Principle of Non-contradiction being a Law of Thought, to violate it is absolutely impossible. To have succeeded in doing so would be to have thought the unthinkable.

If a proposition is stated in the general form 'S is P,' its contradictory may be stated in the form '$\overline{\text{S is P}}$' or 'Not (S is P).' Some care is needed in the interpretation of this contradiction-formula. It is misinterpreted, for instance, whenever the denial is directed upon some assumption which the proposition 'S is P' takes for granted, instead of being directed upon the proposition itself.

Thus, the denial of the statement that 'Japan's President is revered by his people' cannot be identified with the assertion that 'The ruler of Japan is not its president.' For this simply denies the implicit assumption that Japan is represented by a president; it does not deny the given proposition. So, again, 'The Mikado is the President of the Japanese Republic' is not expressed by asserting that the realm of Japan is not a presidency. This denies, not the proposition itself, but an assumption which the statement has taken for granted.

The denial of 'S is P' must then itself be a statement concerning the relation of S to P, and not the denial of some assumption which the assertion 'S is P' presupposes. But insistence on this should not be carried to the point of asserting that the denial of 'S is P' is simply 'S is-not P,' and that the clumsier form 'Not (S is P)' may therefore give way to the form 'S is-not P,' as the typical form of the denial of 'S is P.'

Those who support this equivalence appear to take the singular proposition as representative of all the rest. The statement that 'Socrates is-not wise' may be taken as the denial of 'Socrates is

wise,' and therefore as the equivalent of 'Not (Socrates is wise).' But when quantitative relations are involved, or when the proposition is indesignate,* nothing but confusion can result from equating 'Not (S is P)' to 'S is-not P.'

Let us consider the case where 'S is P' is indesignate, and take, by way of illustration, the proposition 'Frenchmen are vivacious.' It is surely misleading to assert forthwith that the denial of 'Frenchmen are vivacious' is 'Frenchmen are-not vivacious.' For if the indesignate proposition is here taken as particular, then, in denying 'Some Frenchmen are vivacious' by 'Some Frenchmen are-not vivacious,' we should be mistaking what is a mere subcontrary opposition for logical denial.

We conclude, then, that 'Not (S is P)' or '$\overline{\text{S is P}}$,' and not the declaration 'S is-not P,' is the only general form which the denial of 'S is P' can logically take; and that when this denial is specified in definite propositional form, it is a statement having the same subject and predicate as the proposition which it denies.

As regards the more special application of the laws of Contradictory Opposition, since A and O are contradictories, and also E and I, we see that, in the case of the first of the two laws, it consists in arguing that if a certain proposition is accepted, then another proposition, *having the same subject and predicate*, but *differing from it in quality and quantity*, cannot be accepted; and so, *mutatis mutandis*, for the application of the second law of Contradictory Opposition.

Subcontrariety.

The Rules of Subcontrariety:

Rule 1. Of two Subcontraries, if one is accepted, there is no logical ground either for the acceptance or for the rejection of the other.

Rule 2. Of two Subcontraries, if one is rejected, the other must be accepted.

Proof of Rule 1.—To accept the I proposition is to state that one S at least is a P, while *making no statement* about *all* the S's. In accepting I we do not state or imply either that the proposition 'All the S's are P's' must be rejected, or that it must be accepted. The acceptance of I does not involve either the rejection or the acceptance of A. Consequently the acceptance of I does not involve either the acceptance or the rejection of the contradictory of A. But the contradictory of A is O. Therefore the acceptance of I does not involve either the acceptance or the rejection of O. Similarly it may be shown that the acceptance of O does not involve either the acceptance or the rejection of I.

* Vide p. 165.

Proof of Rule 2 :

If I is rejected, E must be accepted (Contradiction).
If E is accepted, O must be accepted (Identity, *vide infra* on Subalternation).
∴ If I is rejected, O must be accepted.

So, again,

If O is rejected, A must be accepted (Contradiction).
If A is accepted, I must be accepted (Identity).
∴ If O is rejected, I must be accepted.

The rules of Subcontrariety, as above enumerated, cease to hold when, in place of the undistributed reading for 'Some,' we substitute either the exclusive or the indefinite reading, or the reading which expresses a limitation of knowledge.

If 'Some' ≡ 'One at least, but not all' (exclusive), then not only are the I and O propositions not inconsistent—*i.e.*, not only *can* they be accepted together—but if one of them is accepted, then the other *must* also be accepted. For to say that Some S's are P's is to state that not all the S's are P's ; and to say that Some S's are-not P's is to state that not all the S's are-not P's—*i.e.*, that Some S's are P's.

If 'Some' ≡ 'One at least, possibly all' (indefinite), the I and O propositions *cannot* be accepted together. If the proposition 'One S at least is a P, possibly all the S's are P's' is accepted, then the proposition 'One S at least is not a P, possibly all the S's are not P's' must be rejected, and *vice versa*. For the statement 'One S at least is a P' is inconsistent with the statement 'possibly all the S's are-not P's'; and the statement 'One S at least is-not a P' is inconsistent with the statement 'Possibly all the S's are P's.' If I state that at least one member of a class is certain to pass (or not to pass) a specified examination, I cannot consistently go on to say that all may possibly fail (or pass).

In the case also of the reading 'One at least, but not assuredly (or "knowedly") all,' there is an inconsistency involved in accepting the I and O propositions together as true, though the inconsistency is here less easy to detect. Thus, as a despairing member of a class of examinees, I might assert : 'One at least of us will fail, though I do not know that we all shall,' and then, meeting a fellow-candidate in whose ability I have full confidence, I might say to him : 'One at least of us will pass,* though I don't know that we all shall,' and my two statements would be verbally compatible, for, knowing that I shall fail, I certainly *do not know* that we all shall pass. But my second statement, if not actually disingenuous, at least cannot be said to express a *limitation of knowledge*. My knowledge as to the truth of the statement that we shall all pass is

* Here 'pass' is used as an abbreviation for 'not fail.'

not limited if I know that one at least will fail. But it is precisely this limitation of knowledge which the reading we are considering professes to express. There is, therefore, a contradiction involved, on this reading, in accepting the I and O propositions as true together. To harmonize the two I must exchange *statement of limitation* (of knowledge) for *limitation of statement.* I must adopt the undistributed reading, and say : 'One at least of us will pass, but more than that I don't say ; about the question of our all passing *I make no statement.*'

Contrary Opposition.

The Rules of Contrary Opposition are the following :

Rule 1. Of two contraries, if one is accepted the other must be rejected.

Rule 2. Of two contraries, if one is rejected, there is no logical ground either for the acceptance or for the rejection of the other.

These rules may now be justified as follows :

1. Contraries cannot be accepted together ; for, supposing this possible, then
   As A is accepted, I must be accepted (Principle of Identity : *vide infra* on Subalternation).
   And, as E is accepted, O must be accepted (*ibid.*).
   ∴ A and O are accepted together, as also are E and I.
   But this is impossible, by the Principle of Non-Contradiction.
2. It may be quite consistent to reject both contraries ; for their contradictories (which are subcontraries) may both be accepted. See Subcontrariety, Rule 1.

The question may be asked why, in controversy, it is preferable to attempt the refutation of a statement by proving its contradictory rather than by proving its contrary. The simple answer is that the contrary of a proposition is harder to prove than its contradictory. But this is not all. The additional element of assertion which the strengthened opposition superinduces upon the pure contradiction not only plays no part in the refutation of the statement, but may give the adversary an opening for counter-attack which he would not have possessed had the refutation taken place through contradiction simply.

Subalternation.

The following are *the Rules of Subalternation :*

Rule 1. If the universal proposition is accepted, the particular proposition must also be accepted.

If A is accepted, I must be accepted (Principle of Identity).

If E is accepted, O must be accepted (*ibid.*).

Rule 2. If the universal proposition is rejected, there is no logical ground either for accepting or for rejecting the particular proposition.

Proof of Rule 2 :

The acceptance of O affords no logical ground either for the acceptance or for the rejection of I (Subcontrariety, 1). But the acceptance of O is logically equivalent to the rejection of A (Contradiction).

∴ the rejection of A affords no logical ground either for the acceptance or for the rejection of I.

Rule 3. If the particular proposition is accepted, there is no logical ground either for accepting or for rejecting the universal proposition.

For a justification of this Rule, see above, p. 174 (Proof of the First Rule of Subcontrariety).

Rule 4. If the particular is rejected, then the universal must also be rejected.

Proof : If I is rejected, E must be accepted (Contradiction).

∴ A must be rejected (Contrariety).

If O is rejected, A must be accepted (Contradiction).

∴ E must be rejected (Contrariety).

The following Table of Opposition will serve to summarize the results of the previous discussion :

	Given	A	E	I	O
1	A accepted	accepted	rejected	accepted	rejected
2	A rejected	rejected	neither	neither	accepted
3	E accepted	rejected	accepted	rejected	accepted
4	E rejected	neither	rejected	accepted	neither
5	I accepted	neither	rejected	accepted	neither
6	I rejected	rejected	accepted	rejected	accepted
7	O accepted	rejected	neither	neither	accepted
8	O rejected	accepted	rejected	accepted	rejected

N.B.—'Some' = 'One at least, but not statedly all.' By 'neither' we mean that there is no logical ground either for the acceptance or for the rejection of the proposition in question.

The Opposition of Disjunctive Propositions.

The contradictory of the disjunctive proposition 'Either P or Q' in its non-exclusive form is 'Neither P nor Q.' When the 'Either . . . or' is read exclusively its contradictory is 'Either both P and Q, or else neither P nor Q.' In this second case 'Either one or the other' is contradicted by 'Either both or else neither.' According to the exclusive view, 'Neither P nor Q' could not be accepted as the contradictory of 'Either P or Q,' since both propositions could be rejected, the accepted statement being 'Both P and Q.'

Again, according to the exclusive reading, 'Neither P nor Q' and 'Both P and Q' may both be considered as contraries of the disjunctive proposition 'Either P or Q.' For 'Either P or Q' and 'Neither P nor Q' cannot both be accepted, and yet, as we have already seen, it may be quite consistent to reject both. Similarly, 'Either P or Q' and 'Both P and Q' cannot both be accepted, but it may be quite consistent to reject them both—namely, when the accepted statement is 'Neither P nor Q.'

The Opposition of Hypotheticals.

The Scheme of Opposition here takes different forms corresponding to the different types of Hypothetical Proposition.

1. (*a*) *The Apodeictic Scheme* (Formal).

H_a. If P is accepted, the acceptance of P is necessary.
H_e. If P is accepted, the rejection of P is necessary.
H_i. If P is accepted, the rejection of Q is not necessary.
H_o. If P is accepted, the acceptance of Q is not necessary.

Example.—H_a. If 'All S's are P's' is accepted, the acceptance of 'All $\bar{S}$'s are $\bar{P}$'s' is necessary.

H_e. If 'All S's are P's' is accepted, the rejection of 'All $\bar{S}$'s are $\bar{P}$'s' is necessary.

H_i. If 'All S's are P's' is accepted, the rejection of 'All $\bar{S}$'s are $\bar{P}$'s' is not necessary.

(*I.e.*, Either 'All $\bar{S}$'s are $\bar{P}$'s' must be accepted, or else there is no ground either for accepting or for rejecting it.)

H_o. If 'All S's are P's' is accepted, the acceptance of 'All $\bar{S}$'s are $\bar{P}$'s' is not necessary.

(*I.e.*, Either 'All $\bar{S}$'s are $\bar{P}$'s' must be rejected, or else there is no ground either for accepting or for rejecting it.)

1. (*b*) *The Apodeictic Scheme* (Material).

H_a. If P is true, then implicitly Q is true.
H_e. If P is true, then implicitly Q is not true (*i.e.*, is false).
H_i. If P is true, then it is not implied that Q is false.
(*I.e.*, Either it is implied that Q is true, or else there is no implication as to the truth or falsity of Q.)
H_o. If P is true, then it is not implied that Q is true.
(*I.e.*, Either it is implied that Q is false, or else there is no implication as to the truth or falsity of Q.)

2. *The Assertoric Scheme.*

Affirmative universal : If P, then (*i.e.*, in that case it is a matter of ascertainable fact that) in all cases we have Q.
Or, more briefly : If P, then in all cases Q.
Negative universal : If P, then in no case Q.
Affirmative particular : If P, then in some cases Q (*i.e.*, In some of the possible instances of P's presence Q will also be present).
Negative particular : If P, then in some cases not Q.

In discussing the Opposition of Hypotheticals, Dr. Keynes contends that, where the hypothetical is *assertorial** (the truth of Q following *upon* the truth of P, but not *necessarily from* it), the true contradictory of 'If P, then Q' is 'P, and not Q.'† He argues that, if the antecedent is not categorically posited in the contradictory, and the contradictory of 'If P, then Q' is stated in the form 'If P, then not Q,' we are not in a position to say that one or other of these must be false. For let us suppose that, as a matter of fact, P happens not to be true. Then no consequent which has P as its antecedent can be definitely labelled as false. When once the assumption represented by the 'if' clause is discredited, we may say anything we like on the imaginative supposition that the antecedent which the 'if' introduces is true. Thus, to take Dr. Keynes' own example,‡ the two propositions 'If this pen is not cross-nibbed, it is corroded by the ink,' and 'If this pen is not cross-nibbed, it is not corroded by the ink' cannot be contra dictories, since, if, as a matter of fact, the pen happens to be cross-nibbed, we cannot regard either proposition as false.

The difficulties which beset the foregoing argument disappear so soon as we insist—(1) on testing contradictories by the principle

* Dr. Keynes makes use of the word 'assertoric.'
† 'Studies and Exercises in Formal Logic,' fourth edition, Part II., ch. ix., pp. 262, 263.
‡ *Ibid.*, p. 267, footnote.

of Logical Consistency instead of by an appeal to the Law of Excluded Middle,* and (2) on respecting the true nature of the Hypothetical Proposition as a connexion of *possibilities.*

Dr. Keynes maintains that, when the antecedent in the 'if' clause is not credible in the light of actual facts, we cannot say of the two statements 'If P, then Q,' 'If P, then not Q' that one or the other must be false. But the interconsistency of the two statements does not depend on the testimony of actual fact. The question is whether the *acceptance* of the one statement is consistent with the acceptance of the other. When once the connexion of possibilities asserted by the proposition 'If P, then Q' is accepted, the further connexion of possibilities asserted by the proposition 'If P, then not Q' must be rejected.† From the point of view of consistent thinking the relation in which the possibilities may stand to realities is entirely irrelevant.

And this brings us to our second criticism, which is that Dr. Keynes fails to respect the nature of the Hypothetical Proposition as a connexion of possibilities. For a main objection that he brings against the *hypothetical* contradictory is that we can conceive cases in which the antecedent clause in both the contradictories cannot be seriously intended as an assumption at all. Thus, when I say 'If this pen is not cross-nibbed,' though all the time aware that the pen *is* cross-nibbed, what I am really saying is 'If this pen is *not* cross-nibbed, as I know it *is*, then . . .,' etc. But the hypothetical proposition, so understood, no longer asserts a connexion of possibilities. It no more deserves the title of 'hypothetical' than the proposition 'Some S's are P's' ('some' being taken to mean 'one at least, possibly all') deserves the title of 'particular' when the 'possibly all' is replaced by 'actually all' (*vide* p. 156).

When Mr. Grimwig in 'Oliver Twist' says 'If ever that boy returns to this house, sir, I'll eat my head,' he does not—argues Dr. Keynes—intend the antecedent to be taken seriously; for what he really means is that the boy is quite certain not to return. But in this case—and the reasoning holds good for all the parallel instances Dr. Keynes adduces—the proposition is hypothetical in form only, not in function. It is a 'rhetorical' hypothetical, of which the true form is the categorical statement: 'That boy's return is an event as impossible as my eating my own head.'

We conclude, then, that the objection to the hypothetical contradictory has not been sufficiently justified.

* The disjunctive statement that 'of two contradictory propositions one must be false' is, in our view, an expression of the Law of Excluded Middle (*vide* p. 99).

† Dr. Keynes' qualification of hypothetical propositions as 'true' and 'false' is a further difficulty in his treatment of this problem. In the proposition 'If P, then Q' is it not only P and Q that can be either true or false?

EXAMPLES.

Example 1.—Give all the logical 'opposites' of the proposition, 'All officers are citizens.'

By its logical 'opposites' are meant the corresponding propositions, in the forms E, O, and I, which have the same subject and predicate, and are related to it respectively as its contrary, contradictory, and subaltern :

Contrary, E : All officers are-not citizens.
Contradictory, O : Some officers are-not citizens.
Subaltern, I : Some officers are citizens.

Example 2.—Give the logical 'opposites' of the proposition, 'It is too late to mend.'

Opposition presents a certain difficulty in the case of singular propositions. Thus, it is not at first sight quite obvious whether the counter-assertion 'It is-not too late to mend' should be regarded as the contrary or as the contradictory of the given proposition. That it is an E proposition, whereas the given proposition is an A proposition, constitutes a *Formal* ground for reckoning it as the contrary. But this merely technical argument has no weight as against the contention, based on logical principle, that the propositions must be contradictories, since they cannot both be rejected. The contradictory of 'It is too late to mend' is, therefore, 'It is-not too late to mend.' There is no technical contrary, though the proposition 'It is too soon to mend' would be the natural contrary. There is no subaltern.

If, however, there is what Professor Keynes has called 'secondary quantification,' then, though the propositions will no longer be opposites in the sense already defined, we may apply the Square of Opposition in a slightly modified form.

We have secondary quantification whenever the attribution of a predicate to a subject is limited with reference to times or conditions. Thus, in the following sentence from the opening of 'The Pied Piper of Hamelin'—

'When begins my ditty—
Almost five hundred years ago—
To see the townsfolk suffer so
From vermin—'twas a pity'—

the first two lines express a secondary quantification of the predicate, and we have secondary quantification also in the singular proposition, 'It is never too late to mend.'

Here the Contradictory is: 'It is sometimes too late to mend.'

The Contrary is : 'It is always too late to mend.'

And the Subalternate is : 'It sometimes is-not too late to mend.'

Example 3.*—Give the contradictory of the U proposition.

The U proposition takes the form, 'All S's are P's and all P's are S's.' The contradictory of this is 'Either some S's are-not P's, or some P's are-not S's.' If either of these alternatives is accepted, the original proposition must be rejected; and, on the other hand, if the latter is rejected, one, at least, of these alternatives must be accepted.

We may observe that we cannot say that 'Some S's are-not P's' is *a* contradictory of 'All the S's are all the P's,' for it may be quite consistent to reject both propositions, and to accept the proposition, 'Some P's are-not S's.'

What is true of the U proposition holds good of all compound propositions. A compound proposition can have only *one* contradictory—not more than one—and the contradictory must affirm a number of alternatives, of which one or other must be accepted if the original proposition is rejected.

Example 4.—Test the following: 'Epimenides says that the Cretans are liars, and Epimenides is a Cretan. Therefore what he says is not true. Therefore the Cretans are not liars. Therefore Epimenides is not a liar, and what he says is true.'—Fallacy of Mentiens.

This argument obviously involves a fallacy, for a proposition is here deduced from its own contradictory. 'What Epimenides says is true' is deduced from 'What Epimenides says is not true.'

We may point out the fallacy most clearly as follows:

The following are the different propositions involved in the argument:

(*a*) Epimenides says that the Cretans are liars.
(*b*) Epimenides is a Cretan.
(*c*) Therefore what he says is not true.
(*d*) Therefore the Cretans are not liars.
(*e*) Therefore Epimenides is not a liar.
(*f*) Therefore what Epimenides says is true.

Let us concentrate attention on the logical connexion between (*c*) and (*d*). If we can show that this connexion is logically invalid when (*c*) and (*d*) are so interpreted as to free the rest of the argument from fallacy, we shall have sufficiently proved the logical invalidity of the given argument.

The logical connexion in question depends on the assumption that what Epimenides says in (*a*) finds its true contradictory in (*d*).

Can we, then, find out without ambiguity—

(1) What Epimenides does say in (*a*);
(2) What (*d*) means?

* After Dr. Keynes.

We can do neither of these things. But we can find out what Epimenides *must* mean in (*a*) if there is to be no fallacy in the inference of (*c*) from (*b*), and what (*d*) *must* mean if there is, again, to be no fallacy in the inference of (*f*) from (*e*).

For (*c*) only follows from (*b*) provided that by (*a*) we mean 'Epimenides says that *all* statements made by Cretans are false statements.' Unless (*a*) means this, the acceptance of (*b*) does not necessitate the acceptance of (*c*).

Similarly, (*f*) only follows from (*e*) provided that by (*d*) we mean 'Therefore, *all* statements made by Cretans are-not false statements.' Unless (*d*) means this, the acceptance of (*e*) does not necessitate the acceptance of (*f*).

If, then, there is to be no fallacy in the above-mentioned inferences, the original statement of Epimenides must be 'All statements made by Cretans are false statements'; and the statement (*d*), which is put forward as its contradictory, must take the form, 'All statements made by Cretans are-not false statements.'

The sole remaining question, then, is whether the proposition 'All statements made by Cretans are-not false statements' is the true contradictory of 'All statements made by Cretans are false statements.' It is obviously not its contradictory, but its contrary, the true contradictory being 'Some statements made by Cretans are-not false statements.'

What we have shown, then, is that the logical connexion between (*c*) and (*d*) is invalid when the interpretations given to (*c*) and (*d*) are the only meanings that render the rest of the argument free from fallacy. The argument, therefore, contains a fallacy which can be driven home in the manner above described.

VI.

IMMEDIATE INFERENCE.

CHAPTER XX.

IMMEDIATE INFERENCE.

Formal Inference and its Logical Principle.

FORMAL Inference is reasoning from accepted statements or premisses to conclusions implied in them. The Principle of Formal Inference may be called *the Law of Formal Validity*. It may be formulated as follows :

If a given proposition or set of propositions is accepted, then the further propositions which are implied in what is thus admitted, these, and these only, must also be accepted ; and the further propositions which are in contradiction with any one of the admitted propositions, or with any one of their implications, these, and these only, must be rejected.

The Law of Formal Validity is no new addition to the Laws of Thought. It simply interprets the Laws of Formal Identity and Non-contradiction as formulated in relation to Inference.

Identity, in its relation to Inference, is a matter of Implication. If one statement is *implied* in another, the two must belong to one and the same identical system. This systematic intimacy between them constitutes their logical Identity. *The Law of Formal Identity* in its relation to Inference may be formulated as follows :

If a given proposition or set of propositions is accepted, then the further propositions which are implied in what is thus admitted, these, and these only, must also be accepted.

So the Law of Non-contradiction in its relation to Inference may be formulated thus :

If a given proposition or set of propositions is accepted, then the further propositions which are in contradiction with any one of the admitted propositions, or with any propositions implied by them, these, and these only, must be rejected.

The Law of Formal Inference is sometimes stated in the form of the concise though negative precept 'Not to go beyond the premisses.' We may connect this injunction with the Law of Formal Validity by showing that in going beyond the premisses we necessarily contradict certain propositions implied by these

premisses. Thus, from the accepted statement 'All S's are P's' we might conclude that 'All P's are S's.' In drawing this conclusion we should be going beyond the accepted premiss, though we should not be contradicting it. But though we do not contradict the premiss 'All S's are P's' when we accept the conclusion 'All P's are S's' as if this were an inference from it, we do thereby contradict certain implications of that premiss. Thus, when we posit 'All S's are P's'—*i.e.*, 'All S's are some P's'—we expressly don't state anything about all the P's. Hence no statement dealing with all the P's is implied in the original statement. Consequently, no statement dealing with all the P's can be disimplicated from the original statement. But the incorrectly drawn conclusion tells us that at least one statement dealing with all the P's can be disimplicated from the original statement. Thus we see that the proposition '"All P's are S's" is implied in the original statement "All S's are P's"' contradicts an implication of the accepted premiss, and must be rejected as inconsistent with it. (It is not, of course, the proposition 'All P's are S's' which contradicts an implication of the accepted premiss. Were this the case, we should be compelled, after accepting 'All S's are P's,' to reject 'All P's are S's' as inconsistent with it.)

We see, then, that, in going beyond the premiss, we have disregarded the Law of Validity, and fallen, at one point at least, into meaningless self-contradiction.

The most simple expression of the Principle of Formal Identity considered as a principle of Inference, an expression but one degree removed from the blank formula of Tautology—'If A is accepted, then A is accepted'—is provided by the First Law of Subalternation, the law which states that if the universal proposition is accepted, the particular proposition of the same quality must be accepted also.

The attempt to prove this law is instructive, as it serves to bring out the fact that the Principle of Formal Identity cannot be proved by means of the Principle of Non-contradiction.

Let us suppose that in accepting A we do not thereby logically bind ourselves to accept I. On this supposition, when we accept A we disable ourselves from rejecting the contradictory of I. But this disability involves us in a logical inconsistency, since A and E are contraries.

This proof, however, presupposes the truth of the law it is endeavouring to prove; for it assumes a law of Contrary Opposition which can be proved only by the help of the very law of Subalternation which we are considering. We have therefore committed the fallacy of reasoning in a circle. Hence the Principle of Identity, in this its simplest form, cannot be proved by means of the Principle of Non-contradiction.

Inference may be mediate or immediate. With Mediate Inference, or Syllogism, we shall be concerned in due course. We propose to make a beginning with the study of Immediate Inference. An Immediate Inference may be defined as the inference from the acceptance or the rejection of a proposition to the acceptance or the rejection of a further proposition on the sole basis of the Laws of Identity and Non-Contradiction. Thus we see that *the relations of 'Opposition' between the four propositions A, E, I, O are established through processes of Immediate Inference.* There is at least one other form of immediate inference, as above defined, which we shall presently consider.

It is customary to include, under the name of *Eductions,* two *processes* of inference of which one alone, as we shall see, is an immediate inference according to the definition as above stated. These two processes are respectively known as Obversion and Conversion. Of these Conversion alone can strictly be called an immediate inference. All other so-called immediate inferences—*e.g.*, Contraposition and Inversion—simply involve alternating repetitions of Conversion and Obversion. We should not, therefore, refer to Inversion, for instance, as an eduction. The two sole eductions are Obversion and Conversion.

Educts—*i.e.*, the propositions inferred through processes of Eduction—may be either 'strong' or 'weak.' A *strong* educt is one which, for purposes of Inference, may be taken as equivalent in meaning to the proposition from which it is inferred. In order to be 'strong' an educt must be of the same *quantity* as the original proposition.

A *weak* educt is one which presents the meaning of the original proposition in a weakened form, and therefore cannot be substituted for the original proposition without weakening the content. Whenever a proposition and its educt differ in quantity, the educt is *weak.* Thus, if the original proposition be 'All candidates are-not examiners,' then 'All examiners are-not candidates' is a strong educt, and 'Some non-candidates are examiners' is a weak educt. We shall lay especial stress on the strong educts, as, from the point of view of Inference, they are the more serviceable and important.

The processes of Immediate Inference and Eduction have, from our present strictly Formal point of view, a purely Formal interest. We are solely interested in discerning what are the logical implications of a single proposition. The whole attention being concentrated on the validity of the reasoning, the matter of the proposition ceases to interest us, and, as an important consequence, the whole question of the existential import of the propositions involved in the inference is appropriately ignored (*vide* p. 145). It is, of course, always possible to concentrate interest on the premiss or premisses of an inference as material evidence. But in that case,

as we have already appealed to knowledge, more or less systematic, such knowledge must inform us whether the subjects of our propositions exist or not, and, if existent, in what sense they exist. There seems to be no ground for perplexing a purely Formal treatment of the kind we are at present undertaking with a theory of existential implication.

Our sole concern is with Formally stated propositions and the inferences that can be necessarily drawn from these. We therefore assume, in our treatment of Eduction, the extensive view of Import and the fourfold division of propositions according to quantity and quality which is based upon it.

EDUCTIONS.

1. *Obversion.**

It has been said that Obversion is the process of substituting for any affirmative proposition its 'equivalent' in negative form, or else the process of expressing the meaning of a negative proposition as an affirmative; thus the obverse of 'A is B' is 'A is-not not-B,' and the obverse of 'A is-not B' is 'A is not-B' We may take this as a correct account of the process as eductive. But to accept it as describing a process of *Immediate Inference* implies a misconception of the genuine meaning of Immediate Inference which it is important to notice.

As we have seen, an inference is immediate if it can be drawn from a single proposition on the sole ground of the principles of Formal Identity and Non-contradiction. But when, for instance, we say that the proposition 'All S's are-not not-P's' is the obverse of 'All S's are P's,' we are maintaining what cannot be supported by reference to these principles alone. The use of the term 'not-P's' implies a universe of discourse G (let us say 'coloured object,' P being 'blue-coloured object'), and the proposition on which the reasoning rests is the following: 'Either an S (in respect of G) is a P, or else it is a not-P.'† With this we combine the statement 'All S's are P's,' and thence infer that 'All S's are-not not-P's.' This is what is known as a *disjunctive* inference, and the process of Obversion, as we find it presented in the ordinary Theory of Immediate Inference, is really not an immediate inference at all, but an inference based on a given disjunction.

In this criticism the essential points are (1) that the statement

* More or less obsolete equivalents are 'Permutation,' 'Æquipollence,' 'Immediate Inference by Privative Conception.'

† This is not infrequently expressed in the form: 'All S's are either P's or not-P's'; but, as we have seen (p. 132), the meaning of this categorically expressed proposition is disjunctive, and therefore can be appropriately stated only in disjunctive form.

'Either an S is a P, or else it is a not-P' cannot be identified with the Law of Excluded Middle; and (2) that even if it could be so identified, Obversion would still rest on a Reality-Principle and not on a mere Validity-Principle. Let us consider these two objections in turn, beginning with the first.

As 'not-P' means 'some specification of G which is other than P' (*vide* p. 49), the statement in question presupposes the more fundamental statement 'All S's are G's.' Thus the statement 'Either a spirit is a pink being, or it is a not-(pink being)' presupposes the proposition 'All spirits are coloured beings.' But the integrity of our thought does not depend on our believing that spirits have colour. It is not meaningless to deny that all spirits are coloured. And a statement that it is not meaningless to deny cannot represent a presupposition involved in the enunciation of a Law of Thought. We may, however, press this point home in a more radical manner. We have seen that the positing of the Law of Excluded Middle in the form given above necessitates the conclusion 'All S's are G's.' But evidently, from the Law of Excluded Middle, we cannot infer more than 'All S's are either G's or not-G's' *in a sense which gives no hint as to which alternative should be selected.* There is, then, a latent contradiction involved in the statement of the Law in the form 'Either an S is a P, or else it is a not-P.' Hence the statement 'Either an S is a P, or else it is a not-P' cannot be identified with the genuine law of thought to the effect that 'Either S is P or $\overline{\text{S is P}}$.'

There remains the further objection that even the genuine Law of Excluded Middle is not a Validity-Principle. To appeal to it is therefore to appeal, beyond the Principles of Formal Inference, to the Intelligibility of Reality. But this is to transgress not only the limits of a Formal treatment of Logic, but also the limits of the Law of Formal Validity, apart from which we possess no criterion for testing the soundness of any Formal inference.

The practical application of the principle of Obversion to the four propositions, A, E, I, O, gives the following results:

Obvertend.		Obverse.	
A	All S's are P's.	All S's are-not not-P's.	E
E	All S's are-not P's.	All S's are not-P's.	A
I	Some S's are P's.	Some S's are-not not-P's.	O
O	Some S's are-not P's.	Some S's are not-P's.	I

These inferences follow at once from the disjunctive major Either an S is a P, or else it is a not-P,' where the P's and the

not-P's are mutually exclusive, and, taken together, exhaustive alternatives of some given universe of discourse. Thus we have :

A.
Either an S is a P, or else it is a not-P.
But all S's are P's.
∴ All S's are-not not-P's.

E.
Either an S is a P, or else it is a not-P.
But all S's are-not P's.
∴ All S's are not-P's.

And so also for I and O.

Obversions, therefore, are not immediate, but *mediate* inferences.

The rule whereby these disjunctive obverses may at once be obtained from the corresponding obvertends is usually enunciated thus :

Negative the predicate, and change the quality of the proposition, but leave the quantity unchanged.

The fact that in Obversion the quantity is left unchanged shows that all obverses are *strong* educts.

If we apply the rule of Obversion to the four obverses of A, E, I, and O, we find that the obverse of the obverse of each proposition —*i.e.*, its obverted obverse—is the proposition itself. Generally, if Y is the obverse of X, then X is the obverse of Y. Obversion is a reciprocal process.

EXAMPLES OF OBVERSION.

	Obvertend.	Obverse.	
A	All men are mortals.	All men are-not non-mortals.	E
E	All men are-not monkeys.	All men are non-monkeys.	A
I	Some men are misers.	Some men are-not non-misers.	O
O	Some men are-not magistrates.	Some men are non-magistrates.	I

*Exercise.**—Express the following propositions in their simplest forms :

1. Not all S's are not-P's.
2. Not all S's are-not not-P's.
3. Not some S's are not-P's.
4. Not some S's are-not not-P's.

A 'not' placed before the sign of quantity contradicts the whole proposition. Thus (1) is equivalent to the contradictory of 'All S's

* Borrowed from Minto's 'Logic,' pp. 149, 150, footnote.

are not-P's '—*i.e.*, to the contradictory of this proposition's obverse, 'All S's are-not P's '—*i.e.*, to 'Some S's are P's,' the I proposition.

Similarly, 2, 3, 4 are respectively equivalent to the O, A, and E propositions.

2. *Conversion.*

Pure Conversion, as applied to categorical propositions, is the educing from a proposition expressed in strict logical form, and on the sole ground of the Law of Logical Validity, another proposition in strict logical form in which the original subject and predicate terms are transposed.

The Rules of Conversion :

Rule 1. The converse must be of the same quality as the convertend.

Rule 2. No term must be distributed in the converse if not distributed in the convertend.

We have now to see what is the connexion between these rules and the fundamental principle of Logical Validity ; for the logical justification of rules can lie only in their being rooted in rational principles.

The first of these rules appears, at first sight, to be nothing more than an arbitrary convention. But, if so, it has obviously no place in the Theory of Immediate Inference, which is based solely on the Principles of Identity and Non-contradiction. Prof. Welton attempts to meet the difficulty by saying that as the converse simply makes the same assertion as the convertend, looked at, as it were, from the other side, 'it is clear' that the quality of the two propositions will be the same.* But is this, then, so clear ? Is it quite obvious, for instance, that 'Some P's are S's' may not be a converse of 'Some S's are-not P's'?

The justification of the first rule of Conversion depends on the recognition (based on the privative view of negation as *bare* negation, blank denial) that affirmation and negation do not imply each other. The fact that the obverse of an affirmative proposition is negative (and *vice versa*) does not show that an affirmative can imply a negative. If the inference of 'All S's are-not not-P's' from 'All S's are P's' rested solely on the Laws of Thought, then the statement that the inference of a negative from an affirmative is *contrary* to the Laws of Thought would obviously be untenable. To accept the validity of the first rule of Conversion is therefore to admit that Obversion does not rest exclusively on the Laws of Thought. And this we have already admitted—indeed, we have

* J. Welton, 'A Manual of Logic,' Bk. III., ch. iii., vol. i., p. 256.

insisted on it. We are therefore prepared to admit the necessary character of the first rule of Conversion, and to regard it not as a convention, but as an expression of the fundamental logical fact that a negation cannot be immediately inferred from an affirmation.

The second rule of Conversion is clearly fundamental, for it simply expresses the essential requirement of the guiding principle of Inference. It is characteristic of all Formal Inference, mediate or immediate, that the conclusion must not go beyond the premiss or premisses. This requirement would be ignored were we to allow any term undistributed in the convertend to appear distributed in the converse. Hence the second rule of Conversion is logically justified, being, in fact, the Law of Formal Identity expressed in terms of distribution.

The Conversion of the A Proposition.—That the proposition 'Some P's are S's' is *a* converse of the proposition 'All S's are P's' follows at once from the Principle of Logical Identity in its simplest form. If we accept the statement 'All S's are P's,' we must accept 'Some S's are P's' (Subalternation). But we cannot accept 'Some S's are P's' without accepting 'Some P's are S's' (Identity).

Moreover, it is the only possible converse, for 'All P's are S's' breaks the distribution-rule, 'Some P's are-not S's' breaks the quality-rule, and 'All P's are-not S's' breaks both.

Thus the only converse of 'All S's are P's' is 'Some P's are S's.' This is a *weak* educt. It is technically called a 'converse by limitation' or a 'converse *per accidens*'*: it would be simpler to call it a 'weak converse.'

The Conversion of the I Proposition.—That 'Some P's are S's' is *a* converse of 'Some S's are P's' may be shown as above. That it is the *only* converse may again be shown precisely as above. Thus the only converse of the proposition 'Some S's are P's' is the *strong* equivalent 'Some P's are S's.'

The Conversion of the E Proposition.—The converse of 'All S's are-not P's' is the E proposition 'All P's are-not S's.' For if we accept the statement 'All S's are-not P's,' we must reject the statement 'Some S's are P's,' and therefore also (by the Principle of Identity) the converse statement 'Some P's are S's.' But if we reject the statement 'Some P's are S's,' we must accept the contradictory statement 'All P's are-not S's.'

Moreover, 'All P's are-not S's' is the only converse possible if we exclude the *weak* converse 'Some P's are-not S's.' For, since the acceptance of 'All S's are-not P's' necessitates the acceptance of

* For the justification of this expression, see H. W. B. Joseph, 'An Introduction to Logic,' ch. x., pp. 211, 212. The justification, however, does not extend to the use of 'accidental' in our sense of the word (*vide supra*, p. 27), and we therefore adopt the phrase 'weak converse' instead of 'converse *per accidens*.'

'All P's are-not S's,' it necessitates also the rejection of the contrary and the contradictory of 'All P's are-not S's'—*i.e.*, the rejection of 'All P's are S's' and of 'Some P's are S's.' These, therefore, cannot be converses of 'All S's are-not P's.' Moreover, they are affirmative propositions, and therefore ruled out by virtue of their quality.

The true converse of E is thus a *strong* educt. It is technically called—as the converse of I also is called—a *simple* converse. It would be better to call them both *strong* converses.

Converse of O.—The O proposition is inconvertible. Given the proposition 'Some S's are-not P's,' we are unable to infer Formally anything about the P's. We can, as we shall see, draw an inference concerning the 'not-P's'—viz., 'Some not-P's are S's'; but this is not a pure converse—*i.e.*, its subject is not the same as the predicate of the convertend.

That O is inconvertible may be shown by a direct application of the two rules of Conversion, for of the possible converses the negatives break the distribution-rule, and the affirmatives break the quality-rule. It may, however, be useful to point out other methods of proving the inconvertibility of O. Thus we may proceed as follows: Writing down the various possible converses—

(1) Some P's are-not S's,
(2) Some P's are S's,
(3) All P's are-not S's,
(4) All P's are S's,

we see at once that if we can prove (1) and (2) to be illegitimate or 'non-illative' converses, it will follow, *a fortiori*, that (3) and (4) are illegitimate.

(1) Is the proposition 'Some P's are-not S's' inferible from 'Some S's are-not P's'? The distribution-rule of Conversion shows that this is impossible; but we may vary the proof thus: The two propositions are not inconsistent. For instance—

'Some men are-not swimmers' . . . (i.)

is not inconsistent with

'Some swimmers are-not men.' . . . (ii.)

But the second of these statements is not *inferible* from the first. For if (ii.) followed necessarily on (i.), its contradictory would be inconsistent with (i.)—*i.e.*, the two statements—

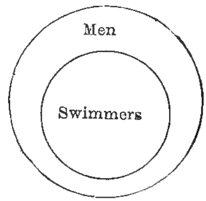
'All swimmers are men'

and

'Some men are-not swimmers'

could not be entertained together. But that they obviously can be accepted together is shown by the accompanying diagram,

which represents, not, indeed, the statements in question, but a single fact about which they might both be truly made.



(2) 'Some P's are S's' cannot be a converse of 'Some S's are-not P's.' This may be proved as above, or by a direct application of the quality-rule of Conversion; or we may argue thus:

If the statement 'Some P's are S's' can be validly inferred from 'Some S's are-not P's,' then so can 'Some S's are P's'; for the statement 'Some S's are P's' must be accepted whenever the statement 'Some P's are S's' is accepted. But we know that the O proposition does not imply its subcontrary I. Therefore, 'Some P's are S's' is not a valid inference from 'Some S's are-not P's.'

We now proceed to give some examples of Conversion. The following extract from Professor Bowen's 'Logic' (p. 157) will here serve as a useful introduction:

'In Conversion of Judgments, the learner must remember that *the whole* Predicate must change places with *the whole* Subject—that is, whatever belongs to the Predicate must be transferred to the Subject's place, and whatever relates to the Subject to the Predicate's place. For example, *Some temple is in the city* is not converted into *Some city is in the temple*, but into *Something in the city is a temple*. Again, the Predicate of *Every old man has been a boy* is not *boy*, but *has been a boy*; therefore it is not converted into *Some boy has been an old man*, but into *Some one who has been a boy is an old man*. To avoid mistakes of this sort, every proposition before Conversion—or, indeed, before it is subjected to any logical treatment whatever—should be reduced to its simplest logical form—that is, to the formula *A is B*, or *A is not B*. Then no error can arise, if we remember that *all** which precedes the Copula *is* or *is not* is the Subject, and that *all** which follows the Copula is Predicate.'

Examples of Conversion:

1. All organic substances contain carbon.
   Some substances which contain carbon are organic substances.

* The 'all' here must not be taken as including the quantity-mark.

2. The poor have few friends.
   Some persons who have few friends are poor people.
3. A wise man makes more opportunities than he finds.
   Some persons who make more opportunities than they find are wise men.
4. Warm-blooded animals are without exception air-breathers.
   Some air-breathers are warm-blooded animals.
5. Some good men have not the courage to appear as good as they are.
   Some persons who have not the courage to appear as good as they are are good men.
6. Some crystals are-not symmetrical.
   No converse.
7. All men have not faith.
   No converse.
8. All mathematical works are not difficult.
   No converse.
9. Mrs. Brown was Miss Smith.
   One who was Miss Smith is Mrs. Brown.
10. An equilateral triangle is a plane rectilinear figure of three equal sides.
    This is a logical definition, which should apply *only* to the class defined. Therefore simple transposition is in the case of definitions not only admissible, but logically requisite. If our purpose is definition, we stultify that purpose if we attempt to convert by limitation. Therefore the converse of the given proposition is :
    A plane rectilinear figure of three equal sides is an equilateral triangle.
11. A is followed by B.
    Something that is followed by B is A.
12. P struck Q.
    Somebody who struck Q is P.

Contrapositive Converse, or Converse by Negation.*

The contrapositive, or Contrapositive Converse, of a given proposition may be defined as the converse of the obverse, or as the converted obverse. The definition is genetic, informing us how the contrapositive may be obtained. Thus we have the following table :

* The nomenclature here varies greatly in different logical treatises. Mr. Joseph ('An Introduction to Logic,' ch. x., pp. 215, 216) restricts 'Converse by Negation' to the converted obverse, and reserves the term 'Contrapositive' for the 'Obverted Contrapositive' of our own scheme (*vide infra*, p. 201).

	Given Proposition.	Obverse.	Contrapositive.	
A	All S's are P's.	All S's are-not not-P's.	All not-P's are-not S's.	E
E	All S's are-not P's.	All S's are not-P's.	Some not-P's are S's.	I
I	Some S's are P's.	Some S's are-not not-P's.	(No contrapositive).	
O	Some S's are-not P's.	Some S's are not-P's.	Some not-P's are S's.	I

The rule of Contraposition may be succinctly stated thus :

'First obvert, then convert.'

We have seen that only E and I have *strong* converses. But if we first substitute for A and O their *obverse* equivalents, and *then* convert these, we find that these indirect converses, which are called *contrapositive converses*, do render the original propositions in unweakened forms. On the other hand, the contrapositive converses of E and I are not strong converses. One is *weak*, the other non-existent.

Table of Strong and Weak Converses.

Given Proposition.	Strong Converse.	Weak Converse.
A	Contrapositive Converse.	Simple Converse.
E	Simple Converse.	Contrapositive Converse.
I	Simple Converse.	(None.)
O	Contrapositive Converse.	(None.)

Exponible Propositions in Relation to Eduction. (*After Dr. Keynes.*)

An *exponible* proposition is a proposition which can be resolved into two or more simpler propositions which are *independent* of one another. For instance, the U proposition 'All S's are all P's' is an exponible, since it may be resolved into the two simpler and mutually independent propositions, 'All S's are P's' and 'All P's are S's.'

Now, we ask, is the Exclusive Proposition 'Only S's are P's' an exponible proposition ?

1. Can we say that it is equivalent to the two propositions 'All not-S's are-not P's' and 'Some S's are P's' ? Yes, argues Dr. Keynes, but the two propositions are not mutually independent.

The statement 'Some S's are P's' is an educt from 'All not-S's are-not P's' ($\bar{S}eP$, ∴ $Pe\bar{S}$, ∴ PaS, ∴ SiP). 'If a proposition were considered exponible simply because it could be resolved into two propositions, whether or not these propositions were independent of one another, then every proposition would be exponible.'*

2. Can we say that the given exclusive proposition is equivalent to the two propositions 'All P's are S's' and 'Some S's are P's'? No doubt; but the latter is the mere converse of the former.

We conclude, then, that the Exclusive Proposition is not exponible.

Examples in Eduction.—Give, where possible, the Converse, Obverse, and Contrapositive of the following propositions:

1. Only Protestant princes can sit upon the throne of England.
   Given proposition = All persons who can sit upon the throne of England are Protestant Princes. A.
   Converse: Some Protestant Princes are persons who can . . . etc. I.
   Obverse: All persons who can . . . etc. are-not non-(Protestant Princes). E.
   Contrapositive: All non-(Protestant Princes) are-not persons who can . . . etc. E.

2. Unasked advice is seldom acceptable.
   Given proposition = Some instances of unasked advice are-not instances of acceptable counsel. O.
   Converse: None.
   Obverse: Some instances of unasked advice are not-(instances of acceptable counsel). I.
   Contrapositive: Some suggestions which are not instances of acceptable counsel are instances of unasked advice. I.

3. No admittance except on business.
   Given proposition = All persons unintent on business are-not persons admitted. E.
   Converse: All persons admitted are-not persons unintent on business. E.
   Obverse: All persons unintent on business are non-(admitted persons). A.
   Contrapositive: Some non-(admitted persons) are persons unintent on business. I.

* J. N. Keynes, 'Studies and Exercises in Formal Logic,' 3rd edition, p. 75.

4. The writer of the document was A.B.

This is a singular proposition with both terms distributed. It is therefore not an ordinary A proposition. Our most straightforward course will be to convert it into a proposition of the same type—*i.e.*, into—

Converse : A.B. is the person who wrote the document.

We shall then have as the

Obverse : The person who wrote the document is-not a person other than A.B. E.

Contrapositive : All persons other than A.B. are-not the person who wrote the document. E.

5. More haste, less speed.

This might be treated as follows :

Given proposition = All cases of increased haste are cases that tend to diminish speed. A.

Converse : Some cases that tend to diminish speed are cases of increased haste. I.

Obverse : All cases of increased haste are-not cases not tending to diminish speed. E.

Contrapositive : All cases not tending to diminish speed are-not cases of increased haste. E.

It seems, however, more satisfactory to regard the given proposition as having an abstract subject. Thus :

Given proposition = Greater haste is a circumstance tending to produce correspondingly less speed. A.

Converse : A circumstance tending to produce correspondingly less speed is greater haste. I.

And so on.

6. Some portraits of some celebrated men are rare.

Given proposition = Some portraits of certain celebrated men are rarities. I.

Converse : Some rarities are portraits of certain celebrated men. I.

Obverse : Some portraits of certain celebrated men are-not non-rarities. O.

Contrapositive : (None).

Systematic Eduction.

The problem of systematic eduction may be stated as follows : Having taken the propositions A, E, I, and O, plot down all the legitimate educts, strong or weak, which can be obtained by successive processes of conversion and obversion.

For the convenient carrying out of this requirement, a suitable symbolic nomenclature is essential. The following substitutions, as used by Dr. Keynes, are now generally recognized :

A. All S's are P's = SaP.
E. All S's are-not P's = SeP.
I. Some S's are P's = SiP.
O. Some S's are-not P's = SoP.

The term ' not-x ' or ' non-x ' is represented by the symbol $\bar{x}$.

Eduction-Scheme.

A.		E.		I.		O.	
SaP		SeP		SiP		SoP	
(PiS)	$\mathrm{Se\bar{P}}$	PeS	$\mathrm{Sa\bar{P}}$	PiS	$\mathrm{So\bar{P}}$		$\mathrm{Si\bar{P}}$
$\mathrm{(Po\bar{S})}$	$\mathrm{\bar{P}eS}$	PaS	$\mathrm{(\bar{P}iS)}$	PoS			$\mathrm{\bar{P}iS}$
	$\mathrm{\bar{P}a\bar{S}}$	$\mathrm{(\bar{S}iP)}$	$\mathrm{(\bar{P}o\bar{S})}$				$\mathrm{\bar{P}o\bar{S}}$
	$\mathrm{(\bar{S}i\bar{P})}$	$\mathrm{(\bar{S}o\bar{P})}$					
	$\mathrm{(\bar{S}oP)}$						

N.B.—The process in every case is checked by the inconvertibility of the O proposition. ( ) indicates a weak educt.

Nomenclature :

S-P : Original proposition, the convertend or obvertend.
P-S : Converse.
P-$\bar{\mathrm{S}}$: Obverted Converse.
S-$\bar{\mathrm{P}}$: Obverse.
$\bar{\mathrm{P}}$-S : Contrapositive (Converse), or Converted Obverse.
$\bar{\mathrm{P}}$-$\bar{\mathrm{S}}$: Obverted Contrapositive.
$\bar{\mathrm{S}}$-$\bar{\mathrm{P}}$: Contrapositive of Contrapositive, or Obverted Inverse.
$\bar{\mathrm{S}}$-P : Inverse.

Note that from SaP, as accepted premiss, we are able to infer both $\mathrm{Po\bar{S}}$ and $\mathrm{\bar{S}oP}$, so that, given SaP, the proposition $\mathrm{Po\bar{S}}$ and $\mathrm{\bar{S}oP}$ (its pseudo-converse) must both be accepted. But this does not prove that the O proposition is convertible. For $\mathrm{Po\bar{S}}$ and $\mathrm{\bar{S}oP}$ are accepted, not unconditionally, but only on the condition that SaP is first accepted. The point to notice is that when two propositions

by means of the acceptance of a third proposition, can be shown to be necessarily accepted together, it does not therefore follow that the acceptance of one of the two, without the acceptance of the third proposition, necessitates the acceptance of the other.

Examples in Systematic Eduction.

1. Examine the inference :

All men are mortals.
∴ All non-men are non-mortals.

The inference is obviously faulty, but its examination will serve to illustrate the general method of solving problems of this kind.

The problem is equivalent to the following : Given SaP as the accepted statement of the relation between S (subject) and P (predicate), find the inferible statement of the relation between $\bar{S}$ (subject) and $\bar{P}$ (predicate).

The educt here required is the obverted inverse. Therefore, having posited the given proposition SaP, we start with an obversion, and then, by means of alternate processes of conversion and obversion, we push forward to the inferible statement which has $\bar{S}$ for subject and $\bar{P}$ for predicate.

$$\begin{array}{c} \underline{SaP} \\ | \\ Se\bar{P} \\ | \\ \bar{P}eS \\ | \\ \bar{P}a\bar{S} \\ | \\ \bar{S}i\bar{P} \end{array}$$

Thus, since $\bar{S}i\bar{P}$ is the strongest form of stated relation between $\bar{S}$ (subject) and $\bar{P}$ (predicate) that can be inferred from SaP, it follows that the subalternans $\bar{S}a\bar{P}$ cannot be inferred.

2. ‘ All that love success love work.’

Arrange the following propositions in the three groups :

(1) Those which can be inferred from the proposition given above.

(2) Those which are not inconsistent with it, but cannot be inferred from it.

(3) Those which are inconsistent with it.
  (*a*) None that loves not success loves work.
  (*b*) All that love work love success.
  (*c*) All that do not love work love success.
  (*d*) None that does not love work loves success.
  (*e*) Some that do not love success love work.
  (*f*) Some that do not love success do not love work.
  (*g*) Some that do not love work love success.
  (*h*) Some that do not love work do not love success.

Condensed solution :

(i.) *Symbolic Dictionary.*

Lovers of success $\equiv$ S.
Lovers of work $\equiv$ P.

(ii.) *Symbolic equivalents of the propositions given above.*

(*a*) $\bar{S}eP$.	(*e*) $\bar{S}iP$.
(*b*) PaS.	(*f*) $\bar{S}oP$.
(*c*) $\bar{P}aS$.	(*g*) $\bar{P}iS$.
(*d*) $\bar{P}eS$.	(*h*) $\bar{P}oS$.

(iii.) *Scheme of Eductions from the original proposition SaP.*

$$\begin{tikzcd}[arrows=dash]
 & \underline{SaP} \arrow[dl] \arrow[dr] & & \\
PiS \arrow[d] & & Se\bar{P} \arrow[d] & \\
Po\bar{S} & & \bar{P}eS \arrow[r] \arrow[d] & \bar{P}oS \\
 & & \bar{P}a\bar{S} \arrow[d] & \\
 & & \bar{S}i\bar{P} \arrow[d] & \\
 & & \bar{S}oP &
\end{tikzcd}$$

(iv.) *Classification of the given propositions, as per problem, on the basis of the Rules of Opposition.*

RESULT : (*d*), (*f*), (*h*) can be inferred from SaP.
(*a*), (*b*), (*e*) are neither inconsistent with SaP nor inferible from it.
(*c*), (*g*) are inconsistent with SaP.

3. Arrange the following propositions so as to show whether or not the acceptance* or the rejection* of one can be inferred either from the acceptance* or from the rejection* of another :

(*a*) No intelligent persons are prejudiced.
(*b*) All unprejudiced persons are intelligent.
(*c*) Some unintelligent persons are unprejudiced.
(*d*) Not every prejudiced person is unintelligent.
(Intermediate Arts Examination, London, 1902.)

(i.) *Symbolic Dictionary.*

Intelligent person $\equiv$ S.
Prejudiced person $\equiv$ P.
Unintelligent person $\equiv \bar{S}$.
Unprejudiced person $\equiv \bar{P}$.

This assumes that 'un' is equivalent to the 'contradictory' particle 'non,' and is not the prefix of a 'contrary term.'

(ii.) *Scheme of Strong Eductions*, drawn out for the purpose of selecting those *strong* educts from the given propositions, SeP, $\bar{P}aS$, $\bar{S}i\bar{P}$, $Po\bar{S}$, which are requisite for a straightforward solution of the problem.

SeP	$\bar{P}aS$	$\bar{S}i\bar{P}$	$Po\bar{S}$
PeS $Sa\bar{P}$	$\bar{P}e\bar{S}$	$\bar{P}i\bar{S}$ $\bar{S}oP$	PiS
$Pa\bar{S}$	$\bar{S}e\bar{P}$	$\bar{P}oS$	SiP
	$\bar{S}aP$		$So\bar{P}$

The purposive selections are : (*a*) $Sa\bar{P}$.
(*b*) $\bar{P}aS$.
(*c*) $\bar{P}oS$.
(*d*) $So\bar{P}$.

(iii.) *Solution.*

Arranging these educts in the two groups :

$$\text{I.} \begin{cases} Sa\bar{P}\ (a) \\ So\bar{P}\ (d) \end{cases} \qquad \text{II.} \begin{cases} \bar{P}aS\ (b) \\ \bar{P}oS\ (c), \end{cases}$$

* We have substituted the words 'acceptance' and 'rejection' for the original 'truth' and 'falsity.'

we see that there can be no inference from the acceptance or rejection of any proposition in the first group to the acceptance or rejection of any proposition in the second, and *vice versa*. This follows at once from the Rules of Conversion and of Subcontrariety.

Hence, with the additional help of the Law of Contradiction, we can at once tabulate our results thus :

Given.	(a)	(b)	(c)	(d)
(a) accepted	accepted	neither	neither	rejected
(b) ,,	neither	accepted	rejected	neither
(c) ,,	neither	rejected	accepted	neither
(d) ,,	rejected	neither	neither	accepted
(a) rejected	rejected	neither	neither	accepted
(b) ,,	neither	rejected	accepted	neither
(c) ,,	neither	accepted	rejected	neither
(d) ,,	accepted	neither	neither	rejected

N.B.—By 'neither' we mean that there is no logical ground either for the acceptance or for the rejection of the proposition in question.

4. What is the logical relation between the first and each of the following propositions given below ?

(*a*) Only the sensitive are sympathetic.
(*b*) Some sensitive people are-not unsympathetic.
(*c*) All unsympathetic people are sensitive.
(*d*) No sensitive people are sympathetic.
(*e*) No unsympathetic people are insensitive.
(*f*) All sympathetic people are insensitive.
(*g*) Some unsympathetic people are-not insensitive.
(*h*) Some insensitive people are sympathetic.
(*i*) No insensitive people are unsympathetic.
(*k*) Some sympathetic people are sensitive.

Logical Dictionary: (*a*) ≡All sympathetic people are sensitive (people).

Sympathetic person≡S.
Sensitive person≡P.
Unsympathetic≡non-sympathetic.
Insensitive≡non-sensitive.

Eduction-Scheme.

	SaP		
Weak converse.	PiS	$\mathrm{Se\bar{P}}$	Obverse.
Obverted converse.	$\mathrm{Po\bar{S}}$	PeS	Contrapositive.
		$\mathrm{\bar{P}a\bar{S}}$	Obverted contrapositive.
		$\mathrm{\bar{S}i\bar{P}}$	Contrapositive of contrapositive, or obverted inverse.
		$\mathrm{\bar{S}oP}$	Inverse.

Symbolic equivalents of given propositions, and results :

(*a*) = SaP.

(*b*) = $\mathrm{Po\bar{S}}$ = obverted converse of (*a*). Inferible from (*a*).

(*c*) = $\mathrm{\bar{S}aP}$ = contradictory of $\mathrm{\bar{S}oP}$ = contradictory of inverse of (*a*). Inconsistent with (*a*).

(*d*) = PeS = contradictory of converse of (*a*). Inconsistent with (*a*).

(*e*) = $\mathrm{\bar{S}e\bar{P}}$ = contradictory of obverted inverse of (*a*). Inconsistent.

(*f*) = $\mathrm{Sa\bar{P}}$ = contrary of obverse of (*a*). Inconsistent.

(*g*) = $\mathrm{\bar{S}o\bar{P}}$ = subcontrary of obverted inverse of (*a*). Neither inferible nor inconsistent.

(*h*) = $\mathrm{\bar{P}iS}$ = contradictory of contrapositive of (*a*). Inconsistent.

(*i*) = $\mathrm{\bar{P}e\bar{S}}$ = contrary of obverted contrapositive of (*a*). Inconsistent.

(*k*) = SiP = Subalternate of (*a*). Inferible.

5. What statements about candidates who pass in Logic can you infer from the statement ‘ All candidates who do not pass in Logic are-not successful candidates ’ ?

Logical Dictionary :

Candidates who pass in Logic ≡ S.
Successful candidates ≡ P.

Problem : Given $\mathrm{\bar{S}eP}$, what can we infer about S ?
Solution : The Eduction-Scheme is the following :

$$\begin{array}{ccc} & \bar{\mathrm{S}}\mathrm{eP} & \\ / & & \backslash \\ \mathrm{Pe}\bar{\mathrm{S}} & & \bar{\mathrm{S}}\mathrm{a}\bar{\mathrm{P}} \\ | & & | \\ \mathrm{PaS} & & \bar{\mathrm{P}}\mathrm{iS} \\ | & & | \\ \mathrm{SiP} & & \bar{\mathrm{P}}\mathrm{oS} \\ | & & \\ \mathrm{So}\bar{\mathrm{P}} & & \end{array}$$

Thus we see that from the accepted statement $\bar{\mathrm{S}}\mathrm{eP}$ we can infer SiP and $\mathrm{So}\bar{\mathrm{P}}$—that is :

> 'Some candidates who pass in Logic are successful candidates,'

and

> 'Some candidates who pass in Logic are not non-successful candidates.'

6. From the assertion 'The unknowable is-not unthinkable,' what can we learn, if anything, about (1) the unthinkable, and (2) the not-unknowable ?

Logical Dictionary :

Unthinkable $\equiv$ S.
Not-unknowable $\equiv$ P.

Therefore, given assertion $\equiv \bar{\mathrm{P}}\mathrm{eS}$

$$\begin{array}{ccc} & \overline{\bar{\mathrm{P}}\mathrm{eS}} & \\ / & & \backslash \\ \mathrm{Se}\bar{\mathrm{P}} & & \bar{\mathrm{P}}\mathrm{a}\bar{\mathrm{S}} \\ | & & | \\ \mathrm{SaP} & & \bar{\mathrm{S}}\mathrm{i}\bar{\mathrm{P}} \\ | & & | \\ \mathrm{PiS} & & \bar{\mathrm{S}}\mathrm{oP} \\ | & & \\ \mathrm{Po}\bar{\mathrm{S}} & & \end{array}$$

Hence we learn $\mathrm{Se}\bar{\mathrm{P}}$ and SaP concerning S, and PiS and $\mathrm{Po}\bar{\mathrm{S}}$ concerning P.

I.e.—(1) All unthinkables are-not unknowables.
All unthinkables are knowables.
(2) Some not-unknowables are unthinkables.
Some not-unknowables are-not not-unthinkables.

The startling character of these conclusions is, of course, due to the self-contradictory character of the data. This is concealed by the fact that when we say that 'the unknowable is-not unthinkable' we really mean to say that the unknowable is not *necessarily* unthinkable—that, for instance, the pure subject which, it might be argued, is unknowable, since only objects can be known, is still something of which we can think. We can think the Self, it might be urged, though we cannot know it. But in saying this, we do not wish to intimate—as we logically do when we urge that the unknowable is-not unthinkable—that *whatever* is unknowable is still thinkable. This is, in fact, self-contradictory. For the unthinkable itself must be unknowable, and therefore, according to our statement, still thinkable.

7. Examine the inference 'A St. Bernard is a dog. Therefore a small St. Bernard is a small dog.'

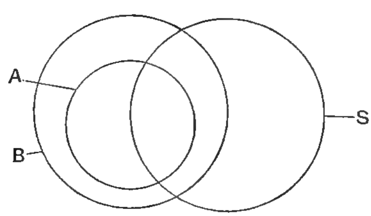
This is a fallacious instance of what is known as Inference by added Determinants, the added determinant here being the epithet 'small.' These inferences are customarily classed as immediate; but, as we shall presently see, they are really mediate inferences.

The given argument runs thus:

'A's are B's. Therefore small A's are small B's,'

'small A's' being the objects common to the extension of class A and the extension of the class 'small things,' or class S.

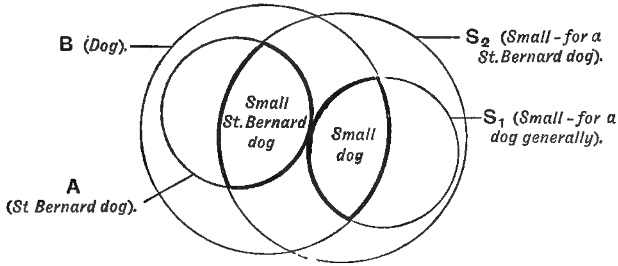
Now, if this class had an absolute, fixed extension, the reasoning would be perfectly correct. The accompanying diagram shows the



facts as the premiss and conclusion would represent them. The validity of the argument may be shown analytically thus:

All A's are B's.
Certain S's are A's.
Therefore *these same* S's are B's—*i.e.*, the small A's are also small B's.

But if the class S is ambiguously S_1 or S_2, according as it concerns the class A or the class B, there is no reason why any part of the extension of AS_1 should coincide with any part of the extension of BS_2.



[This diagram exhibits not the argument itself, but the *facts* which the statements of the argument *might* collectively represent.]

Hence, if an inference by added determinant is to be valid, the determinant added must have precisely the same application in both cases. Its application must not vary with the significance of the term it qualifies. The following inference by added determinant is perfectly valid :

A St. Bernard is a dog.
Therefore a hungry St. Bernard is a hungry dog.

8. Examine the inference :

'All judges are lawyers.
Therefore a majority of judges is a majority of lawyers.'

This is a fallacious instance of a kind of inference (miscalled immediate) usually known as Inference by Complex Conception. Here the subject and predicate are made *determinants of a third expression.* Here again the validity of the inference depends on the unambiguous fixity in the application of this third expression. The following inference 'by complex conception' is valid :

Oranges are fruit.
Therefore a barrel of oranges is a barrel of fruit.

But fallacy arises so soon as the expression (like the word 'majority' in the given example) is used in a relative, adjustive sense, varying in its import with the words that determine it.

VII.

THE SIMPLE CATEGORICAL SYLLOGISM.

CHAPTER XXI.

VII. (i.) FORMAL PRELIMINARY.

INSTEAD of considering the valid inferences which, with the help of the Principles of Consistency and Identity, or an assumed disjunction, we can make from some one given proposition, let us now take *two* propositions, and see what can be inferred from these taken together.

It is not, of course, possible to draw a conclusion from *any* given pair of propositions. Thus, from the two propositions

All bullfinches are birds,
All flounders are fishes,

no conclusion can be drawn. We might suppose that the conclusion 'All flounders are-not bullfinches' legitimately followed, but this is not the case. To render that conclusion logically sound, a further premiss would be required—namely, the statement 'All fishes are-not birds.'

To put the matter more generally, nothing can be inferred from the premisses 'All S_1's are P_1's' and 'All S_2's are P_2's' unless another premiss is given which states some connexion between P_1's and P_2's. This is the central postulate involved in Aristotle's great discovery of the Syllogism, for his discovery essentially consisted in finding out that, if a conclusion is to be logically drawn from a pair of given propositions, these must include a common element, *i.e.*, they must contain a common term. Thus, from the two statements

All birds are vertebrates,
All bullfinches are birds,

where the two ideas 'vertebrates' and 'bullfinches' are connected through the mediating link 'birds,' we at once infer the conclusion 'All bullfinches are vertebrates.'

We may say, then, that a conclusion can be drawn from two premisses,* only when the premisses have a common term. The common term is called the Middle Term. The terms related through the Middle Term are known as the Major and the Minor Term respectively.

* The propositions from which the conclusion is drawn are technically known as premisses, *propositiones præmissæ*.

The Conclusion in a Syllogism is the statement of a relation between the major and minor terms. The minor term is always the subject, and the major term the predicate, of the conclusion.

Of the two premisses, one is called the major and the other the minor premiss. The major premiss is that which contains the major term (P), and the minor premiss is that which contains the minor term (S). The middle term, M, occurs in both. The major premiss, then, states a relation between P and M, the minor a relation between S and M, and the conclusion a relation between S (subject) and P (predicate).

In order to be valid, the conclusion in a syllogism must follow necessarily and exclusively from the premisses—*i.e.*, it must be implied in the premisses themselves. Consider the argument

All Roses are Flowering-plants.
All Sweetbriars are Flowering-plants.
∴ All Sweetbriars are Roses.

This is no valid syllogism, though the truth of the conclusion is indisputable; for the conclusion is not a conclusion *from the premisses*, but from other extraneous sources of knowledge.

Again, take the argument

All bats are birds.
All bantams are bats.
∴ All bantams are birds.

This is a perfectly sound syllogism, though both the premisses are entirely false. If we accept the statement 'All bats are birds,' and also the statement 'All bantams are bats,' we are bound to accept the further statement 'All bantams are birds.'

We must carefully note the relation in which this purely Formal reasoning of the Syllogism stands to the question of Definition. For where the nature of the reference to Reality becomes—as it does in the Formal Syllogism—an irrelevant consideration, and the truth or falsity of the premisses a matter of indifference, all distinctions in the definition of terms—*e.g.*, the distinction between formal and real—become correspondingly irrelevant. The purely Formal treatment is also the merely verbal in this sense: that the words may take on any desired meaning, provided that the meaning is so given as not in any way to endanger the interests of Non-ambiguity and logical Consistency.

The Form of the Syllogism.

The Syllogism, in its most schematic form, may be represented thus:

$$\begin{array}{c} M \sim P \\ S \sim M \\ \hline S - P \end{array}$$

'~' indicating that the order of terms is reversible, '—' indicating that the order of terms is fixed.

The form of a syllogism, however, is not definitely fixed till we know its 'figure' and its 'mood.'

The four figures represent the four possible types of Syllogism, each type being distinguished by a certain order of the terms in the two premisses. For, as in each premiss *M* may be either subject or predicate, we have the four possible figures :

I.	II.	III.	IV.
M—P	P—M	M—P	P—M
S—M	S—M	M—S	M—S
S—P	S—P	S—P	S—P

Figure, then, is 'the form of a syllogism as determined by the position of the middle term in the two premisses.'

In Fig. 1, *M* is subject in the major premiss, and predicate in the minor.

In Fig. 2, *M* is predicate in each premiss.

In Fig. 3, *M* is subject in each premiss.

In Fig. 4, *M* is predicate in the major premiss, and subject in the minor.

Mood is the form of a syllogism as determined by the quality and quantity of the three constituent propositions. Thus, the scheme

$$\begin{array}{c} \text{M—P} \\ \text{S—M} \\ \hline \text{S—P} \end{array}$$

becomes indicative of mood as well as figure when it takes such a form as the following :

$$\begin{array}{c} \text{MaP} \\ \text{SaM} \\ \hline \text{SaP} \end{array}$$

which is the mood AAA in Fig. 1.

Each of the four figures may be tentatively developed into sixty-four different forms of Syllogism. For the major premiss may be A or E or I or O. Suppose it is A. Then with major premiss A we may have four different minor premisses—A or E or I or O. So also with major premiss E or I or O. Thus we have sixteen conceivable combinations of premisses in Fig. 1. And as each of these combinations, again, may conceivably be associated with a conclusion that is A or E or I or O, we have sixty-four conceivable forms in the First Figure. So also in each of the other figures. Thus there are 256 conceivable forms in the four figures taken together.

CHAPTER XXII.

VII. (ii.) THE RULES OF THE SYLLOGISM—THE VALID FORMS.

From these 256 conceivable forms we have now to eliminate those which, in one way or another, violate the requirements of Syllogistic Inference.

We do not propose at this stage to enter into any detailed discussion of the theory of Syllogistic Inference, but it is essential that we should develop it sufficiently to provide ourselves with the requisite practical criteria for discriminating the valid from the invalid forms of syllogistic reasoning.

The fundamental principle of all Inference we have seen to be embodied in the injunction not to go beyond the premisses. To infer syllogistically, in the widest sense of the process, is *to disimplicate from certain interrelated premisses such conclusions as the said premisses collectively necessitate.*

But for our present purpose a more restricted definition is called for. We are dealing with a certain restricted type of Syllogism, the Simple Categorical Syllogism, and the definition must, accordingly, include the further marks which stamp the Syllogism as simple and categorical. *Qua* ' simple,' the Syllogism must contain three propositions, and three only ; and *qua* ' categorical,' its propositions must consist of elements which are not clauses, but terms. Finally, in the Simple Categorical Distributional Syllogism these elements, or terms, are read in extension, and may be referred to as distributed or as undistributed.

The Syllogism, then, with whose definition we are here concerned may be labelled as the S.C.D. Syllogism — ' S.' standing for ' Simple,' ' C.' for ' Categorical,' and ' D.' for ' Distributional.'*

From the point of view of Inference, and *accepting the definition of propositions in terms of distribution,* we may define the Simple Categorical Syllogism as *A form of reasoning according to which from two accepted propositions called premisses, which contain a common term constituting a common link between them, we infer or disimplicate a third proposition, called the conclusion.*

This definition embodies not only the general Principle of Inference but also the Postulate of Mediation or Mediate Inference in its simplest form—the postulate of Aristotle : the two terms of the conclusion, S and P, are brought together through the mediation of the third or common term to which they are severally related. Reasoning of this kind is accordingly known as Mediate Inference.

* The term ' Syllogism,' as used in Division VII., should, *from this point onwards,* be understood as an abbreviation for ' S.C.D. Syllogism,' and the word ' Syllogistic ' should be similarly understood. Throughout the treatment of *Formal* Inference, ' Syllogism ' will be used in the sense of ' Formal Syllogism.'

The definition posited, we have simply to unravel its import in order to obtain, in explicit form, the so-called Rules of the Syllogism, by the aid of which we propose to undertake the logical sifting of the 256 abstractly possible forms of syllogistic reasoning.

THE RULES OF THE S.C.D. SYLLOGISM.

(i.) *Structural Rules.*

Rule I. : Every S.C.D. syllogism contains three propositions, and three only.
Rule II. : Every S.C.D. syllogism contains three different terms, and three only.

(ii.) *Rules of Distribution.*

Rule III. : The middle term must be distributed once at least.
Rule IV. : If a term is distributed in the conclusion, it must have been previously distributed in one of the premisses.

(iii.) *Quality Rules.*

Rule V. : If both premisses are affirmative, the conclusion, if any, is affirmative.
Rule VI. : If one premiss, and one only, is negative, the conclusion, if any, is negative.
Rule VII. : From two negative premisses nothing can be inferred.
Rule VIII. : If the conclusion is negative, one of the premisses is negative.
Rule IX. : If the conclusion is affirmative, both premisses are affirmative.

(iv.) *Quantity Rules.*

Rule X. : Two particular premisses prove nothing.
Rule XI. : If one premiss be particular, the conclusion is particular.

(i.) *Structural Rules.*

The requirement that the Syllogism shall be *simple* provides us with the first rule.

Rule I. : Every S.C.D. Syllogism contains three propositions, and three only.

The second rule is similar in character to Rule I.

Rule II.: Every S.C.D. Syllogism contains three different terms, and three only.

Qua categorical, the Syllogism must contain terms as its structural elements; nor can it contain more than four terms, since, according to the Principle of Formal Validity, the conclusion can add no new term to those contained in the premisses. But (given that the Syllogism is distributional*) the Postulate of Mediation requires that two of these terms shall coalesce so as to mediate between the two extremes. There must, therefore, be three terms, and three only, in the S.C.D. Syllogism.

For the keeping of this rule it is obviously necessary that the middle term be not ambiguous; for if it is ambiguous, it is not one term, but two, so that the syllogism contains four terms in all. The fallacy caused by using four terms in a syllogism is technically called *quaternio terminorum*, or *the fallacy of four terms.*

(ii.) *Rules of Distribution.*

A third rule—the Rule of Mediation (as expressed in terms of distribution)—is ordinarily and conveniently known as the First Rule of Distribution.

Rule III.: The middle term must be distributed once at least.

The fallacy involved in breaking this rule is known technically as *the fallacy of Undistributed Middle.*

When we say that the middle term has not been distributed in either premiss, our statement, in brief, comes to this: that *some* of the objects extensively indicated by the middle term have been identified with, or distinguished from, objects indicated by one of the extremes, and *some*, again, identified with objects indicated by the other; but the statement does not imply that objects indicated by the two extremes have been referred to the *same* objects indicated by the middle term—does not, in fact, imply that there has been any mediation at all.

Consider the following premisses:

'All swallows are fond of insects.'
'All hedgehogs are fond of insects.'

Here the predicate 'creatures fond of insects' is undistributed in both premisses. Swallows are stated to be *some* of these creatures, and so are hedgehogs; but there is no ground for supposing that the 'some' in the two cases refers to the same creatures. Between 'All P's are M_1's' and 'All S's are M_2's' there is no logical connexion possible. To insist that there is would be to violate the Postulate of Mediation and to go beyond the accepted data.

* See Chapter XXVIII., 'Unorthodox Syllogisms.'

The Principle of *Inference*, though implied in all the other rules, finds its explicit embodiment in what is customarily called the SECOND RULE OF DISTRIBUTION.

Rule IV.: If a term is distributed in the conclusion, it must have been previously distributed in one of the premisses.

To say that no term must be distributed in the conclusion that was not distributed in one of the premisses is only another way of saying that we must not, in the conclusion, make any statement about *all* the objects indicated by a term when in the premisses we have only referred to *some* of them—*i.e.*, that we must not introduce in the conclusion a reference to objects concerning which the premisses make no statement. We must not trespass beyond the accepted data.

The fallacy involved in breaking this rule is called an *illicit process of the major* when it is the major term that is used first in its partial, then in its total, extension, and an *illicit process of the minor* when it is the minor term that is treated in this way. Both fallacies are included under the name of *Illicit Process.*

(iii.) *Quality Rules.*

In discussing these five rules we propose, for brevity's sake, to use the expressions 'a class,' 'two classes,' etc., in the sense of 'the objects contained in a class,' 'the objects contained in two classes respectively,' etc.; and where such expressions as 'statement of relation,' 'statement of identity,' 'statement of non-identity' would be inconveniently cumbersome, we propose to substitute for them the words 'relation,' 'identity,' 'non-identity,' etc.

We may justify the Quality Rules of the Syllogism by the following considerations:

There are but two possible ways in which, from statements regarding the relations of two classes to one and the same third class, we can infer a relation of identity or non-identity between those two classes. We must argue from *identity of relation* to a *relation of identity*, or from *non-identity of relations* to a *relation of non-identity.*

Either the relations of P to M and S to M, as stated in the premisses, must be so far *identical* as to necessitate the inference that the classes S and P are (partly or wholly) *identical*,

or those same relations must be so far *non-identical* as to necessitate the inference that the classes S and P are (partly or wholly) *non-identical.*

An inference from identity of relations to a relation of non-identity is as inconceivable as an inference from non-identity of

relations to a relation of identity. From statements which imply the identity of the relations of P to M and S to M it is evidently impossible to infer a relation of non-identity between S and P; and from statements implying the non-identity of those relations it is evidently impossible to infer that S and P stand to one-another in a relation of identity.

Thus, if premisses of the *same* quality give any conclusion, it must be *affirmative;* and if premisses of *differing* quality give any conclusion, it must be *negative.* And, *vice versa*, an affirmative conclusion can be inferred only from premisses of like quality, and a negative conclusion only from premisses of unlike quality. This justifies Rules V., VI., and VIII.

With regard to Rule VII., we see that if two negative premisses can give any conclusion, that conclusion must be affirmative—that is, it must state an identity-relation between S and P. Now, such an identity-relation can be inferred only from premisses which imply the identity of the relations between P and M and between S and M. But two negative premisses imply no more, in this respect, than that these relations so far resemble one another that they are both relations of non-identity. And the relation of non-identity, being purely negative, admits of infinite variety, so that the implied statement of such resemblance as this involves no implication that the relations between P and M and between S and M have with one another any positive identity whatever. Thus, two negative premisses do not imply an identity of relations from which any relation of identity can be inferred. An identity-relation between S and P can be inferred only from statements that the classes S and P both stand in *identity*-relations with one and the same third-class, M; and these statements are not furnished by negative premisses. Thus, from two negative premisses no affirmative conclusion can be drawn. And since the premisses are of like quality, and therefore, as we have seen, a negative conclusion is impossible, it follows that no conclusion can be drawn at all. This justifies Rule VII.

With regard to Rule IX., we have seen that if the conclusion is affirmative the premisses must be of like quality—*i.e.*, they must be either both affirmative or both negative. But in discussing Rule VII. we have shown that they cannot both be negative. Hence they must both be affirmative, and Rule IX. is justified.

Of the five quality-rules, the two last are derivative. Rule VIII. is implied in Rule V., for if the conclusion is not affirmative, then, according to Rule V., both premisses cannot be affirmative; were they so, the conclusion would be affirmative.

Again, Rule IX. is implied in Rule VI., for if the conclusion is not negative, then, according to Rule VI., there can be no negative premiss; were there such a premiss, the conclusion would be negative.

The Rule that nothing can be inferred from two negative premisses is apt to cause difficulty. Arguments can be framed which, though they violate this rule, seem at first sight to be syllogistically sound.

E.g., What is not M is not P.
S is not M.
∴ S is not P.

Here, however, the middle term is not 'M,' but 'Not-M,' so that the minor premiss is really affirmative. Otherwise we should have a *quaternio terminorum ;* and though the argument would, no doubt, in this case be valid—the two middle terms being here reducible to one—yet it would not be stated in the correct form of an S.C.D. syllogism.

The Rule, in fact, applies only so long as we keep to strict syllogistic form, and deal with three terms only. If we loosely allow four terms, the rule ceases to hold.

E.g., All bats are-not birds.
All bats are-not unable to fly.
∴ Some creatures that can fly are-not birds.

This argument makes use of four terms—'birds,' 'bats,' 'creatures unable to fly,' and 'creatures that can fly.' And the reasoning is, therefore, not amenable to rules which apply only to arguments expressed in strict syllogistic form. Obverting both the premisses and also the conclusion, we obtain the following strict and valid syllogism :

All bats are not-birds.
All bats are not-(unable to fly)
(*i.e.*, are creatures that can fly).
∴ Some creatures that can fly are not-birds.

But now the premisses are no longer negative.

(iv.) *Quantity Rules.*

Rule X. : Two particular premisses prove nothing.

The abstractly possible combinations of two particular premisses are OO, II, IO, OI.

OO breaks the rule of Negative Premisses (Rule VII.).

II breaks the first rule of Distribution (Rule III.).

IO and OI, containing, as they do, only one distributed term apiece, leave no term to be distributed in the conclusion if the first rule of Distribution (Rule III.) is to be observed. Hence the conclusion must be I (Rule IV.)—an affirmative conclusion inferred from a negative premiss—which is impossible (Rule IX.).

Though nothing can be inferred from two particular propositions in the usual sense of the word 'particular,' yet from the two *pluratives* 'Most M's are P's,' 'Most M's are S's,' the conclusion 'Some S's are P's' can be drawn,—'most' being equivalent to 'more than half.' The conclusion, however, is *really* drawn, in last resort, from two universal propositions. For let C represent the M's that the premisses assert to be both S's and P's,* and which have thus been implicitly stated to belong to the extensions of both classes. Then the above-given argument is equivalent to—

All C's are P's.
All C's are S's.
∴ Some S's are P's (Fig. 3, Darapti).

Example :

Most dogs are fond of worrying cats.
Most dogs are fond of fetching sticks.
∴ Some creatures fond of fetching sticks are creatures fond of worrying cats.

Let C represent the dogs that are implicitly asserted in the premisses to be members of *both* the majorities in question. Then the argument is virtually equivalent to the following :

All the dogs C are fond of worrying cats.
All the dogs C are fond of fetching sticks.
∴ etc.

Again, in the syllogism—

Some M's are P's.
Some M's are S's.
∴ Some S's are P's.

if the 'Some M's' in the one premiss are intended by the arguer to be the same M's as the 'Some M's' in the other premiss, then the argument, though incorrectly expressed, is valid. 'Some M's' represents a definite class, and *all* the objects which this class contains are stated to be both P's and S's. Thus the fallacy of Undistributed Middle does not occur. Let C represent the common middle term which has been incorrectly labelled 'Some M's.' Then the argument runs : 'All C's are P's,' 'All C's are S's'; therefore 'Some S's are P's'—a syllogism in Fig. 3 (Darapti).

* If two majorities within one and the same extension did not overlap, the whole extension would be less than the sum of its parts, and this is impossible. Therefore some of the M's have been implicitly asserted to be both S's and P's, and *these* M's we call C's.

Rule XI.: If one premiss be particular, the conclusion is particular.

The premisses must be either both affirmative, or one affirmative and one negative (Rule VII.).

1. If both are affirmative, then A and I, as II is impossible (Rule X.).

This combination contains *one* distributed term, necessarily the middle term (Rule III.), leaving no term to be distributed in the conclusion.

Therefore the conclusion is I (Rule IV.).

2. If one is affirmative and one negative, then either O and A or I and E. In either case the premisses distribute only *two* terms. One of these must be the middle term (Rule III.), leaving one term, and one only, to be distributed in the conclusion (Rule IV.). But the conclusion must be negative (Rule VI.).

Therefore it must be O.

N.B.—The reverse of this rule—*i.e.*, the statement that a particular conclusion necessitates a particular premiss—is not true. The only cases, however, in which we find a particular conclusion without a particular premiss are those in which the premisses assume more than is required in order to prove the conclusion.

Our development of the rules of the S.C.D. Syllogism from its definition will have served to bring out the following main point: that these rules are an expression of the Postulate and the Principle of Inference, the Postulate insisting on the necessity of a mediating link, and the Principle forbidding us to draw any conclusion not fully implied in the premisses. The two fundamental rules of the S.C.D. Syllogism are the two Rules of Distribution which respectively embody the requirements of the Postulate and the Principle.

But our presentment of the unity of syllogistic process may be carried one stage further. The Postulate and the Principle are not independent of each other. The Principle, in its application to the Syllogism, involves the Postulate; and the Principle of Inference (in association with the Postulate, where the reasoning is Mediate) is just the Principle of Formal Validity which, in its more positive aspect, as an Identity Principle, may be formulated thus: *If a given proposition or set of propositions is accepted, then whatever further propositions are implied in what is thus admitted, these, and these alone, must be accepted.* And the significance of the word 'implied' is twofold. It includes a direct reference to the rule not to go beyond the accepted data; and, as applied to Mediate Inference, indirectly presupposes the conditions for there being any data at all—*i.e.*, it implies the Postulate of Mediation.

We proceed now to the practical application of the criteria of logical validity which we possess in the Rules of the S.C.D. Syllogism to the logical sifting of the 256 possible forms of Syllogism.

If any one of these forms of reasoning is such as to break none of the rules of the S.C.D. Syllogism, the form is valid; if it breaks *any* one of them, it is invalid.

The Discovery of the Valid Forms.

I. Disregarding varieties of figure, we first plot down the sixty-four abstractly possible moods, with a view to eliminating those which are illogical in every figure.*

AAA	IAA	EAA	OAA
AAI	IAI	EAI	OAI
AAE	IAE	EAE	OAE
AAO	IAO	EAO	OAO
AIA	IIA	EIA	OIA
AII	III	EII	OII
AIE	IIE	EIE	OIE
AIO	IIO	EIO	OIO
AEA	IEA	EEA	OEA
AEI	IEI	EEI	OEI
AEE	IEE	EEE	OEE
AEO	IEO	EEO	OEO
AOA	IOA	EOA	OOA
AOI	IOI	EOI	OOI
AOE	IOE	EOE	OOE
AOO	IOO	EOO	OOO

We now proceed to cancel all moods guilty of the following fallacies:

(1) Two negative premisses.
(2) Two particular premisses.
(3) One negative premiss, affirmative conclusion.
(4) Negative conclusion, two affirmative premisses.
(5) One particular premiss, universal conclusion.

As the result of these eliminations, we are left with twelve moods still to be tested in each of the four figures.

So far we have not had recourse to the two rules of distribution, the reason being that they are not generally applicable unless the figure is known as well as the mood. But in the single case of the mood IEO we have a mood which may be rejected for breaking a rule of distribution apart from any reference to a particular figure.

* For a much neater, but, from our point of view, less appropriate method, see Welton, 'A Manual of Logic,' Bk. IV., ch. iii., vol. i., pp. 319-322.

For the conclusion O distributes its predicate, the major term; but as this term cannot in any figure be distributed in the major premiss, which is an I proposition, there is invariably Illicit Process of the Major.

Eliminating IEO, we are left with the following eleven unrejected moods:

AAA, AAI, AEE, AEO, AII, AOO; EAE, EAO, EIO; IAI; OAO.

II. We have now to find out in which of the four figures these eleven unrejected or 'legitimate' moods are valid—*i.e.*, we have to find out how many forms of Syllogism are valid in any given figure.

Plotting down the figure-schemes, we have—

$$\begin{array}{cccc}
\text{I.} & \text{II.} & \text{III.} & \text{IV.} \\
M\text{—}P & P\text{—}M & M\text{—}P & P\text{—}M \\
S\text{—}M & S\text{—}M & M\text{—}S & M\text{—}S \\
\hline
S\text{—}P & S\text{—}P & S\text{—}P & S\text{—}P \\
\hline
\end{array}$$

We now take each of the eleven moods in turn, and put it in distributive form.

$$\text{Thus, AAA} \equiv \begin{array}{c} du \\ du \\ \hline du \\ \hline \end{array}; \quad \text{EIO} \equiv \begin{array}{c} dd \\ uu \\ \hline ud \\ \hline \end{array}, \text{ etc.}$$

Comparing the distribution-scheme of AAA with each of the figure-schemes in turn, we cancel this mood in any figure in which it is guilty either of undistributed middle or of illicit process of the major or the minor term.

$$\text{Thus, comparing} \quad \begin{array}{c} du \\ du \\ \hline du \\ \hline \end{array} \quad \text{with} \quad \begin{array}{c} M\text{—}P \\ S\text{—}M \\ \hline S\text{—}P \\ \hline \end{array}$$

we see that none of these rules is broken. AAA is therefore valid in Fig. I.

$$\text{Again, comparing} \quad \begin{array}{c} du \\ du \\ \hline du \\ \hline \end{array} \quad \text{with} \quad \begin{array}{c} P\text{—}M \\ S\text{—}M \\ \hline S\text{—}P \\ \hline \end{array}$$

we see that there is undistributed middle. AAA is therefore invalid in Fig. II.

Again, comparing $\begin{array}{c} du \\ du \\ \hline du \\ \hline \end{array}$ with $\begin{array}{c} M—P \\ M—S \\ \hline S—P \\ \hline \end{array}$ and with

$\begin{array}{c} P—M \\ M—S \\ \hline S—P \\ \hline \end{array}$ we see that there is in each case the fallacy of

illicit minor. AAA is therefore invalid in Figs. III. and IV.

Proceeding in this way, we find that in each figure six of the legitimate moods are valid, so that we have in all twenty-four valid forms of Syllogism. Of these, five are known as subaltern or weakened forms—*i.e.*, forms which draw particular conclusions from premisses which warrant conclusions that are universal. This can, of course, be realized only when a universal conclusion is legitimate ; and to *every* form of Syllogism that has a universal conclusion there will correspond a weakened form, with conclusion of the same quality.

Thus, AAA gives AAI in Fig. I.
EAE gives EAO in Fig. I.
EAE gives EAO in Fig. II.
AEE gives AEO in Fig. II.
AEE gives AEO in Fig. IV.

Since all conclusions in Fig. III. are particular, Fig. III. can have no weakened form.

The nineteen forms which are not only valid but strong are found conveniently tabulated in the following mnemonic verses of the Traditional Logic :

Bārbără, Cēlārēnt, Dărĭī, Fĕrĭōquĕ prĭorīs ;
Cēsārĕ, Cāmēstrēs, Fēstīnŏ, Bărōcŏ sĕcūndæ ;
Tērtĭă Dārāptī, Dĭsămĭs, Dātīsĭ, Fĕlāptōn,
Bōcārdō, Fērīsŏn hăbēt ; Quārta īnsŭpĕr āddīt
Brāmāntīp, Cămĕnēs, Dĭmărīs, Fēsāpŏ, Frĕsīsŏn.

Each of these time-honoured names contains three vowels, which inform us, concerning each of the three propositions of the form of Syllogism that answers to the name, whether it is A, E, I, or O. Thus, the mnemonic verses tell us that Ferison, for instance, is a form in Fig. III., having major premiss E, minor premiss I, and conclusion O.

Of these nineteen forms four are known as *strengthened* forms, or *strengthened syllogisms.* A strengthened form is a strong or

non-weakened form which employs *two* universal premisses to prove a particular conclusion when only *one* is needed. Darapti and Felapton in Fig. III., Fesapo and Bramantip in Fig. IV. are strengthened forms. If these are excluded from the list we are left with fifteen 'fundamental' forms, valid forms that are neither weakened nor strengthened. The following comparison between each of the strengthened forms and the fundamental forms of the same figure that have the same conclusion will serve to bring out the superfluous character of the former.

Thus, *Darapti*, $\begin{array}{l}\text{MaP}\\ \underline{\text{MaS}}\\ \underline{\underline{\text{SiP}}}\end{array}$, is obviously not so adequate or economical a form as Disamis, $\begin{array}{l}\text{MiP}\\ \underline{\text{MaS}}\\ \underline{\underline{\text{SiP}}}\end{array}$, or Datisi, $\begin{array}{l}\text{MaP}\\ \underline{\text{MiS}}\\ \underline{\underline{\text{SiP}}}\end{array}$.

Again, *Felapton*, $\begin{array}{l}\text{MeP}\\ \underline{\text{MaS}}\\ \underline{\underline{\text{SoP}}}\end{array}$, is less effective than Bocardo, $\begin{array}{l}\text{MoP}\\ \underline{\text{MaS}}\\ \underline{\underline{\text{SoP}}}\end{array}$, or Ferison, $\begin{array}{l}\text{MeP}\\ \underline{\text{MiS}}\\ \underline{\underline{\text{SoP}}}\end{array}$.

So, in Fig. IV., *Fesapo*, $\begin{array}{l}\text{PeM}\\ \underline{\text{MaS}}\\ \underline{\underline{\text{SoP}}}\end{array}$, is less effective than Fresison, $\begin{array}{l}\text{PeM}\\ \underline{\text{MiS}}\\ \underline{\underline{\text{SoP}}}\end{array}$; and *Bramantip*, $\begin{array}{l}\text{PaM}\\ \underline{\text{MaS}}\\ \underline{\underline{\text{SiP}}}\end{array}$, less effective than Dimaris, $\begin{array}{l}\text{PiM}\\ \underline{\text{MaS}}\\ \underline{\underline{\text{SiP}}}\end{array}$.

CHAPTER XXIII.

VII. (iii.) EXERCISES ON THE STRUCTURE OF THE S.C.D. SYLLOGISM.

Exercise 1.—Show that there cannot be more than four syllogistic figures.

The number of figures is limited to the number of ways in which the position of the middle term can be varied in the two premisses. M may be subject or predicate in either premiss, thus offering four possible variations in position.

Exercise 2.—Prove that there must always be in the premisses one distributed term more than in the conclusion.

If no term is distributed in the conclusion, yet one term—the Middle—must be distributed in the premisses. And if the major or the minor term is distributed in the conclusion, it must be distributed also in the premisses, as well as the middle term.

Exercise 3.—If the major premiss in Fig. I. were particular affirmative, what fallacy would be committed?

P, being undistributed in the major, must be undistributed in the conclusion. The conclusion must therefore be affirmative, and therefore the minor must also be affirmative. The middle term is therefore undistributed in both premisses, and the fallacy is that of undistributed middle.

Exercise 4.—If the major premiss in Fig. II. were particular, what fallacy would be committed?

P, being undistributed in the major, must be undistributed also in the conclusion, which is therefore affirmative. Both premisses are therefore affirmative, and we have the fallacy of undistributed middle.

Exercise 5.—Why is it that the moods EAO, EIO are valid in all the four figures?

The question is equivalent to the following: Given that the conclusion is O, why is it that the premisses EA, EI can be utilized to prove it in all the four figures?

The conclusion being O, P is distributed in the major, but, as the major is E, P may be either subject or predicate there. Again, *M*, being already distributed in the major, need not be distributed in the minor. Therefore the minor may be either A or I; and be either MS or SM, as S is undistributed in the conclusion.

Hence the major may be MP or PM, and the minor MS or SM. Therefore, etc.

Exercise 6.—Detect the fallacy in the following solution of the problem: Given the major premiss particular, find out whether the minor is A, E, I, or O.

Since the major is given particular, the minor must be universal. Again, since the major is particular, the conclusion is particular. Therefore S is undistributed in the conclusion, and therefore also in the minor. Therefore the minor must be an affirmative universal, since the negative universal distributes *both* terms. Therefore the minor is A.

This conclusion, though correct, has been incorrectly drawn. The fallacy occurs in inferring that, because S is undistributed in the conclusion, it must therefore be undistributed in the minor

premiss.* This inference would, however, be valid if the conclusion were known not to be a weakened conclusion. It is true that in the case of Bramantip the major term, which is undistributed in the conclusion, is distributed in the major premiss. But Bramantip may be regarded as a weakened form in this sense: that its conclusion is the weak converse of the conclusion which can be drawn in Fig. I. from the same premisses.

In connexion with the working out of exercises on syllogistic structure in relation to the S.C.D. Syllogism, the following hints may be found to be of service.

(i.) In the working out of these exercises, it is permissible to take for granted the Rules of the S.C.D. Syllogism, and these only.

(ii.) What is *not* to be taken as given:

(*a*) Knowledge of the valid forms in each figure. The mnemonic verses are to be used only as a *reference*, for purposes of identification and nomenclature, or as a check in testing the correctness of one's answers. Hence a result based solely on a mere reference to the list of the moods which are valid in any given figure is of no value at all. Results must always be reasoned out from the rules of the S.C.D. Syllogism.

(*b*) The Rules of the Figures, sometimes called the Special Rules of the Syllogism. These should not be used as a substitute for direct reasoning based on the general Rules of the Syllogism.

(iii) The solution of a problem in Formal Logic should not be regarded as an experimental process. It should proceed by *developing* the implications of the given data till the required solution actually unfolds itself. The discussion of a syllogistic problem should always take the form of a direct series of necessary inferences, and should not consist in experimentally testing a number of moods.

Thus if, in an exercise on the S.C.D. Syllogism, we are given certain quite general data, which do not involve any specific reference to mood or figure, then the best way of discovering the figure or figures in which the conditions are fulfilled is to *infer from the given data the respective positions of the middle term in the*

* *Cf.* H. W. B. Joseph, 'An Introduction to Logic,' ch. xii., p. 252, footnote: 'Beginners imagine sometimes that the fallacy of illicit process is committed if a term which is distributed in the premiss is undistributed in the conclusion. This is, of course, not the case. I must not presume on more information than is given me, but there is no reason why I should not use less.'

two premisses. This is equivalent to discovering the figure from first principles. It is invariably the shortest, neatest, and surest method of procedure, and should always be aimed at whenever possible.

The two following exercises may serve to illustrate this important point :

1. If the minor premiss of a syllogism be O, what is the figure and the mood ?

The minor is O. Therefore the conclusion is O. Therefore the major term must be distributed in the major premiss. But since the minor premiss is negative and particular, the major must be affirmative and universal. Thus the major is A ; and, since P is distributed, the order of its terms is PM.

Again, *M*, being undistributed in the major, must be distributed in the minor, and must therefore be the predicate in the minor, since the minor is an O proposition. Hence the order of terms in the minor is SM, and the syllogism is *Baroco* in Fig. II.

2. Prove that a universal affirmative proposition can form the conclusion in Fig. I. only.

Both premisses must be universal affirmative, and consequently distribute only their subjects.

Now S, being distributed in the conclusion, must be distributed in—*i.e.*, in this case, be the subject of—the minor premiss.

This leaves *M* to be distributed in the major, of which it is therefore the subject.

We therefore get MaP
SaM, *i.e.*, *Barbara* in Fig. I.
SaP

CHAPTER XXIV.

VII. (iv.) THE ANALYSIS OF SYLLOGISMS, AND THE REDUCTION OF ARGUMENTS INTO SYLLOGISTIC FORM.

A Categorical argument, as presented for logical handling, may either be already expressed in syllogistic form or require reducing into such form. Reduction to *strict* syllogistic form is an indispensable preliminary to the application of the Rules of the S.C.D. Syllogism as tests of logical validity.

An important form of unsyllogized argument goes by the name of *Enthymeme*.

An Enthymeme is usually defined as a defective syllogism, as a syllogistic argument with one proposition suppressed. The suppressed proposition may be either premiss or conclusion.

Enthymemes are of three orders :

In an *Enthymeme of the first order* the *major premiss* is omitted.

Example.—The soul is a simple substance, consequently it is indestructible.

Here the omitted major is 'All simple substances are indestructible substances.'

In an *enthymeme of the second order* the *minor premiss* is omitted.

Example.—Some plants are insectivorous, for the Sundew eats insects.

Here the completed syllogism runs :

The Sundew is an insectivorous organism.
The Sundew is a plant.
∴ Some plants are insectivorous organisms (Darapti).

In an *enthymeme of the third order* the *conclusion* is omitted.

Example.—A man of fashion is a fop, and all fops are fools.

In our definition of the Enthymeme we have restricted the omission or defect to a single proposition. There is, however, an important variety of 'defective' argument in which *two* out of the three propositions essential to a syllogism are omitted. Here the single proposition does duty for an inference.

By way of illustration we may take the answer of the railway ticket-clerk who, when asked by on old lady whether she need take a separate ticket for her pet tortoise, promptly replied : 'No, ma'am ; cats is dogs, and rabbits is dogs, but tortoises is hinsecs.'

Here the two propositions, 'Cats is dogs' and 'Rabbits is dogs,' are arguments reduced each to a minor premiss. For the major premiss, 'Dogs are creatures that must have tickets,' and the conclusion,

$$\left.\begin{array}{l}\text{‘Cats}\\ \text{‘Rabbits}\end{array}\right\}\text{are creatures that must have tickets,’}$$

are intended, but not expressed. As for the sentence, 'No, ma'am, Tortoises is Hinsecs,' which is equivalent to 'Since tortoises are insects, they do not need tickets,' it is an enthymeme of the first order, the omitted major being 'Insects are-not creatures that need tickets.'

The importance of the Enthymeme lies in its being the natural mode of reasoning, the Syllogism being comparatively artificial. 'The argument,' says Dr. Gilbart, 'first occurs to our mind in the form of an enthymeme, but when we wish to make it clearer, we extend it to a syllogism.'* From the point of view of Formal analysis, however—and we are not here concerned with any other

* 'Logic for the Million,' p. 254.

interest save that of validity—the Enthymeme is a defective syllogism, and its logical correction consists in supplying the omitted proposition in such a way that the argument becomes syllogistically valid. This is, of course, not always possible. If a particular premiss is given, for instance, with a universal conclusion, the pseudo-enthymeme cannot possibly be expanded into a valid syllogism. If a negative premiss is given with an affirmative conclusion, there is not the same hopelessness. *E.g.*, 'All children are non-combatants, for no children are soldiers.' Here the conclusion may be obverted into the form 'All children are-not combatants,' and the omitted major is 'All combatants are soldiers,' the argument being in Camestres (Fig. II.).

As transitional between the Enthymeme and the Syllogism proper we may note such arguments as the following :

'Some plants are parasites—for example, Mistletoe.'

Here the expression 'for example' implies both that Mistletoe is a plant and also that it is a parasite. Hence there is no premiss actually omitted ; the argument is abbreviative rather than defective. It may be fully stated thus :

Mistletoe is a parasite.
Mistletoe is a plant.
∴ Some plants are parasites (Darapti).

A similar argument is supplied by the following sentence :

The example of Demosthenes shows that some orators are made, not born.

Where the given argument is not logically defective, it may or may not be valid. Enthymemes reduced to syllogisms are necessarily valid, for they are made valid through the very process of remedying their defectiveness. But it is, of course, otherwise where the argument is from the outset completely stated. In this case the first step must be to reduce the argument into strict syllogistic form, particular care being taken to place the three propositions in their proper relative places.

Example.—No patience is pleasant, for no painless experience is patience, and no painful experiences are pleasant.

Substituting for the proposition 'No painful experiences are pleasant (experiences)' its obverted converse, 'All pleasant experiences are non-painful experiences,' we obtain the argument :

All pleasant experiences are non-painful experiences.
All painless experiences are-not instances of patience.
∴ All instances of patience are-not pleasant experiences.

If we may assume that 'painless' is equivalent to 'non-painful,' this is a valid argument in Camenes (Fig. IV.). Otherwise it exhibits *quaternio terminorum.*

When an argument is presented in what appears to be syllogistic form, we must not suppose that the first, in order, of the premisses is necessarily the major premiss. The major premiss is the premiss that contains the major term.

Example: All fungi are plants.
Some fungi are microscopic organisms.
∴ Some plants are microscopic organisms.

By considering the conclusion of this argument we are able at once to state which is the major and which the minor term, for the major term is always the predicate of the conclusion—in this case the term 'microscopic organisms.' Thus the second premiss, since it contains the major term, must be the major premiss. The true syllogism, then, stands thus:

Some fungi are microscopic organisms.
All fungi are plants.
∴ Some plants are microscopic organisms (Disamis, Fig. III.).

The order of the premisses cannot, of course, affect the validity of the argument or the applicability of the rules of the S.C.D. Syllogism; nor does it even affect the labelling of the mood, provided that we take the preliminary precaution of deciding, as above, which premiss is the major and which the minor.

In the reduction of arguments to strict syllogistic form, the mood and figure should be stated if the argument is valid, and the mood, figure, and Formal fallacy if it is invalid.

EXAMPLES.

Example 1.—Since we must admit that all plants are not petunias, and that no poodles are petunias, it follows that some plants at least are poodles.

This argument may be written thus:

All poodles are-not petunias.
Some plants are-not petunias.
∴ Some plants are poodles.

Here we have the pseudo-syllogism EOI in Fig. II., the fallacy being that of two negative premisses.

Example 2.—No one can be a great logician without being a philosopher as well. Now we must admit that Aristotle was a philosopher. It follows, then, that Aristotle is a great logician.

This argument, when expressed in strict logical form, runs thus:

All great logicians are philosophers.
Aristotle was a philosopher.
∴ Aristotle was a great logician.

This is AAA in Fig. II. Fallacy: Undistributed middle.

Example 3.—Only suns are stars. Therefore Sirius, being admitted to be a sun, is also a star.

Logical form :

(*a*) All stars are suns.
Sirius is a sun.
∴ Sirius is a star.

This, again, is AAA in Fig. II. Fallacy : Undistributed middle.

Or—

(*b*) All not-suns are-not stars.
Sirius is-not a not-sun.
∴ Sirius is a star.

This is EEA in Fig. I. Fallacy : Two negative premisses.

Example 4.—Since only plants are seaweeds, it follows that some plants are not green, for all seaweeds are not green.

Here the conclusion of the argument must be the proposition introduced by 'it follows that.' Therefore the major term is 'green (things).' Therefore the major premiss is 'All seaweeds are not green things,' which, in strict logical form, reads 'Some seaweeds are-not green things'; and we obtain as our syllogism :

Some seaweeds are-not green things.
All seaweeds are plants.
∴ Some plants are-not green things.

This is a valid argument in Bocardo (Fig. III.).

The Fallacy of Four Terms.

A given argument is not necessarily invalid because the form in which we happen to express it has four terms. It may be that what is incorrect is not the argument, but our pseudo-distributional syllogism. If so, the syllogism must be recast into a form that does not involve the fallacy. Thus an argument will frequently be found to contain the four terms S, P, M, and not-M. In this case the 'not' may frequently be transferred to the copula by obversion, leaving a three-term syllogism. Again, a not infrequent mistake is that of stating, with regard to a syllogism, that it not only has four terms, but also breaks such and such rules. But if by syllogism we mean an S.C.D. Syllogism,* then, when once we have convicted a so-called 'syllogism' of a *quaternio terminorum*, it is sufficiently stamped as non-syllogistic, and it must then be futile to convict it of breaking any other rule of the Syllogism.

* *Vide supra*, p. 216.

Example :

No one is contemptible but the contemptuous.
Some cowards are not contemptuous.
∴ Some cowards are not contemptible.

Formal restatement :

All non-contemptuous persons are-not contemptible persons.
Some cowards are-not contemptuous persons.
∴ Some cowards are-not contemptible persons.

Here we have four terms ; but the inference is not that the 'syllogism' is invalid, but simply that we have not yet a syllogism at all. We must recast this seeming syllogism into the correct form of an S.C.D. Syllogism :

All non-contemptuous persons are-not contemptible persons.
Some cowards are non-contemptuous persons.
∴ Some cowards are-not contemptible persons (Fig. I., Ferio).

Or—

All contemptible persons are contemptuous persons.
Some cowards are-not contemptuous persons.
∴ Some cowards are-not contemptible persons (Fig. II., Baroco).

CHAPTER XXV.

VII. (v.) USES AND CHARACTERISTICS OF THE FOUR FIGURES—THE SPECIAL RULES.

FIGURE I.

A GLANCE at the mnemonic verses will show that Fig. I. gives conclusions in all the four forms, and therefore serves every purpose of affirmation or denial, partial or total. We are therefore able, in Fig. I., to make *universal* statements, affirmative or negative, and to support them ; we are also able to contradict any statement, affirmative or negative, and to support our denial in this same figure.

Further, Fig. I. is the only figure in which the subject and predicate of the conclusion are respectively subject and predicate in the premisses in which they occur, so that, as Professor Carveth Read puts it, 'the course of argument has, in its mere expression, an easy and natural flow.'

Figure II.

In Fig. II. only negative conclusions can be proved, so that its use is greatly restricted as compared with that of Fig. I. Still, where suggested assertions about a given subject have to be rejected, the use of the Second Figure is more natural than that of the First. Thus the question may be 'Can we accept the statement that this plant is a Dicotyledon?' and the answer 'No, we must reject it; for we have already accepted the statement that this plant does not possess a character common to all Dicotyledons.'

All Dicotyledons are plants possessing two primary leaves, and only two.
This plant is-not a plant possessing two primary leaves and only two.
∴ This plant is-not a Dicotyledon (Camestres).

Thus Fig. II. is essentially the figure of denial. By means of it we can go on denying a succession of disjunctively accepted predications until, by a process of exclusion, we are enabled to accept the one predication that remains. Thus Fig. II. may play an important part in the service of the Disjunctive Syllogism.

Figure III.

In this figure only particular conclusions can be proved. Its one undoubted advantage is seen in dealing with singular propositions; for it is the only figure in which the singular name can be subject in both premisses.

Example :

Socrates is a warrior.
Socrates is a philosopher.
∴ One philosopher at least is a warrior (Darapti).

Figure IV.

The Fourth Figure is tolerated by logicians only because its inclusion is necessary for the structural completeness of syllogistic theory. It formed no part in the logical system of Aristotle, but was added by Galen some centuries later. It is, however, a perfectly valid form of reasoning, and, as such, resists the effort to exclude it from the Syllogism. Thus, Mr. Joseph, who starts by rejecting the forms of the fourth figure, is compelled to introduce them as 'indirect moods of Fig. I.'*

It is easy to show that in no case can Fig. IV. improve on Fig. I.

* See 'An Introduction to Logic,' ch. xii., pp. 246, 258, 261; ch. xiv., p. 301.

For, in the case of Bramantip, Camenes, and Dimaris, the premisses, when transposed, at once give conclusions in Fig. I. in the forms Barbara, Celarent, and Darii respectively, and the reasoning in these latter forms has that easy flow which is entirely lacking in Fig. IV., where Subject and Predicate in the conclusion are respectively Predicate and Subject in the premisses. Moreover, in the case of Bramantip, the conclusion is weaker than the conclusion drawn in Fig. I. from precisely the same premisses. Thus, given the statements, 'All grasses are monocotyledons,' and 'All monocotyledons are flowering-plants,' the conclusion in Fig. IV. is 'Some flowering-plants are grasses,' the weak converse of the conclusion in Fig. I., 'All grasses are flowering-plants.'

As regards Fesapo and Fresison, the former, as a 'strengthened' form, is inferior in effectiveness to Fresison; and Fresison itself is Ferio over again with the premisses simply converted; but it is Ferio with its easy flow rendered awkward, and to that extent is inferior to it. Hence Fig. IV. is in all its forms a less effective instrument than Fig. I.

The Special Rules of the Four Figures.

Each of the four figures has its own special rules, deducible from the more general rules of the Syllogism in conjunction with the specific form of the figure.

FIGURE I.

Rule 1. *The minor premiss must be affirmative.*

Proof: If possible, let it be negative.

Then the major must be affirmative (Rule VII.), and the conclusion must be negative (Rule VI.).

M—P
S—M
———
S—P

Therefore the major term will be undistributed in the premiss and distributed in the conclusion, and we have the fallacy of illicit major (Rule IV.). Hence the minor premiss cannot be negative in Fig. I., and must therefore be affirmative.

Rule 2. *The major premiss must be universal.*

Proof: The minor premiss being affirmative, the middle term is there undistributed, and must therefore be distributed in the major premiss (Rule III.).

But it is subject in this premiss. Hence the major premiss must be universal.

Figure II.

Rule 1. *One premiss must be negative.*

P—M
S—M
S—P

Otherwise there will be undistributed middle. Hence the conclusion in Fig. II. must always be negative (Rule VI.).

Rule 2. *The major premiss must be universal.*

Proof: The conclusion being negative, *P* is distributed there, as, therefore, also in the major premiss (Rule IV.). Hence, since P is subject there, that premiss is universal.

Figure III.

Rule 1. *The minor premiss must be affirmative.*

M—P
M—S
S—P

Proof: If it were negative, the conclusion would be negative, and the major premiss affirmative. But this involves illicit major (Rule 4). Hence the minor premiss must be affirmative.

Rule 2. *The conclusion must be particular.*

Proof: As the minor premiss is affirmative, S is undistributed there, being predicate, and is therefore undistributed in the conclusion also (Rule IV.), which must therefore be particular.

Figure IV.

Rule 1. If the major is affirmative, the minor is universal.
Rule 2. If the minor is affirmative, the conclusion is particular.
Rule 3. If the minor is negative, both premisses are universal.
Rule 4. If either premiss is negative, the major is universal.

The proof of these rules may suitably be taken as an exercise.

CHAPTER XXVI.

VII. (vi.) THE DICTA.

THE view of propositional import which we have assumed as fundamental for a strictly Formal treatment of Logic has been an Identity-view. According to this view, the proposition 'All S's are P's' means 'Each of all S's is ($\equiv$ "is identical with") a P.' The Identity-view, so interpreted, requires (*a*) that both subject and predicate terms be read in extension, and (*b*) that the extensive reading be itself understood in a distributive, and not in a collective, sense.

At this point it will be convenient to consider a view of the import of a categorical proposition which is in some respects akin to that of Formal Identity-import, though by no means to be confused with it. The two views of Import are akin in this respect, that in each the two terms of the proposition are read in extension. The difference between these views depends on the way in which this extension is referred to. In the case of the Identity-view both terms are referred to *distributively*. In the case of the other view—usually known as the Class-inclusion view—the extension of the subject-term may be referred to either distributively or collectively, but that of the predicate-term is always *collectively* understood.

The definition of the Class-inclusion view of Propositional Import includes the following essential marks :

1. The relation between subject and predicate must be that of inclusion in, or exclusion from, a class.
2. The extension of this predicate-class must be referred to collectively.

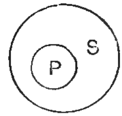
The proposition 'All S's are P's,' read on this view, is equivalent to the statement 'All the S's (distributively) are included in the class P,' or, 'All S (collectively) is included in the class P.' If we assume that *both* terms are read collectively, the fourfold scheme of propositions will be thus expressed :

A. All S (collectively) is included in P.
E. All S (collectively) is excluded from P.
I. Some S (collectively) is included in P.
O. Some S (collectively) is excluded from P.

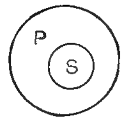
These propositions do not profess to express the relations which, in fact, may hold between two classes, S and P ; what they formulate is not 'relations,' but statements about relations. Were it the intention to consider the relation directly, we should require, for

exhaustively representing the possible relations between S and P, a fivefold scheme of propositions :

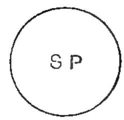
1. S wholly includes P.



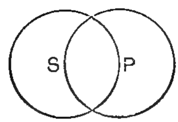
2. S is wholly included by P.



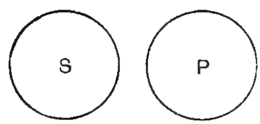
3. S coincides with P.



4. S partially includes and partially excludes P.



5. S entirely excludes P.



THE SO-CALLED 'PRINCIPLES' OF THE SYLLOGISTIC FIGURES.

1. *Formal Enunciations of the Dicta, on the Class-inclusion View of Statement-import.*

The principle of Fig. I. has been called the *Dictum de Omni et Nullo,* and may be enunciated as follows :

> *If we can assert of the whole of a class that it is included in, or excluded from, a second class, we can assert the same of any part of that whole class.*

The principle of Fig. II. has been called the *Dictum de Diverso,* and may be enunciated as follows :

> *If we can assert of one class that it is included in, and of another that it is excluded from, a third class, we may also assert the mutual exclusion of the two classes.*

The principle of Fig. III. has been called the *Dictum de Exemplo et de Excepto*, and may be enunciated as follows :

> *If we can assert of two classes that they both include one and the same third class, we can also assert that they partially include each other ; and if we can assert of two classes that the one includes a third class which the other excludes, we can also assert that part of one of these classes is excluded from the other.*

The principle of Fig. IV. has been called the *Dictum de Reciproco*. Its formulation is necessarily so cumbrous that it seems idle to present it.

These dicta have been very variously enunciated, the enunciations varying with the view of import adopted. In the formulations given above the class-inclusion view of statement-import has been adopted, both subject and predicate being treated as classes, and *both* referred to *collectively*. It follows from this way of interpreting the Dicta that the subject-class is the class as indicated by the subject-term *as quantified* in the premiss. Thus, in the premiss, ' All S is included in P,' ' All S ' is the one class, and ' P ' the other. So, again, in the premiss, ' Some S is included in P,' ' Some S ' is the one class and ' P ' the other. Where, as sometimes in Fig. III., the subject-term M is differently quantified in the two premisses, the ' third class ' referred to in the Dictum must be taken to be that indicated by ' Some M.'

2. *The Logical Status of the Dicta.*

A logical principle, in order that it may be efficiently regulative, must satisfy two conditions :

(i.) Its application must lead to unambiguous results.
(ii.) It must admit of being used as a criterion to test the soundness of all relevant cases.

Of these two requirements it is the first alone which is satisfied by the Dicta.

(i.) That the application of the Dicta furnishes unambiguous results may, in the case of the *Dictum de Omni et Nullo*, be set forth as follows :

	All M is P		The whole class (All M) is statedly included in the class P.
Barbara :	All S is M	$\equiv$	∴ Since (All S) is statedly contained in the whole class (All M), we must admit that it is included in the class P.
	∴ All S is P		

Celarent:

$$\begin{array}{l} \text{All M is-not P} \\ \underline{\text{All S is M}} \\ \therefore \underline{\underline{\text{All S is-not P}}} \end{array} \equiv$$

The whole class (All M) is statedly excluded from the class P.
∴ Since (All S) is statedly contained in the whole class (All M), we must admit that it is excluded from the class P.

Darii:

$$\begin{array}{l} \text{All M is P} \\ \underline{\text{Some S is M}} \\ \therefore \underline{\underline{\text{Some S is P}}} \end{array} \equiv$$

The whole class (All M) is statedly included in the class P.
∴ Since (Some S) is statedly contained in the whole class (All M), we must admit that it is included in the class P.

Ferio:

$$\begin{array}{l} \text{All M is-not P} \\ \underline{\text{Some S is M}} \\ \therefore \underline{\underline{\text{Some S is-not P}}} \end{array} \equiv$$

The whole class (All M) is statedly excluded from the class P.
∴ Since (Some S) is statedly contained in the whole class (All M), we must admit that it is excluded from the class P.

Thus the *Dictum de Omni et Nullo,* when applied to the four valid forms of Syllogism in Fig. I., leads in each case to an unambiguous result; it enables us to draw from the premisses the correct conclusion in regard to both quality and quantity. 'The whole class' in all four moods is 'All M.' The 'part' is either 'All S' or 'Some S,' and is stated in each minor premiss to be included in 'All M.' And we see that in all four moods, in strict accordance with the Dictum, what is asserted of 'All M' may be asserted in the same sense of the part 'All S' or 'Some S.'

The *Dictum de Diverso* may similarly be shown to fulfil the first requirement of a logical principle.

Thus, if the class All P is statedly contained in, and the class All S statedly excluded from, a third class M, we must admit that they are excluded from each other—*i.e.*, All S is excluded from All P, or All S is-not P. This is the reasoning of the form Camestres.

The application of the Dictum may similarly be shown to yield unambiguous results in the case of the remaining forms in the Second Figure. Thus, in the case of Baroco, the conclusion is that All P and Some S exclude each other—*i.e.*, Some S is-not P.

The first part of the principle of Fig. III., known as the *Dictum de Exemplo,* concerns the three affirmative forms of Syllogism in the Third Figure. We may illustrate its application in the case of *Darapti:*

We assert of the two classes S and P that each includes a third

class (All M). Hence S and P must partially include each other—*i.e.*, Some S is P.

The second part of the principle, known as the *Dictum de Excepto*, concerns the three negative forms of the Third Figure. We may illustrate its application in the case of Bocardo :

We assert of the two classes, S and P, that S includes a third class (Some M) which P excludes. Hence we can also assert that part of S is excluded from P—*i.e.*, that Some S is-not P.

(ii.) We have now to point out that as *tests* or regulative principles, even within the narrow limits of their own appropriate provinces, the Dicta have no value whatsoever. They cannot be used to sift the pseudo-forms of Syllogism in their respective figures from the logically valid forms. They are, at best, mere descriptive statements of the type of argument characteristic of the *valid* forms in their several figures. They are abstracts *from* these forms, not rules *for* them. In a word, they are not principles but figure-heads.

Let us take the *Dictum de Omni et Nullo*. If it be a logical principle, we must expect it to be regulative with regard to all syllogisms, valid or invalid, that are of the form characteristic of Fig. I. ; for is it not put forward as the 'principle of Fig. I.' ?

But suppose the pseudo-syllogism AEE, Fig. I., expressed as follows : 'All M is P, All S is-not M, therefore All S is-not P,' desires to be tested by this principle. The Dictum can only reply that AEE, Fig. I., having, unfortunately, a negative minor, does not satisfy the conditions requisite for the Dictum's applicability. 'But,' says AEE, Fig. I., 'am I not in Fig. I., and are you not the *principle* of Fig. I. ?' 'No,' answers the Dictum, 'I am not the principle of Fig. I. altogether, but only a principle of the *valid forms* in Fig. I. I was only put in office over these moods after they had been sifted from the others. You therefore do not come under my jurisdiction.'

This is perfectly just. AEE, Fig. I., should have applied to the 'Special Rules,' Fig. I., to have its inference tested. The Special Rules of any figure, so far as they serve any useful function other than that of the General Rules of the Syllogism, constitute the test of admission to the figure in question. No mood can be admitted among the valid forms of Fig. I. unless (1) its major premiss is universal, and (2) its minor premiss is affirmative.

Precisely similar reasoning would show that the Dicta of the Second and Third Figures are but 'principles' of the *valid* forms in their respective figures, and therefore cannot be employed to sift the valid forms from the invalid.

Our conclusion, therefore, is that the sole function of the Dicta is to draw the proper conclusions from whatever premisses successfully run the gauntlet of the Rules of the Syllogism and the Special Rules of the Figures.

The right *order of precedence*, then, is the following :

1. The *Principle of Logical Validity* and the *Postulate of Mediate Inference* as embodied in the definition of the S.C.D. Syllogism. *These* are the true principles of syllogistic reasoning.
2. The *General Rules* deduced from these.
3. The *Special Rules of the figures*, deduced from the General Rules.
4. *The Dicta.*

N.B.—The *Dictum de Omni et Nullo* should not under any circumstances be claimed as the source of the authority of the special Rules of the First Figure. It is impossible to infer the latter from the former (as it is customary to do) without putting the cart before the horse. The Special Rules can be justified only by the General Rules *taken collectively—i.e.*, by the *system* of general rules.

3. *The Dictum de Omni et Nullo in its relation to Deduction.*

The function of this Dictum has been identified with that of Deduction or Deductive Method. Deduction is defined as 'the applying of a general law or rule to particular cases,' and it is then pointed out that this is precisely the function of the Dictum. It is, therefore, important for us to examine whether this contention can be reasonably supported. Is it, we ask, the function of the Dictum to apply general laws to particular cases ?

Assuming for the moment that it is logically permissible to characterize the function of a principle of Inference as that of *applying* a universal to particulars, we have still to ask whether the universal which the Dictum applies is *any* universal or only a special kind of universal.

This question can be unambiguously answered. In so far as the dictum is enunciated on the class-inclusion view, it is clear that the only universals or laws which it can possibly be made to apply are the laws which can be expressed in class-inclusion form. Where a law can be adequately represented, diagrammatically, by the total inclusion of one circle in another, or by the exclusion, mutual and total, of both, its application to particular cases may then be guaranteed by the Dictum.

Now, if Science were still based on the scholastic metaphysics of the Middle Ages, it might be possible to apply its laws uniformly in this very simple way. But as Science stands to-day the suggestion is ridiculous. Laws that could be effectively applied on the Dictum principle would have to be sought, if anywhere, in those classification-tables in which species are still subsumed under

genera ; but as they are 'connotation-tables,' the Dictum has even here no natural, direct applicability.

But there is a further and a more deep-reaching limitation which makes it impossible to allow that the Dictum can apply, in an effective scientific way, even the small group of laws to which, on any given view of propositional import—be it the Class-inclusion view, the Identity view, or any other—its function is restricted. We express this limitation when we say that a law, applied in accordance with the Dictum, *cannot account for the differences* among the particulars to which it is intended to apply. The distinction between the ways in which the application of Law is understood by the Dictum and by Science respectively is in this respect *radical*.

Let us imagine that the Law of Gravitation had been 'applied' to the movements of Jupiter and Saturn in the way prescribed by the Dictum. It could have been applied to these movements only so far as they *resembled* each other. It could have given no account whatsoever of the points of difference. It would have been a law of the heavenly bodies in general, but of no heavenly body in particular. It would have enabled astronomers to conclude that Jupiter, moving as it did, moved like a heavenly body, and that Saturn did the same. Can we suppose that the science of Astronomy could have profited much from a theory of gravitation so applied as to be unable to explain the differences in the movements of the different planets ? It must surely be obvious that the Dictum has no power of giving effect to the explanatory function of the laws which it pretends to apply.

If we ask for the reason why the application of a general law to particular cases is understood so differently by the Dictum and by the principles of Science, and why it is that the function of the Dictum cannot be styled 'deductive' in the sense already indicated, the answer is to be found in the fundamental importance which Deductive Science attaches to System. Hence arises the impossibility of identifying with the Principle of Deduction any principle which represents the process as an inference proceeding from some *single, isolated generality*. Thus, the movements of Saturn and of Jupiter cannot be deduced from the sole theory of gravitation, apart from the laws of motion and of equilibrium with which it is inextricably involved. So, again, the theorem of Euclid I. 47 could not be deduced from any other single geometrical theorem taken in isolation from the rest.

It is true that, in reasoning from a plurality of theories, the fact that we do not reason from any one of them *in isolation* in no way precludes our reasoning from each of them *singly*. As Professor G. F. Stout has said : 'The logical interconnexion of the theories lies precisely in the fact that each supplies specifying conditions for the application of the others. Consider the theory of gravitation. Here laws of motion and of equilibrium, etc., are already recognized

in understanding what this theory means. So far as this is the case, the single theory of gravitation is treated as being in itself a system of theories. But so far as this is not the case, the laws of motion, equilibrium, etc., with their special applications, supply determining conditions which logically specify the mode in which the law of gravitation must work in this or that instance. They define the conditions of its application. Doubtless it also defines the conditions of *their* application. But we may start from the single theory of gravitation, and regard these others merely as specifying conditions, just as we may start from the others, and regard the theory of gravitation as a specifying condition.'

So far we have been proceeding on the assumption that the function of a logical principle might be adequately characterized as that of *applying* universals to particulars. The assumption, however, involves an important ambiguity, and tends to misleading conceptions as to the nature of Logical Inference. The work of 'application' involves, in fact, imaginative and constructive activities of mind which cannot be reckoned as processes of logical inference, so long as Inference is defined as a process of disimplication, pledged not to trespass beyond the data. We must distinguish between Method and Inference, between Deduction-process and Deductive Inference. If, dissatisfied with the Dictum, we seek for a conception that can inspire the systematic application of Law to Fact, we must turn, not to any mere principle of Deductive Inference, but to that larger process of Deduction of which the aim may correctly be defined as 'the valid application of systematized knowledge to unsystematized fact.'

'Application,' then, is a function of Deduction. As such, it is not limited by the Principle of Logical Inference. Through Deduction we bring laws to bear at the right points, and so combine in fruitful co-operation the premisses from which we then draw our deductive inferences. The application of System to Fact demands not only inference from given evidence, but also the purposive arrangement of the evidence itself. Thus, the proof of Euclid I. 47, or of any geometrical theorem, cannot be reduced, without remainder, to a chain of deductive inferences. It includes constructions and combinations, apart from which the purely inferential process would be directionless and ineffective.

That the definition of *Deduction* which we have given does not apply to Deductive *Inference* may also be shown as follows: Premisses may be given which cannot be said to stand to each other in the relation of statement of law to statement of fact. Thus, from the two premisses 'The Nile is a blessing to Egypt' and 'The Nile is infested with crocodiles' we may infer a necessary conclusion; but in so doing we cannot be said, in any sense of the words, to have applied a general law to a particular fact. So,

again, in considering the two premisses 'Socrates was a warrior,' 'Socrates was a philosopher,' we ask in vain which of them represents the universal or the general law, and which the particular to which it is applied.

CHAPTER XXVII.

VII. (vii.) THE PROBLEM OF REDUCTION.

REDUCTION may be defined as the process through which the reasoning in a given syllogistic form is restated in another syllogistic form.

There is, however, a narrower sense in which the term is customarily used. According to the more restricted definition, Reduction is the process through which a syllogism in the Second, Third, or Fourth Figure comes to be restated in the form of a syllogism of the First Figure. Regarded from this point of view, the First Figure becomes the standard or the 'perfect' figure, and the others are called 'imperfect.' There is no reason why we should quarrel with the preference here given to Fig. I., so long as the reasons given for it are reasons of convenience only. Fig. I. expresses the principle of mediate inference in its most straightforward form, and is for that reason to be preferred. It should not be supposed, however, that the reasoning in Fig. I. is a whit more valid than the reasoning in Figs. II. and III., or even than that in Fig. IV. The validity of the reasoning in Fig. IV. is as surely guaranteed by the Rules of the Syllogism as is the validity of the reasoning in Fig. I.

We accept Fig. I., then, as the standard or 'perfect' figure on the secondary grounds already alleged. We also take it as proved that the only four strong and valid forms of Syllogism in Fig. I. are Barbara, Celarent, Darii, and Ferio. It is on the basis of these presuppositions that the processes of Reduction are developed. It has been customary to recognize two kinds of Reduction: (1) Direct or Ostensive Reduction; (2) Indirect Reduction, or *Reductio per Impossibile.* But of these two types the latter is inapplicable within the limits of a strictly Formal treatment. In its essence it is equivalent to Indirect Proof, and seeks to establish the truth, and thence the validity, of a syllogistically inferred conclusion in an 'imperfect figure' by proving that the denial of the same leads to contradiction with the assumed truth of one or other of the premisses. This whole procedure, however, presupposes the truth-interest, and cannot appropriately be considered till we come to treat of the theory of Deductive or Truth-Inference in a

later chapter. We shall then be meeting it under the name of 'Indirect Proof,' or *Reductio ad Absurdum.* We need only add that the fact that the Scholastic Logicians resorted to Indirect Reduction in their attempt to confirm the validity of two 'imperfect' forms of Syllogism, Baroco and Bocardo, is sufficient evidence that their treatment of Logic was not so strictly and consistently Formal as has sometimes been supposed.

Direct or Ostensive Reduction.

We may illustrate the operation of direct Reduction by reducing Baroco and Bocardo to the First Figure.

Baroco may thus be represented in symbolic form: $\begin{array}{c} \mathrm{PaM} \\ \mathrm{SoM}. \\ \hline \mathrm{SoP} \end{array}$

We wish to reduce this to the form: $\begin{array}{c} \mathrm{M—P} \\ \mathrm{S—M}. \\ \hline \mathrm{S—P} \end{array}$ Conversion '*per accidens*' of the major premiss would readily give the desired reduction, did it not leave us with two particular premisses. Reduction, however, to the form $\begin{array}{c} \overline{\mathrm{M}}\mathrm{—P} \\ \mathrm{S—}\overline{\mathrm{M}} \\ \hline \mathrm{S—P} \end{array}$ would, if logically possible, answer the requirement just as well. From the eduction-schemes of PaM and SoM we select those inferences that suit the above form. They are $\overline{\mathrm{M}}$eP, the contrapositive of PaM, and Si$\overline{\mathrm{M}}$, the obverse of SoM; and since from our new premisses we are able to draw the old conclusion SoP, we have a syllogism in *Ferio,* Fig. I.

Bocardo may be represented thus in symbolic form: $\begin{array}{c} \mathrm{MoP} \\ \mathrm{MaS}. \\ \hline \mathrm{SoP} \end{array}$

We wish to reduce this to the form: $\begin{array}{c} \mathrm{M—P} \\ \mathrm{S—M}. \\ \hline \mathrm{S—P} \end{array}$ But as in Fig. I. the major premiss must be universal, and we cannot reduce the given particular major into a universal, our first step must be to transpose the premisses. This gives— $\left\{\begin{array}{l} \mathrm{MaS} \\ \mathrm{MoP} \end{array}\right.$. In order to place the middle term in the position of predicate in the

minor premiss, we must convert this premiss ; but, as it is an O proposition, we cannot do this without obverting it first.

We take, then, the contrapositive of our minor premiss, and we obtain $\left\{\begin{matrix}\text{MaS}\\ \bar{\text{P}}\text{iM}\end{matrix}\right.$. From these premisses in Fig. I. we are able to draw the conclusion $\bar{\text{P}}$iS, of which the obverted converse is the original conclusion SoP.

Thus, the valid syllogism $\begin{array}{c}\text{MaS}\\ \bar{\text{P}}\text{iM}\\ \hline \bar{\text{P}}\text{iS}\end{array}$ is the equivalent of Bocardo,

and it is in the First Figure (Darii). Bocardo has therefore been reduced to the First Figure, as desired.

The reduction of the remaining forms is simpler than that of either Baroco or Bocardo, and may readily be obtained by reasoning out the steps as above. It seems ungracious, however, to pass over without remark the ingenious key to these reductions which the traditional Logic has preserved for us in the Mnemonic Verses.

Each of the curious names preserved in these Latin hexameters contains, in addition to the vowels which give the mood, other letters significant for the purpose of reduction. Thus, to take the typical case of CAmEstrEs, the initial letter C indicates the initial letter of that form of Syllogism in the First Figure—namely, Celarent—to which this form can be reduced. The small letters *m* and *s* refer to the vowels just before them, and tell us how we must handle the propositions represented by these vowels in order to effect the desired reduction.

Thus : *s* (= *simpliciter*) indicates simple conversion of the E proposition in premiss and conclusion.
m (= *muta*) indicates that the premisses have to be 'changed'—*i.e.*, transposed.

Following out these indications, we see at once that $\begin{array}{c}\text{PaM}\\ \text{SeM}\\ \hline \text{SeP}\end{array}$ reduces to $\begin{array}{c}\text{MeS}\\ \text{PaM}\\ \hline \text{PeS}\end{array}$ —*i.e.* to Celarent in Fig. I.

Of the other letters in the mnemonic verses, we should note the significance of *p*, which signifies 'conversion per accidens.'*

It will have been noticed that Baroco and Bocardo do not reduce directly to Barbara, as the initial letter *B* would lead us to expect.

* For an ingenious revision of the mnemonic verses, which, while freeing the scheme from all meaningless letters, shows at once the figure to which any given form belongs, see Professor Carveth Read, 'Logic Deductive and Inductive,' third edition, pp. 126, 127.

As a matter of fact, the initial B, in the names of these two forms, refers to the Barbara syllogism which enters into their *indirect* reduction. The *c* (= *per contradictionem*) indicates the necessity of resorting to this indirect reduction.

The refusal of the ancient logicians to admit the soundness of the *direct* reduction of Baroco and Bocardo appears to have been due to their distrust of the negative term. It is true that, in so far as the reduction involves the use of obversion, it cannot be justified by the sole, unconditional appeal to the principles of Identity and Non-contradiction, and we are therefore bound to recognize that the validity of these reductions extends no further than the adequacy of the formula : 'Either an S is a P or else it is a not-P.'

CHAPTER XXVIII.

VII. (viii.) UNORTHODOX SYLLOGISMS.

According to Archbishop Whately,* the Syllogism is the form to which all correct reasoning *may* ultimately be reduced. So, also, J. S. Mill writes : 'All valid ratiocination, all reasoning by which, from general propositions previously admitted, other propositions equally or less general are inferred, may be exhibited in some of the above forms '†—*i.e.*, the valid forms of the traditional Syllogism.

Now, if Whately and Mill were right, all Formal Inference whatsoever would ultimately be reducible to three-term syllogisms. Dr. Keynes,‡ however, shows quite clearly that *so long as we retain the orthodox copula-mark*, many types of argument are irreducibly four-termed. Consider, for instance, the *argumentum a fortiori :*

B is greater than C.
A is greater than B.
∴ A is greater than C.

The irreducibility of this argument to 'syllogistic' form is sufficiently shown by the fact that it cannot, through reduction or otherwise, be made to conform to the *Dictum de Omni et Nullo*. A new Dictum is required, and one which must be placed on a par with the old, not in subordination to it. The new Dictum may be expressed in the form, '*Whatever is greater than a second thing which is greater than a third thing is itself greater than that third thing.*'§

* 'Logic,' Book I., § 2.
† 'A System of Logic,' Book II., ch. ii., § 1.
‡ 'Studies and Exercises in Formal Logic,' fourth edition, part iii., ch. vii., pp. 384-388.
§ The endless number of such Dicta that would be necessary in order to do justice to all the possible forms of argument is in itself sufficient proof of their futility as principles.

'There are an indefinite number of other arguments,' continues Dr. Keynes, 'which for similar reasons cannot be reduced to syllogistic form. For example: A equals B, B equals C, therefore A equals C; X is a contemporary of Y, and Y of Z, therefore X is a contemporary of Z; A is a brother of B, B is a brother of C, therefore A is a brother of C; A is to the right of B, B is to the right of C, therefore A is to the right of C; A is in tune with B, and B with C, therefore A is in tune with C. All these arguments depend upon principles which may be placed on a par with the *dictum de omni et nullo*, and which are equally axiomatic in the particular systems to which they belong.'* 'The claims,' adds Dr. Keynes in conclusion, 'that have been put forward on behalf of the syllogism as the exclusive form of all deductive reasoning must accordingly be rejected.'

If we endeavour to go a step further, and consider the relation in which these unorthodox arguments stand to the traditional syllogism, we cannot do better than follow the indication given by Professor Keynes,† and, instead of opposing these arguments to the S.C.D. Syllogism as irreducibly 'four-termed'—the S.C.D. Syllogism being three-termed—admit at once that the vital difference between the two forms of argument lies in a difference of *copula*. All simple Categorical Syllogisms are three-termed by virtue of the Postulate of Mediation.‡ For if a 'four-termed' syllogism is a valid mediate inference, how is the mediation effected? If there are two middle terms, where is the 'common link'? In last resort, the two middle terms must themselves be mediated by a third, through a three-term syllogism. But if the various types of Simple Categorical Syllogism do not differ from each other in number of terms, it must be allowed that in these different forms of syllogism the three terms are differently *related*. Thus, in the *a fortiori* syllogism the symbol for the copula is the sign $>$, whereas in the S.C.D. Syllogism the corresponding symbol is the sign $=$.

The conception of a number of syllogistic types, differentiated from each other by distinctions of copula, finds its fruitful development in what is now known as the 'calculus of relations' or the 'logic of relatives.' Such a calculus, however, can be satisfactorily elaborated only by the methods of Symbolic Logic.

* 'Studies and Exercises in Formal Logic,' fourth edition, part iii., ch. vii., pp. 386, 387.

† *Ibid.*, pp. 387, 388.

‡ *Vide* above, pp. 213, 216.

VIII.

OTHER FORMS OF SYLLOGISM.

(i.) Complex Categorical Syllogisms : Sorites and Epicheirema (ch. xxix.).
(ii.) The Disjunctive Syllogism (ch. xxx.).
(iii.) The Hypothetical Syllogism (ch. xxxi.).
(iv.) The Dilemma (ch. xxxii.).

CHAPTER XXIX.

VIII. (i.) COMPLEX CATEGORICAL SYLLOGISMS: SORITES AND EPICHEIREMA.

The Sorites (in Categorical Form).

A Sorites may be generally defined as a chain of reasoning which links syllogism on to syllogism, but without explicitly drawing any conclusions till the argument reaches its final terminus. If we understand by a polysyllogism a series of syllogisms, of which each proves a premiss of the syllogism that succeeds it, we may say, with Mr. Joseph,* that a Sorites is a 'polysyllogism . . . with the intermediate conclusions suppressed.' The premisses which enter into such a train of argument will be of the form either of first figure premisses or of fourth figure premisses. In the latter case, we have the so-called Aristotelian Sorites, better known as the 'Progressive' Sorites, seeing that Aristotle does not discuss the Sorites.† The form, when categorically expressed, is as follows:

$$\begin{array}{l} S—X_1 \\ X_1—X_2 \\ X_2—X_3 \\ \cdots\cdots \\ \cdots\cdots \\ X_n—P \\ \hline \therefore\ S—P‡ \end{array}$$

When the premisses are of the first figure, the Sorites is known as the Goclenian or Regressive Sorites, Rodolphus Goclenius, Professor at Marburg at the end of the sixteenth century, having first called attention to this form of the argument.§ We then get the form or schema:

$$\begin{array}{l} X_n—P \\ \cdots\cdots \\ \cdots\cdots \\ X_2—X_3 \\ X_1—X_2 \\ S—X_1 \\ \hline \therefore\ S—P \end{array}$$

* 'An Introduction to Logic,' ch. xvi., p. 327. † *Ibid.*, p. 327, footnote.

‡ A particularly fine illustration of this type of Sorites is given by Mr. Joseph (*ibid.*, p. 328). § *Ibid.*, p. 327, footnote.

The fact that a sorites may have all its premisses in the fourth figure by no means implies that the argument can be actually analysed out into arguments in Fig. IV. As a matter of fact, the component syllogisms of the Progressive Sorites, no less than those of the Regressive Sorites, are, *as they stand*, syllogisms in Fig. I.

Hence there are four, and only four, possible forms of the Aristotelian, as of the Goclenian Sorites, corresponding respectively to Barbara, Celarent, Darii, and Ferio. Assuming four premisses, the forms may be specified as follows :

1. Forms of the *Aristotelian* Sorites :

(1) All S's are X's. All X's are Y's. All Y's are Z's. All Z's are P's. ∴ All S's are P's.	(2) All S's are X's. All X's are Y's. All Y's are Z's. All Z's are-not P's. ∴ All S's are-not P's.
(3) Some S's are X's. All X's are Y's. All Y's are Z's. All Z's are P's. ∴ Some S's are P's.	(4) Some S's are X's. All X's are Y's. All Y's are Z's. All Z's are-not P's. ∴ Some S's are-not P's.

2. Forms of the *Goclenian* Sorites :

(1) All Z's are P's. All Y's are Z's. All X's are Y's. All S's are X's. ∴ All S's are P's.	(2) All Z's are-not P's. All Y's are Z's. All X's are Y's. All S's are X's. ∴ All S's are-not P's.
(3) All Z's are P's. All Y's are Z's. All X's are Y's. Some S's are X's. ∴ Some S's are P's.	(4) All Z's are-not P's. All Y's are Z's. All X's are Y's. Some S's are X's. ∴ Some S's are-not P's.

Analysis of the Sorites.

The analysis of the Sorites takes place as follows : The Sorites must be broken up into as many syllogisms in Fig. I. as there are propositions between the first and the last. In the analysed Aristotelian Sorites, S, the subject of the conclusion, is the subject in the conclusion of each of the component syllogisms. In the analysed

Goclenian Sorites, P, the predicate of the conclusion is the predicate in the conclusion of each of the component syllogisms.

It is important in these analyses not to confuse the major and the minor premisses. In the Aristotelian Sorites the first syllogism, in order to be in Fig. I., must have as major the second premiss, not the first. Thus, analysing the first form of Aristotelian Sorites given above, we have as the first syllogism :

All X's are Y's.
All S's are X's.
∴ All S's are Y's.

Then, similarly, this conclusion must be the minor and not the major premiss of the next syllogism :

All Y's are Z's.
All S's are Y's.
∴ All S's are Z's.

So, again :

All Z's are P's.
All S's are Z's.
∴ All S's are P's (conclusion).

Prosyllogisms and Episyllogisms.—In analysing a complex syllogism like a Sorites into its component syllogisms, we see that each component (except the last) is a prosyllogism with respect to the syllogism which succeeds it, and that each component (except the first) is an episyllogism with respect to the syllogism that precedes it.

A prosyllogism is a syllogism of which the conclusion is a premiss in another syllogism with which it is connected.

An episyllogism is a syllogism of which one of the premisses is the conclusion of another syllogism with which it is connected.

Special Rules of the Aristotelian Sorites :

1. No premiss but the first can be particular.
2. No premiss but the last can be negative.

Proof of Rule 1.—The only premiss in an Aristotelian Sorites which is a minor in one of the component syllogisms is the first. Each of the others is, in its turn, a major premiss. Hence, if any premiss in the Sorites, save the first, were particular, it would lead to a particular major in the First Figure. A particular major premiss in Fig. I., however, gives illicit process of the major or undistributed middle.

Proof of Rule 2.—Let us take a typical Aristotelian Sorites, of four premisses, say, and analyse it out into its component syllogisms. This will serve to steady the subsequent reasoning.

Thus :

All A's are B's.
All B's are C's.
All C's are D's.
All D's are E's.
∴ All A's are E's.

This reduces to the following series of syllogisms in Fig. I. :

All B's are C's.	All C's are D's.	All D's are E's.
All A's are B's.	All A's are C's.	All A's are D's.
∴ All A's are C's.	∴ All A's are D's.	∴ All A's are E's.

Now, any premiss of the given Sorites would, if negative, require the conclusion of the syllogism into which it enters as a premiss to be negative.*

But this means illicit major in the episyllogism in the case of all but the final premiss, where the component syllogism is not followed by any episyllogism. For the conclusion of each syllogism—except, of course, the last—becomes the minor premiss of the episyllogism ; and in Fig. I., if the minor premiss be negative, the conclusion being negative and the major premiss affirmative, illicit major follows.

Hence it is only that premiss which enters into the final syllogism —*i.e.*, only the last premiss of the Sorites—that can be negative without fallacy.

The special rules of the Goclenian Sorites may be formulated as follows :

1. No premiss but the last can be particular.
2. No premiss but the first can be negative.

We leave the proof of these two rules to the consideration of the reader.

It will be seen that, in resolving this Sorites into first-figure syllogisms, the two leading premisses must not be transposed, and the conclusion of each prosyllogism serves as major, not as minor, premiss of the episyllogism.

A Sorites cannot be framed in Figs. II. and III.—A Sorites differs in this respect from a simple syllogism : that it has many 'middle terms.' Now, *any* two consecutive premisses of a Sorites in a given

* If the *first* premiss were negative, this negative conclusion could not be drawn at all. The attempt to draw it would involve illicit major (or else two negative premisses).

figure must show the form of that figure. Thus, if we start with two premisses in Fig. III.—say, *e.g.*,

All S's are X's,
All S's are Y's,

the third premiss must take the form 'All S's are Z's'; for it is only on condition that *S* is again subject of the premiss that we can arrange that the second and third premisses together shall supply the data in Fig. III. But the premisses—

All S's are X's,
All S's are Y's,
All S's are Z's,

involve, after the first stage, a *quaternio terminorum*. For from the first two premisses we obtain the conclusion 'Some Y's are X's,' and this has no point of contact with the third premiss, so that the third premiss cannot be said to belong to the same train of argument as the first two premisses. The reasoning, that is, breaks off after the first syllogism, and we have consequently no polysyllogism and no Sorites. A precisely similar argument would show that a Sorites could not be framed in the Second Figure.

But though it is not possible to frame a Sorites in the Second or the Third Figure, it is quite possible, by the aid of Conversion, to resolve those forms of the Aristotelian Sorites which have particular conclusions into Third Figure Syllogisms, and those forms of the Goclenian Sorites which have negative conclusions into Second Figure Syllogisms. The resolution of the Aristotelian Sorites into Second Figure Syllogisms is not possible; for, in order to carry it out, we should have to convert the second premiss, and this would give in every case component syllogisms exhibiting the fallacy of undistributed middle. Similarly, it is impossible to resolve any Goclenian Sorites into syllogisms of the Third Figure.

Referring back to the four forms of the Aristotelian Sorites given on page 256, we see that if we convert the first premiss of (3), we shall change the form of the Sorites somewhat, without affecting in any way the nature of the argument, for the I. proposition is simply convertible.

But when we come to analyse the Sorites whose premiss has been thus converted, we shall find that the component syllogisms are no longer in Fig. I., but in Fig. III., as is clearly shown below:

Some X's are S's.
All X's are Y's.

∴ Some Y's are S's.
All Y's are Z's.

∴ Some Z's are S's.
All Z's are P's.

∴ Some P's are S's. ∴ Some S's are P's.

Form (4) of the Aristotelian Sorites may be treated in the same manner.

So, again, the resolution of the Goclenian Sorites :

All Z's are-not P's.
All Y's are Z's.
All X's are Y's.
All S's are X's.
∴ All S's are-not P's,

into its component syllogisms gives a series of syllogisms in Fig. II., if we start by converting the leading premiss, and convert all the negative conclusions as they occur.

All P's are-not Z's.
All Y's are Z's.
∴ All Y's are-not P's.

Converting this conclusion, we have :

All P's are-not Y's.
All X's are Y's.
∴ All X's are-not P's.

Converting this conclusion, we have :

All P's are-not X's.
All S's are X's.
∴ All S's are-not P's.

The Goclenian Sorites with a particular negative conclusion may be similarly resolved.

We see, then, that we can resolve some forms of Sorites into syllogisms of Fig. II., and others into syllogisms of Fig. III. We may also (*e.g.*, by converting the final premiss of the Celarent type of Aristotelian Sorites) resolve a Sorites into syllogisms which are partly of the first and partly of another Figure (*e.g.*, partly Fig. I. and partly Fig. II.).

The Epicheirema.

The Epicheirema is a chain of reasoning of an abridged and concentrated kind. Chains of reasoning may be either progressive or regressive. When, as in the case of the Sorites, the conclusion of each component syllogism becomes a premiss in an episyllogism, the chain is progressive ; when the movement of thought takes place in the contrary direction—namely, from syllogism to prosyllogism—

the chain is regressive. The Epicheirema is essentially regressive in character. The main argument consists of an ordinary syllogism, but the premisses in the argument are, one or both of them, brought forward as conclusions from other premisses. Thus, the Epicheirema is a syllogism of which each premiss, or one of the premisses, is stated as the conclusion of a prosyllogism (as when the premiss 'All S's are P's' is given thus: 'All S's are P's, for All M's are P's, and All S's are M's'), or, as is more usual, is stated as the conclusion of an enthymeme. When there are two prosyllogisms or enthymemes, the Epicheirema is called 'double'; when only one, it is 'single.'

The following illustrates a *single* Epicheirema:

> 'All rational beings are to be treated with respect, inasmuch as they are made in the image of God.
> Slaves are rational beings.
> Therefore slaves should be treated with respect.'
> (Father Clarke.)

The following illustrates a *double* Epicheirema:

> 'All Malays are cruel, because all savages are.
> All the aboriginal inhabitants of Singapore are Malays, because all the natives of that part of Asia are.
> Therefore all the natives of Singapore are cruel.'
> (Welton.)

The following scheme, given by Professor Welton, illustrates what he refers to as a Complex Epicheirema:

Every M is P, because it is X, and every X is Y.
Every S is M.
∴ Every S is P.

We may analyse this Complex Epicheirema into its three component syllogisms. These are:

(1)	(2)	(3)
Every X is Y.	(Every Y is P.)	Every M is P.
Every M is X.	Every M is Y.	Every S is M.
(∴ Every M is Y.)	∴ Every M is P.	∴ Every S is P.

It should be noted, in connexion with the form of the major premiss, that the addition 'and every X is Y' is not *another reason* added to the first reason 'because it is X,' but rather a proposition which, in combination with it, yields a reason as conclusion. Every Complex Epicheirema, in fact, must be either a single or a double Epicheirema. It cannot be construed into a fresh form distinct from both of these two forms.

CHAPTER XXX.

VIII. (ii.) THE DISJUNCTIVE SYLLOGISM.

THE Disjunctive Syllogism, where there are two alternatives only, may be developed from the Disjunctive Proposition, as basis of inference, in *two* ways, giving two distinct moods.

1. THE MODUS TOLLENDO PONENS—*i.e.*, the mood which posits the one alternative by rejecting the other.

Either P or Q.	Either P or Q.
Not P.	Not Q.
∴ Q.	∴ P.

2. THE MODUS PONENDO TOLLENS—*i.e.*, the mood which sublates or rejects the one alternative by accepting or positing the other.

Either P or else Q.	Either P or else Q.
P.	Q.
∴ Not Q.	∴ Not P.

[NOTE.—The minor premiss may posit or reject an alternative in a form which is more definite than that in which it is presented in the major premiss. This is by no means an arbitrary provision, for a syllogism, like any other form of thinking, is essentially a *development* of meaning. What is relatively indefinite in the major premiss may receive clearer definition in the minor premiss and conclusion.

It should be noticed that the Disjunctive Proposition as an integral element in the Disjunctive *Syllogism* requires to be read from the point of view of mere statement-import. Thus, 'Either P or Q' would read: 'Either P or Q, but not statedly both.' So 'Either P or else Q' would read: 'Either P or Q, but statedly not both.']

In a disjunctive syllogism in which there are two, and only two, alternatives, the 'major' is always disjunctive, the 'minor' categorical, and the conclusion categorical. Where there are more than two alternatives in the major, either the minor premiss or the conclusion may be disjunctive. Thus:

Either P or Q or R.
Not P.
∴ Either Q or R.

Either P or else Q or else R.
Not P.
∴ Either Q or else R.

Either P or Q or R.
Neither P nor Q (compound categorical).
∴ R.

Either P or else Q or else R.
P.
∴ Neither Q nor R.

Either P or else Q or else R.
Either P or else R.
∴ Not Q.

Of the two disjunctive moods, the *Modus Tollendo Ponens* is necessarily valid, whether 'either . . . or' be taken to imply 'statedly not both' or 'not statedly both.' It is the mood which brings out the *exhaustive* character of the disjunctive major. The *Modus Ponendo Tollens*, on the other hand, is valid only when 'either . . . or' implies 'not both'—*i.e.*, when the *exclusive* reading is understood. If we have accepted no statement to the effect that So-and-So, while he is either a knave or a fool, *is not both together*, it is invalid to argue that because he is a knave he cannot be a fool.

CHAPTER XXXI.

VIII. (iii.) THE HYPOTHETICAL SYLLOGISM.

HYPOTHETICAL Syllogisms may be either *Mixed* or *Pure*.

A *Mixed* Hypothetical Syllogism has the first premiss hypothetical, the second categorical, and the conclusion categorical; whereas a *Pure* Hypothetical Syllogism has both premisses and the conclusion hypothetical.

The two fundamental forms of the *Mixed* Hypothetical Syllogism are known respectively as Constructive and Destructive. Considered as Moods of the Syllogism, they are respectively known as the *Modus Ponens* and the *Modus Tollens*.

1. *Constructive :*

If A, then C
A
―――――
∴ C

This is the *Modus Ponens*, the mood which in the minor premiss posits the antecedent of the major.

2. *Destructive :*

If A, then C
not C
―――――
∴ not A

This is the *Modus Tollens*, the mood which in the minor premiss removes or sublates the consequent of the major.

The Categorical premiss in the *modus ponens* need not be affirmative, nor, in the *modus tollens*, need it be negative.

Thus :

If not A, then C
not A
∴ C (*modus ponens*).

So again :

If not A, then not C
C
∴ A (*modus tollens*).

The Rules of the Mixed Hypothetical Syllogism.

When once the hypothetical premiss is accepted, then—

1. *If the Antecedent is accepted, the Consequent must be accepted.* This follows at once from the consideration that to disallow the rule is to refuse to accept the hypothetical premiss from which the reasoning starts.

2. *If the Consequent is rejected, the Antecedent must be rejected.* Granted that the Consequent is rejected, then our data are seen to necessitate the rejection of the Antecedent. For the acceptance of the Antecedent would lead to the contradiction of accepting the Consequent which we have just been rejecting. And what it is *contradictory* to accept, we are logically compelled to reject.

Fallacies of the Mixed Hypothetical Syllogism.

1. *Rejection of the Antecedent.*—Rejection of the Antecedent does not entitle us to the inference that the Consequent must be rejected.

For, the rejection of a proposition being identical with the acceptance of its contradictory, we see that to assert that from the rejection of the Antecedent we can infer the rejection of the Consequent is to say that if a proposition C can be inferred from another proposition B, then from the contradictory of B we can infer the contradictory of C. But this is manifestly not the case. For instance, from an A proposition we can infer the corresponding I ; but from O, the contradictory of A, we cannot infer E, the contradictory of I. Therefore, from the rejection of the Antecedent we cannot infer the rejection of the Consequent.

2. *Acceptance of the Consequent.*—Acceptance of the Consequent does not entitle us to the inference that the Antecedent must be accepted.

For to assert that from the acceptance of the Consequent we can infer the acceptance of the Antecedent is to say that if a proposition C can be inferred from another proposition B, then from C we can infer B. But this is manifestly not the case. For instance, the Subalternate can be inferred from the Subalternans, but not the Subalternans from the Subalternate.

Summing up, we see that, while acceptance of the Antecedent involves acceptance of the Consequent, and rejection of the Consequent involves rejection of the Antecedent, nothing can be logically inferred either from the rejection of the Antecedent or from the acceptance of the Consequent.

The So-called Immediate Inferences from Hypotheticals.

We may, perhaps, venture the following suggestions with regard to the so-called 'Immediate Inferences' from Hypotheticals.

1. There can be no logical '*converse*' of the proposition 'If A, then C,' for there can be no inference from the acceptance of the consequent.
2. Nor can there be any logical '*inverse*' of the proposition 'If A, then C,' for there can be no inference from the rejection of the antecedent.
3. There can be no logical '*obverse*' of the proposition 'If A, then C,' not even the proposition 'If A, then not $\overline{C}$'—*i.e.*, 'If we accept the antecedent, we must reject the contradictory of the consequent.' For, as the rejection of the contradictory of any proposition is precisely identical with the acceptance of that proposition, this obverse has no logical value.
4. There can be no logical '*contrapositive*' of the proposition 'If A, then C,' for where there is neither converse nor obverse, there can manifestly be no converted obverse.

   The drawing of the inference 'If A, then C, ∴ if $\overline{C}$, then $\overline{A}$,' which usually goes by the name of contrapositive inference, is just another form of the hypothetical syllogism

$$\begin{array}{c} \text{If A, then C} \\ \text{Not C} \\ \hline \therefore \text{ not A,} \end{array}$$

   and is therefore not immediate. For the proposition 'If $\overline{C}$, then $\overline{A}$,' as an inference from 'If A, then C,' simply states that granted 'If A, then C,' then, if we also grant $\overline{C}$, we must necessarily grant $\overline{A}$.

It remains, of course, perfectly legitimate to posit the proposition 'If $\overline{C}$, then $\overline{A}$' as the contrapositive of 'If A, then C,' provided that the contrapositive is not here taken as an immediate inference, found by converting the obverse of 'If A, then C.' This reservation should be borne in mind in connexion with the problem which immediately follows.

Give the contrapositive of the following proposition :

If either no P is R or no Q is R, then nothing that is both P and Q is R (Keynes).

This takes the form 'If A is accepted, then C is accepted,' of which the contrapositive is 'If C is rejected, A is rejected.'

The rejection of C means the rejection of 'Nothing that is both P and Q is R,' and is therefore equivalent to 'Some things both P and Q are R.'

The rejection of A means the rejection of 'Either no P is R or no Q is R.'

This disjunctive proposition is of the form 'Either X or Y,' of which the contradictory, on the exclusive view, is : 'Either both X and Y, or neither X nor Y.'

Hence the rejection of A is given by 'Either no (P's and Q's) are R, or some P's and some Q's are R.'

The required *Contrapositive* is then the following :

'If some things that are both P and Q are R, then either no P is R and no Q is R, or some P's are R and some Q's are R.'

On the non-exclusive view it takes the simpler form :

'If some PQ's are R, then some P's are R, and some Q's are R.'

Examples.

1. Which of the following arguments are logically correct ?

(*a*) A is B if it is C ; it is not C, therefore it is not B.
(*b*) A is not B, unless it is C ; as it is not C, it is not B.
(*c*) If A is not B, C is not D ; but as A is B, it follows that C is D.
(*d*) A is not B if C is D ; C, then, is not D, for A is B.

(Jevons.)

Put into strict logical form, these run :

(*a*) If A is C, A is B
A is not C
∴ A is not B

Fallacy of Rejection of Antecedent. (Assuming that 'A is not C' $\equiv$ $\overline{\text{A is C}}$, or not (A is C), and similarly as regards 'A is not B.')

(*b*) If A is not C, A is not B
A is not C
∴ A is not B

Valid. *Modus Ponens.*

(*c*) If A is not B, C is not D
A is B
∴ C is D

Fallacy of Rejection of Antecedent. (Assuming that 'A is B' $\equiv$ $\overline{\text{A is not B}}$, and similarly as regards 'C is D.')

(*d*) If C is D, A is not B
A is B
∴ C is not D

Valid. *Modus Tollens.* (Assuming that 'A is B' $\equiv$ $\overline{\text{A is not B}}$ and 'C is not D' $\equiv$ $\overline{\text{C is D}}$.)

2. Arrange the following in proper logical form, and test their validity :

(*a*) It was agreed that unless the weather turned fine, we were to postpone the match ; so, as the weather has not turned fine, the match must be postponed.
(*b*) Men are not pleased if their meals are not served punctually ; thus, as Mr. X. is never kept waiting for his meals, he must necessarily be pleased.
(*c*) If the cat's away, the mice are everywhere ; the cat must then be about, for the mice are nowhere.

Reduced to proper form, the given arguments read as follows :

(*a*) If the weather does not turn fine, the match is to be postponed.
The weather has not turned fine.
∴ The match is to be postponed.
(*Modus Ponens.*)

(*b*) If men are not served punctually, they are displeased.
Mr. X. is served punctually.
∴ He must be pleased.
(Rejection of Antecedent.)

(*c*) If the cat is away, the mice are everywhere.
But the mice are not anywhere.
∴ The cat must be about.

Here the argument is sound (*Modus Tollens*), though the minor, in rejecting the consequent through its contrary instead of through its contradictory, states more than is necessary for drawing the conclusion. The mood is therefore a 'strengthened mood.'

3. Annex the proper conclusion (if any) to the following premisses :

(*a*) If the earth did not rotate on its axis, there would be no alternation of day and night, whereas this alternation does actually occur (Welton).
(*Modus Tollens.*) *Ergo :* The Earth does rotate on its axis.

(*b*) If no men were mad, asylums would be useless ; but they are not useless (Welton).
(*Modus Tollens.*) *Ergo :* 'Some men are mad,' the *contradictory* of the antecedent.

(*c*) If all men were reasonable, all would be contented ; but some are unreasonable.

This involves rejection of the antecedent ; for even if 'unreasonable' be taken as the term *contrary* to reasonable, the 'unreasonable' would still be included under 'those who are not reasonable.' Thus no conclusion can be drawn.

The Transversion of the Hypothetical Syllogism into Categorical Form.

The *Modus Ponens* can be transverted as follows :*

The case of A (being accepted) is the case of C (being accepted).
This is the case of A (being accepted).
∴ *This* is the case of C (being accepted).
(Barbara, Fig. I.)

* It should not be forgotten that these transversions do not give us, in the place of the given hypotheticals, genuine categoricals—*i.e.*, propositions with a categorical meaning. What they give us are simply hypothetical propositions in categorical form. If taken to be a genuine bridge from one type of proposition to the other, the clumsy artifice through which the transversion is effected is no more than 'a transparent piece of self-deception. "The case of" must be taken to mean "the possible case of." Thus, it is simply a synonym for "in case"—*i.e.*, for "if" ' (Professor Stout).

Again, in so far as the rejection of a statement is taken as equivalent to its non-acceptance, the transverse of the *Modus Tollens* takes the following form :

The case of A (being accepted) is the case of C (being accepted).
This is-not the case of C (being accepted).
∴ *This* is-not the case of A (being accepted).
(Camestres, Fig. II.)

These transversions are useful, if only because they serve to emphasize the *mediate* character of hypothetical inference (which has, curiously enough, been disputed) ; and they afford justification for the convenient practice of referring to the hypothetical premiss as the *major*, and the categorical as the *minor*, since, in the transversion, the hypothetical premiss becomes the proposition containing the major term, and the categorical premiss, the premiss containing the minor term.

Again, by these transversions we are able to show that the fallacy of rejecting the antecedent is analogous to illicit major, and the fallacy of accepting the consequent corresponds to undistributed middle.

Thus (*a*), the *rejection of the antecedent,* in the transverse, gives :

The (case of A being accepted) is a (case of C being accepted).
This is-not a (case of A being accepted).
∴ This is-not a (case of C being accepted).
I.e., a categorical pseudo-syllogism, involving *illicit major.*

So again (*b*), the *acceptance of the consequent* in the transverse gives :

The (case of A being accepted) is a (case of C being accepted).
This is a (case of C being accepted).
∴ This is a (case of A being accepted).
I.e., a categorical pseudo-syllogism, involving *undistributed middle.*

The Pure Hypothetical Syllogism.

This takes the form :

If A, then C_1
If C_1, then C_2
∴ If A, then C_2

The Law of Inference here exemplified may be expressed as follows :

'The consequent of the consequent is the consequent of the ground.'

It is a direct embodiment of the principle of Identity.

This same principle of reasoning applies equally to syllogisms of this form, however we may increase the number of the premisses. Thus:

$$\begin{array}{ll} & \text{If A, then } C_1 \\ & \text{If } C_1\text{, then } C_2 \\ & \text{If } C_2\text{, then } C_3 \\ & \quad \ldots\ldots\ldots\ldots \\ & \underline{\text{If } C_{n-1}\text{, then } C_n} \\ \therefore & \underline{\underline{\text{If A, then } C_n}} \end{array}$$

We may conveniently refer to this form as the Standard Sorites (Hypothetical).

If we alter the order of the premisses, starting with the last and proceeding gradually to the first, we get the following form of reasoning, which we may call the Inverted Sorites.

$$\begin{array}{ll} & \text{If } C_{n-1}\text{, then } C_n \\ & \text{If } C_{n-2}\text{, then } C_{n-1} \\ & \quad \ldots\ldots\ldots\ldots \\ & \text{If } C_1\text{, then } C_2 \\ & \underline{\text{If A, then } C_1} \\ \therefore & \underline{\underline{\text{If A, then } C_n}} \end{array}$$

Example.—If humble-bees are abundant, the red clover which humble-bees alone pollinate is also abundant.
If field-mice are few, humble-bees are abundant.
If cats are abundant, field-mice are few.
∴ If cats abound, red clover is also abundant.
(Darwin, 'Origin of Species,' p. 53.)

The analysis of a Sorites of this inverted form presents no difficulty. When analysed out, the argument runs as follows:

(1) Transposing the first two premises, we have

$$\begin{array}{ll} & \text{If } C_{n-2}\text{, then } C_{n-1} \\ & \underline{\text{If } C_{n-1}\text{, then } C_n} \\ \therefore & \underline{\underline{\text{If } C_{n-2}\text{, then } C_n}} \end{array}$$

(2) Combining this conclusion with the third premiss, we get

$$\begin{array}{ll} & \text{If } C_{n-3}\text{, then } C_{n-2} \\ & \underline{\text{If } C_{n-2}\text{, then } C_n} \\ \therefore & \underline{\underline{\text{If } C_{n-3}\text{, then } C_n}} \end{array}$$

and so on.

The analysis of the Standard Sorites proceeds on similar and still simpler lines, for there is here no need for any transposition of premisses. Thus:

$$\begin{array}{rl} (1) & \text{If A, then } C_1 \\ & \underline{\text{If } C_1\text{, then } C_2} \\ \therefore & \underline{\underline{\text{If A, then } C_2}} \end{array}$$

$$\begin{array}{rl} (2) & \text{If A, then } C_2 \\ & \underline{\text{If } C_2\text{, then } C_3} \\ \therefore & \underline{\underline{\text{If A, then } C_3}} \end{array}$$

and so on.

CHAPTER XXXII.

VIII. (iv.) THE DILEMMA.

'THE Dilemma is an argument in which one or other of two alternatives is offered, the admission of either of which involves something further which an opponent will commonly be unwilling to admit. Thus, the adversary is said to be placed in a "dilemma"—that is, he is in a position in which he cannot help admitting that one or other of the two alternatives offered to him holds good, while yet the admission of either involves him in the acknowledgment of something further as dependent on it, which he is unwilling to allow, and so he has only a choice of evils left' (Karslake, 'Aids to the Study of Logic,' ii.,p. 94).

Formal Definition.

The Dilemma is a compound hypothetical syllogism, and is therefore *governed by the laws of the hypothetical syllogism.* It must, however, be partly disjunctive in form, seeing that the essence of a Dilemma is to present a choice of evils. The disjunction may be offered in the consequent of the hypothetical major premiss, but is normally presented in the minor premiss and in the form of a disjunctive proposition.

A Dilemma may be either Constructive or Destructive. It is *constructive* if the argument proceeds from the acceptance of antecedent to the acceptance of consequent, and *destructive* if the argument is from the rejection of consequent to the rejection of antecedent. These two standard forms of Dilemma may be formulated as follows :

1. *The Constructive Dilemma.*

If P_1, then Q_1; and if P_2, then Q_2.
But either P_1 or P_2.
∴ Either Q_1 or Q_2.

Example.—'If I cross the field, I shall meet the bull; and if I go up the lane I shall meet the farmer.
'Either I shall cross the field or I shall go up the lane.
∴ 'Either I shall meet the bull or I shall meet the farmer.' (Stock.)

2. *The Destructive Dilemma.*

If P_1, then Q_1; and if P_2, then Q_2.
But either not Q_1 or not Q_2.
∴ Either not P_1 or not P_2.

Example.—'If he were clever, he would see his mistake; and if he were candid, he would acknowledge it.
'Either he does not see his mistake or he will not acknowledge it.
∴ 'Either he is not clever or he is not candid.'
(Stock.)

The two fundamental forms of Dilemma—the Constructive and the Destructive—may take certain limiting forms which are sufficiently important to call for special consideration.

Thus, in the case of the Constructive Dilemma, the two consequents Q_1 and Q_2 may coincide, and the Dilemma then takes the following simplified form:

If either P_1 or P_2, then Q.
But either P_1 or P_2.
∴ Q.

Example.—'Whether a man acts in accordance with his own judgment or is guided by the opinions of others, his action will be criticized.
'But either he acts in accordance with his own judgment or he is guided by the opinions of others.
∴ 'In any case, his action will be criticized.'

It should be noticed that the 'either . . . or' of the major premiss does not stand for a genuine disjunction. That the disjunction is merely verbal is shown by the fact that we have but slightly to modify the statement of the major premiss to get rid of the disjunction altogether. Thus, instead of saying 'If either P_1 or P_2, then Q,' we may say 'If P_1, then Q; *and* if P_2, then Q.'

In the case of the Destructive Dilemma the two antecedents may coincide. The major premiss then reads, 'If P, then Q_1; and if P, then Q_2,' and the Dilemma takes the following simplified form :

If P, then both Q_1 and Q_2.
But either not Q_1 or not Q_2.
∴ Not P.

Example.—'If table-rappers are to be trusted, the departed are spirits, and they *also* exert mechanical energy.
'But either the departed are not spirits or they do not exert mechanical energy.
'∴ Table-rappers are not to be trusted.' (Carveth Read.)

These simplified forms of the two standard types of Dilemma are sometimes referred to as the Simple Constructive Dilemma and the Simple Destructive Dilemma respectively ; and by way of contrast the standard forms are respectively referred to as the Complex Constructive Dilemma and the Complex Destructive Dilemma. It is, however, essential to bear in mind, when these titles are used, that the fundamental forms of Dilemma are the complex forms, the simpler forms being mere derivatives from these, owing their simplicity to the coincidence either of the two consequents or of the two antecedents. As a diagnostic mark for distinguishing between the complex and the simplified varieties, we may mention the difference in the form of the conclusion. The conclusion, in the case of the Complex Dilemma, is disjunctive ; in the case of the Simple Dilemma, categorical.

We have spoken of the Simple Destructive Dilemma as derived from the corresponding complex form by making the two antecedents coincide. If, in addition to this coincidence of antecedents, we substitute for Q_1 'Either R_1 or R_2,' and cancel Q_2, the Destructive Dilemma takes the following form :

If P, then either R_1 or R_2.
But neither R_1 nor R_2.
∴ Not P.

The peculiarity of this form of dilemma is that the disjunction, instead of being presented in the minor premiss, is presented in the major.

Example.—'If a body moves, it must either move where it is, or where it is not.
'But a body cannot move where it is ; neither can it move where it is not.
'∴ It cannot move at all.'
(*I.e.*, Motion, being unintelligible, is impossible.)
[Zeno's Dilemma].*

* Vide Chapter XXXIII., p. 290; and *cf.* Joseph, *ibid.*, pp. 332-334.

In testing the Formal validity of dilemmas, we have to bear in mind three essential criteria :

1. The argument must, either in major or in minor premiss, present genuine alternatives. Otherwise, however correct the reasoning may be in itself, we have only a pseudo-dilemma. Thus the following, though valid as an argument, is not a dilemma :

> 'Whether Geometry be regarded as a mental discipline or as a practical science, it deserves to be studied.
> 'But Geometry may be regarded as *both* a mental discipline *and* a practical science.
> ∴ 'It deserves to be studied.' (Dr. Fowler, quoted by Mellone.)

Of this reasoning we cannot say that it is 'an argument in which one or other of two alternatives is offered.' For the major premiss, in which alone there is any semblance of a disjunction, is not genuinely disjunctive, but only a compound hypothetical proposition of the form 'If S_1 or S_2, then P.' The disjunctive form of the antecedent is quite illusory, the whole proposition being equivalent to 'If S_1, then P ; *and* if S_2, then P.' Here there is no choice of alternatives offered as in the case of a disjunctive consequent ('If S, then either P_1 or P_2).

2. The principles of hypothetical reasoning must be strictly applied. Where these are infringed we have a *Formally invalid* dilemma.

3. The *logical* structure of the disjunctive proposition, and its import, whether exclusive or non-exclusive, must be steadily kept in view.

The 'Rebuttal' of a Dilemma.

The metaphors used to describe dilemmatic argument seem to have been taken from the old pastime of bull-fighting. Thus, the adversary who is unable to escape the force of a dilemma is said to be 'fixed on the horns of a dilemma.' He is said to *rebut* the dilemma if he can oppose to the argument with which he is assailed a relevant and equally convincing counterpart.

We must attempt to show in what sense and to what extent such rebuttal is possible. Let us start by considering what we can logically understand by the rebuttal of a typical Constructive Dilemma, —a dilemma, that is, of the following form :

$$\begin{aligned} &\text{If } P_1, \text{ then } Q_1 ; \text{ and if } P_2, \text{ then } Q_2. \\ &\text{But either } P_1 \text{ or } P_2. \\ \therefore\ &\text{Either } Q_1 \text{ or } Q_2. \end{aligned}$$

Now, if we were to suppose that the objector who endeavours to rebut this dilemma is restricted, as regards data, to the statements brought forward in the premisses of the dilemma, the only premisses

which he could relevantly utilize for the purposes of rebuttal—utilize, that is, in such a way as to win his opponent's assent to them—would be *inferences* from the original premisses. But even if it were possible to draw such inferences, they would be useless for the objector's purpose; for starting, as he would then be doing, from the very same premisses as his opponent, though perhaps differently expressed and arranged, he could not possibly extract from these any conclusion that would contrast with the conclusion of the original dilemma.

The possibility of a cogent rebuttal depends, then, on the *data* common to the two disputants *not* being exhaustively stated in the premisses of the original dilemma. The premisses of both the dilemma and its counterpart must be drawn from the same set of facts, each disputant selecting from them the data that suit his purpose.

A merely Formal treatment of rebuttal is therefore necessarily inadequate. We must presuppose a certain common fund of material evidence—a certain given situation. But more than this is necessary to account for the peculiar form which dilemmatic argument takes.

If we consider a classical example of Rebuttal such as the Litigiosus (*vide* pp. 293-295), we see that the Counter-dilemma derives its cogency from an adroit juggling with two standards the determinations of which are in every instance precisely opposed. If we symbolize these standards as S_1 and S_2, and explicitly refer to them wherever reference to them is implied, we may formulate the Dilemma (Constructive) and its Counter-Dilemma as follows:

Dilemma: If P_1, then S_1Q_1; and if P_2, then S_2Q_2.
But either P_1 or P_2.
$\therefore$ Either S_1Q_1 or S_2Q_2.

Counter-Dilemma: If P_1, then $S_2\bar{Q}_2$; and if P_2, then $S_1\bar{Q}_1$.
But either P_1 or P_2.
$\therefore$ Either $S_1\bar{Q}_1$ or $S_2\bar{Q}_2$.

Example—Dilemma:

'If I live for others, neglecting myself, then (to judge by egoistic standards)* I shall make myself unhappy; and if I live for myself, neglecting others, then (to judge by altruistic standards) others will make me unhappy.

'But either I shall live for others or I shall live for myself.

'$\therefore$ Either (to judge by egoistic standards) I shall make myself unhappy, or (to judge by altruistic standards) others will make me unhappy.'

* The additions in parentheses would not, of course, appear in the actual presentation of the Dilemma, nor even in its Retort.

Counter-Dilemma :

> 'If I live for others, neglecting myself, then (to judge by altruistic standards) others will not make me unhappy ; and if I live for myself, neglecting others, then (to judge by egoistic standards) I shall not make myself unhappy.
>
> 'But either I shall live for others or I shall live for myself.
>
> '∴ Either (to judge by altruistic standards) others will not make me unhappy, or (to judge by egoistic standards) I shall not make myself unhappy.'

We seem, then, to be justified in concluding that, so far as the Constructive Dilemma is concerned, it is possible, where two standards are alternately appealed to according to the convenience of the reasoner, to devise a relevant Counter-Dilemma that shall meet the original argument upon its own ground by playing its own game. The Retort, then, serves the useful purpose of equalizing matters, and the onus of taking any further steps is appropriately left to the propounder of the original dilemma. Where there is no such shifting of the point of view, no retort is required, and no relevant counter-dilemma is possible. A Rebuttal, so understood, is not a refutation. It is the logical means for exposing the dilemmatic fallacy of Shifting the Standard of Reference, by making a counter-move of a complementary kind.

If we simplify these results by making Q_1 and Q_2 coincide, we obtain the formula for the Simple Constructive Dilemma and its Counterpart or Retort :

Dilemma : If P_1, then S_1Q ; and if P_2, then S_2Q.
But either P_1 or P_2.
$\therefore$ Either S_1Q or S_2Q ; $\therefore$ $\underline{\underline{Q}}$.

Counter-Dilemma : If P_1, then $S_2\overline{Q}$; and if P_2, then $S_1\overline{Q}$.
But either P_1 or P_2.
$\therefore$ Either $S_1\overline{Q}$ or $S_2\overline{Q}$; $\therefore$ $\underline{\underline{\overline{Q}}}$.

Example—Dilemma :

> 'If I am altruistic, then (to judge by egoistic standards) I am unhappy (because whatever happiness I bring about is other people's, and not mine).
>
> 'If I am egoistic, then (to judge by altruistic standards) I am unhappy (because to seek after one's own happiness, says the Altruist, is the surest way of missing it).
>
> 'But either I shall be altruistic or I shall be egoistic.
>
> '∴ I am bound to be unhappy.'

Counter-Dilemma :

'No! I am bound to be happy.

'For, if I am altruistic, then (to judge by altruistic standards) I am sure to find my own happiness (since I don't seek after it).

'And if I am egoistic, then (to judge by egoistic standards) I am sure to be happy (because whatever happiness I bring about is my own).

'But either I shall be altruistic or I shall be egoistic.

'∴ I am bound to be happy.'

The Rebuttal of the Destructive Dilemma.

We may formulate the Complex Destructive Dilemma and its Counter-Dilemma as follows :

Dilemma : If P_1, then S_1Q_1 ; and if P_2, then S_2Q_2.*
But either $S_1\bar{Q}_1$ or $S_2\bar{Q}_2$.
∴ Either $\bar{P}_1$ or $\bar{P}_2$.

Counter-Dilemma : If $\bar{P}_1$, then S_2Q_2 ; and if $\bar{P}_2$, then S_1Q_1.
But either $S_2\bar{Q}_2$ or $S_1\bar{Q}_1$.
∴ Either P_1 or P_2.

The corresponding formulations in the case of the Simple Destructive Dilemma and its Counter-Dilemma will then be :

Dilemma : If P_1, then S_1Q_1 and S_2Q_2.*
But either $S_1\bar{Q}_1$ or $S_2\bar{Q}_2$.
∴ $\bar{P}_1$.

Counter-Dilemma : If $\bar{P}_1$, then S_2Q_2 and S_1Q_1.
But either $S_2\bar{Q}_2$ or $S_1\bar{Q}_1$.
∴ P_1.

* The reference to the Standards S_1 and S_2 would not, of course, be *explicitly* made either in the presentation of the Dilemma or in that of the Counter-Dilemma.

IX.

FALLACIES.

CHAPTER XXXIII.

FALLACIES.

LOGICAL Fallacies are infringements of logical principle, and any classification of fallacies based upon this definition would be appropriate. But what are we to accept as the logical principles, infidelity to which spells fallacy? Here differences in arrangement are inevitable. But we would suggest the following classification of logical principles as most in harmony with the distinctions upon which, in dealing with the fundamentals of Logic, we would lay the greatest stress :

1. The Principle of Definition.
2. The Principle of Inference.
3. The Principle of Proof.
4. The Principle of Inductive Method.

Corresponding to these we should have four main sources of fallacy :

1. Ambiguity.
2. Invalidity.
3. Inconclusiveness.
4. Breach of Method.

Fallacies are, then, of four main kinds. They may be Verbal, Formal or Inferential, Demonstrational, Methodological. Let us consider these types in turn, begining with the Verbal.

1. VERBAL FALLACIES.

Ambiguity, as we have already seen, is not to be confused with an appropriate *indefiniteness* in our use of words. All indefiniteness does not call for definition, but only such indefiniteness as is not sufficiently definite for our purpose. In practical intercourse it would be pedantic to insist on a purposeless refinement of our meaning. In literature, the infinite—*i.e.*, the indefinite—suggestiveness of a word or phrase is often its main title to excellence. But in scientific terminology and nomenclature, in the enunciation

of all fundamental laws, and in all reasoning that aims at proof, precision in the meanings we give to the terms we use is absolutely indispensable. Here to be indefinite is to be ambiguous and to supply to the reasoned superstructure, that labours to complete itself on the security of undefined meanings, the sandy foundation of the Gospel parable. As an illustration we may take the following argument :

If A is true, I is true.
If I is true, O may be true.
∴ If A is true, O may be true.

Here the conclusion is, to all appearance, self-contradictory, though the premisses are sound and the reasoning valid. The appearance of contradiction vanishes, however, when once the premisses and the conclusion are rid of their ambiguities. The argument then takes the following form :

If A is true, I is true.
If I is true, then (on the ground of I being true) we are unable to state anything certain concerning the truth or falsity of O.
∴ If A is true, then (*on the ground of I being true*) we are unable to state anything certain concerning the truth or falsity of O.

When one and the same word or phrase is used repeatedly (twice or more than twice) in the same argument, it is particularly essential that such indefiniteness as it may legitimately possess should not result in any such variation of its meaning as would amount to its being used in different senses in the various contexts.

When in different parts of an argument the same word or phrase is used in different senses, while the argument proceeds as if these senses were the same, we have the important type of ambiguity known as *Equivocation*.

In illustration of this fallacy we may cite the following example :

'All able men are consistent with themselves.
'He who changes his opinions is not consistent with himself.
'∴ He who changes his opinions is not an able man.'
(Father Clarke.)

Here 'consistent,' in the major, refers to opinions held together and at the same time, whilst in the minor premiss it refers to opinions held at different times. *Cf.* Emerson's dictum that 'a foolish consistency is the hobgoblin of little minds.'

Where the Equivocation arises through a confusion of grammatical form, as between masculine and feminine gender, active and passive

voice, etc., the fallacy is called a fallacy of *Flexion*. Thus the participle of a verb may have acquired a meaning quite distinct from that of other parts of the verb—*e.g.*, 'All presuming men are contemptible ; this man, therefore, is contemptible, for he presumes to believe his opinions are correct' (Jevons).

As a further instance, we may take the following argument of J. S. Mill :

> 'The only proof capable of being given that an object is visible is that people actually see it. The only proof that a sound is audible is that people hear it. And so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable is that people do actually desire it' ('Utilitarianism,' chap. iv., § 2).

In this passage Mill is endeavouring to prove that happiness is the Summum Bonum. He must therefore understand 'desirable' in the sense of 'worthy to be desired.' But if the word, as Mill's argument requires, is to have a meaning analogous to that of the words 'visible' and 'audible,' it must mean 'able to be desired.' Now, if this meaning is given to it, the argument can in no way help Mill to show that happiness is a Moral End or Good.

Where the Equivocation arises through a confusion of the distributive and collective uses of a term, the fallacy is known as a fallacy of *Composition* or of *Division*. There is fallacy of *Composition* when, having predicated something of a term used distributively, we proceed as if the term had been used collectively.

The following will serve as illustrations of this fallacy :

1. All drops of water are small objects.
   The Pacific Ocean is (nothing but) drops of water.
   ∴ The Pacific Ocean is a small object.

2. All atoms are invisible.
   All material objects are (composed of) atoms.
   ∴ All material objects are invisible.

3. No human beings (singly) are 200 years old.
   The British race is (an aggregate of) human beings.
   ∴ The British race is not 200 years old.

There was, doubtless, a latent Fallacy of Composition in Sidney Smith's remark during the discussion as to whether it would be possible to pave St. Paul's Churchyard with blocks of wood. 'The Dean and Chapter,' he said, 'need only put their heads together, and the thing would be done.' But though to infer from those dignitaries' being *individually* blockheads that therefore *collectively* their heads would serve the useful purposes of a wood-pavement

was doubtless an instance of this fallacy, yet another kind of Equivocation also contributed to the argument.

There is fallacy of *Division* when, having predicated something of a term used collectively, we proceed as if the term had been used distributively.

The following may serve as examples of this fallacy :

1. All men are an aggregate including many millions of persons. Hence at least one such aggregate is a man.
2. All coals are atoms of carbon. Therefore some atoms of carbon are coals.
3. English people are not cowardly. Therefore no cowards are English people.
4. The angles of a triangle (collectively) are equal to two right angles. A, B, C (distributively) are the angles of a triangle (distributively). Therefore A, B, C (distributively) are equal to two right angles.

Perhaps the commonest form of the fallacy is that which it takes in such arguments as 'It must be wrong for you to act in this manner, because if every one did so, the consequences would be disastrous.' We start by urging that if A *and* B *and* C . . . (conjunctive) acted in some specified manner—*e.g.*, all studied Logic to the neglect of business—the welfare of the world would be fatally affected, and we go on to argue that no less fatal consequences must follow when A *or* B *or* C . . . (disjunctive) act in the manner specified—*e.g.*, become so enamoured of Logic as to allow the study of it to lead to the neglect of the interests of business.

A good illustration of this fallacy occurs in the 'Imitatio Christi' attributed to Thomas à Kempis (Lib. I., cap. xx., § 8). Here the writer, who is exhorting the 'good monk' to seek no earthly delight, but to remain alone in his cell, reasons with him thus : 'What canst thou see elsewhere which here thou seëst not ? Behold the sky and the earth, and all the elements ; for of these all things are made.' Expressing this argument syllogistically, we see that the fallacy lies in the 'division' of the Middle Term :

The elementary substances (in organized combination) are the whole material world.
The objects included in the prospect from your cell are the elementary substances (not so combined).
∴ The objects included in the prospect from your cell are the whole material world.

A more important illustration of the fallacy of *Equivocation* is supplied by the fallacy of *Accident* and the Converse fallacy of Accident.

The fallacy of *Accident* consists in first employing a term or proposition in a relatively indeterminate or unconditioned sense,

and then proceeding as though it had been used in a sense relatively determinate or conditioned. The nature of this fallacy is sufficiently described by its Latin name, 'Argumentum a dicto simpliciter ad dictum secundum quid'—that is, an equivocal transition from a statement in its general or indeterminate form to the same statement 'with a modification.' Thus, in discussing ethical questions, some one may insist that all men by nature seek after the good, meaning that each man acts *sub specie boni*, and is therefore seeking after some kind of good, even when he is pursuing his own pleasure. But, in continuing the argument, he may use the term 'good' in a differentiated sense—*e.g.*, in the sense of the 'common good,' or the good of humanity—and come thereby, through a fallacy of accident, to the conclusion that all men by nature seek the general good.

The fallacy can be easily exposed, once the ambiguity in the use of the term 'the good' is remedied, and the equivocation thus eliminated. In the light of a Formal criticism, the fallacy of Accident, when used in Syllogism, is seen, like all the other fallacies of equivocation, to reduce to a *quaternio terminorum*. As a further instance of this fallacy, we may cite the following :

The killing of living creatures is sometimes necessary.
Murder is the killing of living creatures.
∴ Murder is sometimes necessary.

In the Converse Fallacy of Accident we first employ a term or proposition in a relatively determinate or conditioned sense, and then proceed as though it had been used in a sense which is relatively indeterminate or unconditioned. We argue 'a dicto secundum quid ad dictum simpliciter.'

Example.—To drink wine in excess is injurious to health. Hence, since it is wrong to injure our health, we see that to drink wine is wrong.

Ambiguity may lie not only in the use to which a word or phrase is put, but in the very structure of a sentence. The corresponding fallacy is then known as the fallacy of *Amphibole*, or Ambiguous Structure, a sentence being 'amphibolic' when it admits of a double interpretation.

Example.—'The wolf the shepherd slew.'
Here we are left uncertain whether it was the wolf or the shepherd that was killed.

As another example of this fallacy we may take the response of the oracle to Pyrrhus : 'Aio te, Æacida, Romanos vincere posse.' 'Pyrrhus the Romans can, I say, subdue.' It was, in fact, the business of the oracle to devise plausible Amphiboles.

The fallacy of *False Parenthesis* is an important form of Amphibole—*e.g.*, 'I will go and return to-morrow.' This may either mean 'I will (go and return) to-morrow,' or it may mean 'I will go (and return to-morrow).' *Cf.* also the following: 'I ruined the Cause and injured my own prospects, which I deeply regret.'

The following advertisements accurately illustrate the fallacy of Amphibole:

(*a*) Wanted: a groom to look after two horses of a pious turn of mind.
(*b*) For sale: a Newfoundland dog; will eat anything—very fond of children.
(*c*) Lost: a valuable silk umbrella belonging to a gentleman with a curiously carved head (Welton).

2. Formal or Inferential Fallacies.

The main breaches of the Principle of Formal Inference, not to go beyond the accepted premisses, we have already considered. To break any rule of the Syllogism is to commit a Formal fallacy. As these breaches of rule have already been discussed and illustrated in dealing with the rules themselves and their application, there can be no call here for their further consideration.

We need only allude to one important point, and that is the relation of Formal to verbal fallacy. To apprehend that relation logically, we must see it in the light of the following rule: 'Never consider whether an argument presents a Formal fallacy or not, until it has been adequately cleared of all its verbal ambiguities.' All verbal fallacies must, in fact, be rectified before the argument can be logically convicted of any Formal fallacy. In other words, the Principle of Logical Consistency depends for its correct application upon a due preliminary observance of the Principle of Non-Ambiguity. Thus, if we wish to discuss the validity of the following argument, 'Some men are selfish; therefore it may be true that some men are not selfish,' we must first of all know precisely what we mean by the words 'some' and 'may.'

A further reason why the verbal fallacies must be disposed of first is that this is part of the process of reducing to strict logical form. Until this is done, it is irrelevant to speak of Formal Fallacies—or, at least, it is quite impossible to detect them.

We may add that all Verbal Fallacies in argument would, if the argument were written syllogistically, reduce to Quaternio Terminorum, except Amphibole, which would simply vanish.

3. Demonstrational Fallacies.

Demonstrational fallacies may be divided under two main heads:

(1) Fallacies of Irrational Evidence.
(2) Fallacies of Illicit Proof.

(1) *Fallacies of Irrational Evidence.*

The characteristic appeal which Science, in its processes of reasoning, makes to the mind is the *argumentum ad judicium*, or appeal to the reason. When the appeal is *not* to the impartial reason but to the feelings, passions, prejudices of men, it is, from the logical point of view, radically *irrelevant*, and involves the fallacy of Irrational Evidence.

In the Essay concerning Human Understanding (Book IV., ch. xvii., § 22) Locke contrasts what he terms the *argumentum ad judicium* with certain so-called arguments which, instead of making an appeal to impartial reason, address themselves to the modesty, ignorance, passions, and prejudices of men. 'Of all the arguments,' he writes, 'that men ordinarily make use of, the argument *ad judicium* alone brings true instruction with it, and advances us in our way to knowledge.' 'For,' he adds, '(1) it argues not another man's opinion to be right, because I, out of respect, or any other consideration but that of conviction, will not contradict him. (2) It proves not another man to be in the right way, nor that I ought to take the same with him, because I know not a better. (3) Nor does it follow that another man is in the right way because he has shown me that I am in the wrong. I may be modest, and therefore not oppose another man's persuasion; I may be ignorant, and not be able to produce a better; I may be in an error, and another may show me that I am so. This may dispose me, perhaps, for the reception of truth, but helps me not to it; that must come from proofs and arguments, and light arising from the nature of things themselves, and not from my shame-facedness, ignorance, or error.'

Varieties of Irrational Evidence—(1) *The Argumentum ad Hominem.*—This fallacy has received two different interpretations that should not be confused with each other.

(i.) The first implies in the so-called argument a very glaring irrelevancy. Instead of defining our position rationally, we merely show that our opponent is not the man to attack it. The chief characteristic of this form of argument consists in its evading the real issue by means of some personal calumny or rejoinder. Professor Minto gives the following illustration: 'A story is told of O'Connell that on one occasion, when he had to defend a man who was clearly in the wrong, the counsel for the prosecution was a certain Mr. Kiefe, who had come in for some money in rather a questionable way, and had taken the name of O'Kiefe. O'Connell commenced his defence by addressing his opponent:

"Mr. Kiefe O'Kiefe,
I see by your brief o' brief
That you are a thief o' thief,"

which so disconcerted Mr. O'Kiefe and so tickled the jury that a verdict was returned for the defendant.'

Occasionally this appeal to the individual is justifiable. Thus, if a man is zealous in some cause which brings him in a large income, we are quite justified in urging his interestedness against his right to speak on the subject. But this simply comes to refusing to discuss the matter with him. It should, moreover, be borne in mind that the rejoinder is a mere retort and not an argument, and should therefore be advanced *as a retort.* If advanced as an argument, it exhibits the fallacy of irrational evidence.

The *argumentum ad passiones*, or the *argumentum ad populum*, is an argument similarly irrelevant with the type of *argumentum ad hominem* we have just been considering. Here it is not the judgment that is convinced, but the inclinations and passions.

(ii.) As a statement of the second interpretation of the *argumentum ad hominem*, I borrow the following from Dr. Gilbart,* who himself refers to Dr. Watts' 'Improvement of the Mind' (*cf.* Part I., ch. x., § xii.). It will be seen that it differs essentially from the interpretation already considered in that it *does* imply an appeal to the reason. It cannot, therefore, be classed as a fallacy of *irrational* evidence. If we call it a fallacy of *irrelevant* evidence, it is because its reasoning is not *ad rem*, but *ad hominem.* But this defect, though it prevents the evidence from having any relevance for *scientific* purposes, does not preclude its possessing a certain *formal* relevance. As a strictly *formal* argument, the *argumentum ad hominem*, thus interpreted, may be perfectly valid. 'Sometimes we may make use of the very prejudices under which a person labours in order to convince him of some particular truth, and argue with him upon his own professed principles as though they were true. This is called *argumentum ad hominem*, and is another way of dealing with the prejudices of men. Suppose a Jew lies sick of a fever, and is forbidden flesh by his physician, but hearing that rabbits were provided for the dinner of the family, desired earnestly to eat of them ; and suppose he became impatient because his physician did not permit him, and he insisted upon it that it could do him no hurt: surely, rather than let him persist in that fancy and that desire, to the danger of his life, I would tell him that these animals were strangled, which sort of food was forbidden by the Jewish law, though I myself may believe that law is now abolished.

'Encrates used the same means of conviction when he saw a Mahometan drink wine to excess, and heard him maintain the lawfulness and pleasure of drunkenness : Encrates reminded him that his own prophet Mahomet had utterly forbidden all wine to his followers, and the good man restrained his vicious appetite by his superstition when he could no otherwise convince him that drunkenness was unlawful nor withhold him from excess !'

* 'Logic for the Million,' p. 125.

(2) *Argumentum ad Verecundiam.*—This is an appeal to a man's modesty. It consists in alleging, as of more weight than a reasoned proof, the opinion of the majority, the wisdom of the aged, or the tradition of the elders. 'When men are established in any kind of dignity,' writes Locke,* 'it is thought a breach of modesty for others to derogate any way from it, and question the authority of men who are in possession of it. This is apt to be censured, as carrying with it too much of pride, when a man does not readily yield to the determination of approved authors, which is wont to be received with respect and submission by others; and it is looked upon as insolence for a man to set up and adhere to his own opinion against the current stream of antiquity, or to put it in the balance against that of some learned doctor, or otherwise approved writer. Whoever backs his tenets with such authorities, thinks he ought thereby to carry the cause, and is ready to style it "impudence" in anyone who shall stand out against them. This, I think, may be called *argumentum ad verecundiam.*'

(3) *Argumentum ad Ignorantiam, or Address to our Ignorance.*—'Another way that men ordinarily use to drive others, and force them to submit their judgments, and receive the opinion in debate, is to require the adversary to admit what they allege as a proof, or to assign a better. And this I call *Argumentum ad Ignorantiam.*'†

(2) *Fallacies of Illicit Proof.*

The two main fallacies to which we are liable when, in the light of a truth-interest, we seek to prove conclusions from material evidence are the Fallacies of *Petitio Principii* and *Ignoratio Elenchi*. The Dilemma, however, is liable to proof-fallacies of a peculiar kind, and we shall therefore treat of these separately.

(i.) *Petitio Principii.*—Mr. Alfred Sidgwick‡ has aptly defined *Petitio Principii* as 'the surreptitious assumption of a truth you are pretending to prove.' The procedure is popularly known as Begging the Question. The terms 'Circular Reasoning' and 'Arguing in a Circle' are sometimes reserved to indicate an argument in which a premiss is used that cannot be proved except by means of the very conclusion that it is to assist in proving. Such reasoning, however, involves precisely the same fallacy as a *Petitio Principii*, though in a form that is less direct and less easy to detect, and we therefore propose to use *Petitio Principii* in the larger sense of the term, which includes both the direct and the indirect forms of begging the question at issue.

That the fallacy is Material and not Formal will be apparent when we consider what we mean by Formal Proof, what we mean

* 'Essay concerning Human Understanding,' ch. xvii., § 19.
† Locke, *ibid.*, ch. xvii., § 20.
‡ 'The Use of Words in Reasoning,' p. 131.

when we say that the conclusion is proved to be valid on the assumption that certain premisses are accepted. There is here no pretence of proving the conclusion *as a matter of fact;* we profess only to prove it as *a conclusion validly inferred.* We cannot, however, in the premisses assume the conclusion *qua conclusion,* for until the premisses are given no conclusion can be drawn. And yet, in a Formal argument, the conclusion cannot be conceived out of relation to the premisses from which it is inferred—cannot be conceived, that is, in any relation other than that of a conclusion from the premisses in question.

What, in fact, is meant by a *petitio principii* is that, in our endeavour to establish the material truth of a proposition, we actually utilize for this proof, at one stage of it or another, the very proposition we wish to prove. The attempt to prove the Principle of Identity by means of the Principle of Non-Contradiction* is a case in point. Another instance is supplied by the famous dilemma of Zeno the Eleatic, in which that paradoxical thinker undertook to prove the impossiblity of motion.

Zeno's Dilemma :

'If a body moves, it must move either where it is or where it is not.

'But a body cannot move where it is; neither can it move where it is not.

'∴ It cannot move at all.

'*I.e.*, Motion is impossible.'

The conclusion cannot be gainsaid if once we grant the major, seeing that the major covertly assumes it. The truth, however, is that bodies move neither where they are nor where they are not, but '*from* where they are *to* where they are not. Motion consists in change of place; the major assumes that the place is unchanged—that is, that there is no motion' (*cf.* Minto, 'Logic,' pp. 224, 225).

The argument is really equivalent to the following :

'If a body moves, it must move under conditions which render motion impossible.

'But a body cannot move under conditions which render motion impossible.

'∴ It cannot move at all.'

Here the *petitio principii* is manifest.

Let us take as a further illustration the following argument, by means of which Father Clarke illustrates this fallacy :

'The Catholic Church is infallible. Therefore its sayings are infallibly true.

'The Catholic Church maintains the inspiration of the Bible.

'∴ Therefore the Bible is inspired.

* *Vide supra*, p. 188.

'Now, the Bible being inspired, all its statements are infallible.
'But the Bible states that the Catholic Church is infallible.
'∴. *The Catholic Church is infallible.*'

If this be taken as a merely Formal inference, there is no *Petitio Principii*. There is, of course, that futile form of tautology which consists in the respondent's saying that, as the infallibility of the Catholic Church is accepted by the questioner, the conclusion that it is so, on that assumption, follows irresistibly. The conclusion is as irresistible as it is tautological.

But if the questioner is anxious to discover a cogent proof of the infallibility of the Catholic Church, based on real evidence, the attempt in question involves *Petitio Principii* in its most unveiled form. The real ground for supposing that the Catholic Church is infallible, it says in effect, is that it is infallible. This is the fundamental fallacy of Dogmatism—the fallacy of mistaking assertion for proof—and this is the fallacy of *Petitio Principii* in its blankest and absurdest form.

As an instance of a *Petitio Principii* only one degree less glaring, we have the case in which the conclusion, in an argument intended as a real proof, is exactly *synonymous* with one of the premisses. Thus, a Sophist may bring forward a proposition expressed in words of Saxon origin, and give as a reason for it the very same proposition stated in words of classical origin.

E.g.: 'To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the State, for it is highly conducive to the interests of the community that each individual should enjoy a liberty perfectly unlimited of expressing his sentiments' (Whately).

(ii.) *Ignoratio Elenchi.*—The *Ignoratio Elenchi* may be described as an evasion of the point at issue. In a long argument, for instance, the ground may be shifted, and the actual conclusion reached may thus be different from the conclusion which the issue requires. Where the issue is deliberately evaded, the *Ignoratio Elenchi* becomes a fallacy of 'surreptitious conclusion.'

This view of the *Ignoratio Elenchi* does not exactly reproduce the original interpretation of the fallacy, of which it is a modernized modification. The 'elenchus' was the technical name given to the final syllogism, in which the contrary or contradictory of the opponent's thesis was shown to be true, and the thesis thereby disproved. Hence *Ignoratio Elenchi* is literally the ignorance of the syllogism required for 'clinching' a point in this special way.

The famous paradox of Achilles and the Tortoise, invented by Zeno the Eleatic, may serve as an illustration of *Ignoratio Elenchi.*

Achilles and a tortoise run a race together. The details of the race are not stated, but we may suppose, for clearness' sake, that

Achilles runs ten times quicker than the tortoise, and accepts, in consequence, a handicap of 100 yards. Under these conditions, says the argument, Achilles will never overtake the tortoise; for when the tortoise has gone 10 yards, Achilles will still be 10 yards behind him. When these 10 yards are caught up, the tortoise will still be ahead by 1 yard. When this yard is caught up, $\frac{1}{10}$ of a yard will still separate the two, the advantage resting with the tortoise. When this $\frac{1}{10}$ yard is covered, the lead dwindles to $\frac{1}{100}$ yard; and yet, though it thus decreases continuously from $\frac{1}{100}$ yard to $\frac{1}{1000}$ yard, and then to $\frac{1}{10000}$ yard, it still finds the tortoise in front and Achilles behind. Achilles, then, though he will be continually drawing nearer to the tortoise, will never actually overtake him.

The main gist of Aristotle's criticism of this paradox is that it involves a confusion of infinite length with infinite divisibility of length. The argument aims at proving that the space which Achilles must cover before overtaking the tortoise is an infinite magnitude; but what it does prove is not this, but simply that the space in question is divisible *ad infinitum*. It is in this failure to lead the reasoning to its right terminus that the *Ignoratio Elenchi* consists.

(iii.) *Proof-Fallacies in the Dilemma.*—When A presents B with a choice between two alternatives, and points out that, whether he choose the one alternative or the other, disastrous consequences will follow, B is threatened with being fixed on the horns of a dilemma. If he can show, however, that the consequences which attach to the acceptance of the alternatives are not really disastrous, or that there are legitimate alternatives other than the two to which he is restricted, he will have freed himself from an embarrassing situation. In technical phraseology, he will have succeeded in his attempt either 'to take the dilemma by the horns' or 'to escape between the horns of the dilemma.' He takes it by the horns when he successfully disputes the truth of its hypothetical propositions; he escapes between the horns when he shows that the disjunctive premiss is false.

For instance, if A informs B that if he takes wine he will gradually become a drunkard, and that if he refrains from wine he will necessarily become dull and anæmic, B may justly deny the truth of both the propositions. He may insist that he may still take wine and not lose his self-control, and that he may also cease taking wine and yet remain spirited and full of life. Or, again, A may urge that men are either fools or knaves, and point out that whether they belong to the one class or the other, the outlook for humanity is a dark one. In this case B may attack A's division of men into cheaters and cheated, and protest that the non-cheated need not be cheaters, but rather men who refuse to allow themselves to be cheated because they have a right sense of the dignity of human nature, and believe in a common good.

In further illustration of a dilemma which can be defeated only by escaping between its horns, we may cite the Fatalistic Dilemma, commonly known as the *Ignava Ratio*.

The Fatalistic Dilemma (*Ignava Ratio*) :

> 'If it is fated that you die, you will die whether you call in a doctor or not ; and if it is fated that you recover, you will recover whether you call in a doctor or not.
>
> 'But it must be fated either that you die or that you recover.
>
> '∴ You will either die or recover, independently of the question whether you will call in a doctor or not.'

The disjunctive premiss in this dilemma is faultily constructed. The action of an inexorable fate is assumed as the basis upon which the disjunction is built up. When the alternatives are stated in the correct form, 'Either it is fated that you die, or it is fated that you recover,' it is clear that a third alternative is possible—namely, '*or* men's destinies are not predetermined by an inexorable fate.' But the admission of this third alternative entirely breaks up the logical force of the dilemma.

The mere *rebuttal* of a dilemma, as we have seen, is not a refutation of it. It serves, however, as an effective check upon the pretensions of the argument rebutted, and if the counter-dilemma set up to meet the original dilemma is as cogent as its rival, the result will be a logical dead-lock.

To transcend the dead-lock, we must weigh the respective claims or values of the two standards which are at once used and abused by the two dilemmas. If we can decide which of the two standards is the more obligatory, our logical duty will be to apply it rigorously to the case at issue, and so, by the substitution of a Single for a Double Standard of Reference, effectively clear up the situation. If the two standards should turn out to be equally obligatory in relation to the issue with which the dilemmas are concerned, we are confronted with a perplexity which, in its abstractly logical way, is analogous to the perplexities from which tragedies arise, both in drama and in real life. We can but deepen the issue, and revise in the light of our clearer insight.

Many dilemmas, however, lead to no such tragical dead-lock. There are comedy-dilemmas as well as tragedy-dilemmas. As an instructive example of the Comedy-dilemma, we may cite the famous and ancient dilemma known as *Litigiosus*.

Litigiosus.—Protagoras the Sophist is said to have engaged with his pupil Euathlus that half the fee for instruction should be paid down at once, and the other half remain due till Euathlus should win his first cause. Euathlus deferred his appearance as an advocate till Protagoras became impatient and brought him into court. The sophist then addressed his pupil as follows: 'Most foolish

young man, whatever be the decision, you must pay your money; if the judges decide in my favour, I gain my fee by the decision of the Court; if in yours, by our bargain.' This dilemma Euathlus rebutted by the following: 'Most sapient master, whatever be the decision, you must lose your fee; if the judges decide in my favour, you lose it by the decision of the Court; if in yours, by our bargain, for I shall not have gained my cause.'

Putting the dilemma presented by Protagoras into strict logical form, we get the following:

> 'If the judges decide in favour of Protagoras, then Protagoras gains his fee by the decision of the Court;
> 'And if the judges decide in favour of Euathlus, Protagoras gains his fee by the terms of the Agreement.
> 'But either the judges will decide in favour of the one disputant, or they will decide in favour of the other.
> 'Hence, whether through the decision of the Court or the terms of the Agreement, Protagoras will gain his fee.'

The *Retort* presented by Euathlus will then run as follows:

> 'If the judges decide in favour of Protagoras, Protagoras will *not* gain his fee by the terms of the Agreement;
> 'And if the judges decide in favour of Euathlus, Protagoras will *not* gain his fee by decision of the Court.
> 'But either the judges will decide in favour of the one disputant, or they will decide in favour of the other.
> 'Hence, whether through the terms of the Agreement or the decision of the Court, Protagoras will not gain his fee.'

With a view to meeting the perplexities of the situation, as expressed through these two dilemmas, let us consider the respective values of the two standards relative to the point at issue. The question at issue is whether Protagoras is or is not to gain his fee. The standards are (1) the deed of agreement, (2) the verdict of the Court. With regard to the claims of these two standards, it seems obvious that the deed of agreement has the prior claim. In fact, it is only on the supposition that the terms of the deed have been ignored or violated by Euathlus that the recourse to law can be justified. But it is quite plain that, whilst plaintiff and defendant are pleading their claims before the law, the deed of agreement remains unviolated. Euathlus has not yet won a case, and, until he has done so, the verdict of the Court, whether favourable or unfavourable, can have no legal value.

The root-error, which consists in the appeal to a second standard when the first is still fully competent to meet all the requirements of the situation, may be detected in another form in the very structure of the disjunctive minor premiss from which both dilemmas are developed. For not only were the judges not obliged to decide

in favour of one or other of the disputants, but it was logically impossible that they should give any verdict at all. There was simply no case to be judged ; for the condition that alone could have justified an appeal to law over the matter—the winning of a case by Euathlus—had never been fulfilled. But where there is no case to be heard, there can be no case to be lost or gained. In particular, a refusal to decide on the part of the judges cannot be interpreted as a verdict in favour of Euathlus. For the judges do not refuse to decide on a case they have considered and found baffling ; they refuse to accept the plaintiff's appeal as constituting a legal case at all. Their verdict is not on the case, but on the nature of the conditions requisite to constitute a case. 'There is absolutely no way out of the difficulty except one. The Court can only postpone the case until the conditions on which a decision depends are fulfilled. Until then, a verdict is impossible. Protagoras had made payment depend on the occurrence of a case which could be decided. No such case had occurred. Hence he must still wait till it did occur. And if it never occurred, he would never get his money. The assumption, of course, is that the verdict is based on the facts of the case. If the Court give an arbitrary verdict, they are simply robbing one or other of the disputants' (Professor Stout).

4. METHODOLOGICAL FALLACIES.

The discussion of fallacies in Method falls naturally within the scope of the Theory of Induction. Here we would simply draw attention to the fact that under fallacies in Method we do not include mere inaccuracies in the application of a Method. Such inaccuracies have importance only for the special science which they concern : their significance is not methodological. More generally, errors of fact are not fallacies. 'If the falsity of the premiss,' says Mr. Joseph (*ibid.*, ch. xxvii., p. 532), 'can only be ascertained empirically, there is error, but not fallacy.' Thus it would be an error, but not a fallacy, to assert that the whale is a fish, or that a dromedary has two humps. If, on the other hand, instead of observing all instances *relevant* to the testing of an idea, we were to observe only such as were favourable to it ; or if, again, we were to ignore the limitations of Inductive Method, and speculate as to the purposes of things instead of inquiring after the laws which they obey, we should commit breaches of Method, and the fallacy would be methodological.

X.

THE PROBLEM OF INFERENCE.

(i.) Mill's Estimate of the Syllogism (ch. xxxiv.).
(ii.) The Function and Value of a Formal Discipline (ch. xxxv.).
(iii.) Truth-Inference, formal and real (ch. xxxvi.).

CHAPTER XXXIV.

X. (i.) MILL'S ESTIMATE OF THE SYLLOGISM.

An Inference, according to Mill, if it is to be an inference at all, must lead us to new truth, must bring us from the known to the hitherto unknown. Now since what we *do* know, according to Mill, are the particulars of sense-observation and the observed resemblances between them, these constitute the natural starting-point of Inference ; and the procedure whereby we improve on such initial knowledge is formulated by Mill as follows :

> 'Certain individuals have a given attribute.
> 'An individual or individuals resemble the former in certain other attributes.
> '∴ They resemble them also in the given attribute.'

This, says Mill, may be taken as 'an universal type of the reasoning process.'*

All inference, then, according to Mill, is from individuals to individuals, from particulars to particulars. The child whose fingers have once been burnt makes this particular experience his reason for not touching the grate with his fingers again, and has no thought of any general maxim such as 'Fire burns.' So the village matron, consulted as to how to treat her neighbour's sick child, 'pronounces on the evil and its remedy simply on the recollection and authority of what she accounts the similar case of her Lucy.'†

After stating that all Inference is from particulars to particulars, Mill goes on to identify such inference with inductive inference. All inference, according to Mill, is inductive, and always consists in reasoning from particulars to particulars. Now the usual view of inductive inference is that it consists in arguing from particulars to universals.‡ Mill quite accepts this usual view, harmonizing it with his own through his own peculiar view of the meaning of generalization.

* J. S. Mill, 'A System of Logic Ratiocinative and Inductive,' Book II., ch. iii., § 7, init. † *Id., ib.*, § 3.

‡ This, of course, is not the same as arguing from the particular proposition I or O to the universal proposition A or E. A particular instance would naturally be expressed in the form of a singular proposition.

If we prefer, argues Mill, as a practical convenience, to argue first from a number of known particulars, such as 'Socrates died,' 'Plato died,' etc., to a general proposition which includes these particular statements—'All men are mortal'—and then to argue syllogistically from this general proposition to the particular proposition, 'The Duke of Wellington [then living] is mortal,' we may do so, even with advantage, but we must not fancy that the validity of the inference is in any way increased by taking this circuitous route viâ the universal proposition. For we do not reason *from* this universal proposition, even when we have got it, but only *according to* it. What we do reason from are the particular facts from which the general proposition was first drawn. The universal premiss in a syllogism, according to Mill, contains two elements: a proved part and an unproved part. The proved part is that which registers our previous observations of particular cases; the unproved part adds the inferences from these, and 'instructions for making innumerable inferences in unforeseen cases.'* 'The unproved part is bound up in one formula with the proved part in mere anticipation, and as a memorandum of the nature of the conclusions which we are prepared to prove.'†

With the assertion of the general proposition the *inference*, on Mill's view, is complete. Hence, when we conclude, according to the major 'All men are mortal,' that Socrates is mortal, or that the Duke of Wellington must eventually die, we make no inference, but simply interpret a memorandum. 'The inference is finished when we have asserted that all men are mortal. What remains to be performed afterwards is merely deciphering our own notes.'‡ To syllogize, in a word, is not to infer, but to decipher and interpret.

Mill clinches this statement of his, that Syllogism is not Inference, by showing that, if it be considered as an argument to prove the conclusion, it involves the fallacy of *Petitio Principii*. He takes the example:

'All men are mortal,
Socrates is a man,
∴ Socrates is mortal;'

and points out that 'we cannot be assured of the mortality of all men, unless we are already certain of the mortality of every individual man.'§

In thus accusing the Syllogism (*qua* proof process) of a *Petitio Principii*, Mill does not go so far as to assert that the conclusion to be proved must have formed one of the particular cases through the observation of which the major premiss was first inferred. He expressly guards himself, in fact, against this misinterpretation

* J. S. Mill, 'A System of Logic Ratiocinative and Inductive,' Book II., ch. iii., § 3.

† *Id.*, *ib.*, § 8, last footnote.

‡ *Id.*, *ib.*, § 3.

§ *Id.*, *ib.*, § 2, init.

of his meaning. 'Whoever pronounces the words "All men are mortal,"' he writes, 'has affirmed that Socrates is mortal, though he may never have heard of Socrates; for since Socrates, whether known to be so or not, really is a man, he is included in the words "All men."'* Hence, in asserting all men to be mortal, we are implicitly asserting Socrates to be mortal, though Socrates may have had nothing whatever to do with the *establishment* of the general proposition. Its establishment is an inductive inference from the observations, 'My father and my father's father . . . and . . . other persons were mortal';† and may have been quite independent of Socrates.

CRITICISM.

If we could agree with Mill's use of the term 'Inference,' there would be no reason to quarrel with his views on the function of the Syllogism. For Mill admits that, as a process of *the disimplication of premisses* (the deciphering or interpreting of a memorandum, as he puts it), the Syllogism is a perfectly valid form of reasoning. There is nothing fallacious about the Syllogism, on his view, unless it be regarded as a process of Inference; for Inference, in Mill's sense, is a process of reasoning from the known to the unknown. In his clear insistence on the point that Syllogism is one thing and Induction another, Mill deserves the gratitude of all who profit from his labours.

It would be possible, no doubt, to maintain that Mill goes too far in pressing his charge of *Petitio Principii* against the Syllogism *qua* proof-process; for, granted that the conclusion is implied in the premisses adduced in its support, there is surely some novelty in the transition from the implicit to the explicit. Yet, as Mill justly insists, the major premiss 'All men are mortal' *covers the case* of Socrates, even though we never heard of that particular case, and though the minor premiss has not yet been *stated* and may not yet be known to be true. In asserting the major premiss, we do in fact (whether consciously or not) assert Socrates to be a mortal; for whether we have ever heard of him or not, or whether, having heard of him, we know whether he is a man or not, still, as a matter of fact, he *is* a man. Hence, though we may justly claim that there is a distinction between a 'stated' and a 'covered' case, and that Mill fails to do justice to the distinction, this should not blind us to the fact that the attempt to treat a syllogism as a material proof or inference (in Mill's sense) is fundamentally fallacious, and that if the fallacy is not a *Petitio Principii* strictly so-called, it is a fallacy so closely related to it that it is barely worth while to differentiate the two.

* J. S. Mill, 'A System of Logic Ratiocinative and Inductive,' Book II., ch. iii., § 8, last footnote.

† *Id.*, *ib.*, § 6.

Mill's View of the Nature of Inference.

Inference, says Mill, is from particulars to particulars. But at the same time he recognizes the possibility of making inferences from particular facts of observation to general propositions or laws; indeed, his whole theory of Induction is a setting forth of the methods of such inference. Induction *is* for Mill an inference from particulars to particulars, because he regards the universal propositions arrived at as mere summaries of particular facts—not, however, of the *observed* facts only, but also of all others that may be inferred from these.

Mill clearly recognizes two kinds of inference:

1. Inference from particular facts observed (*e.g.*, 'A, B, C, and others, have died') to a particular conclusion (*e.g.*, 'Socrates is mortal'); and
2. Inference from particular facts observed to a general or universal proposition (*e.g.*, 'All men are mortal'), which, though general, is a *summary* of particulars.

The second kind of inference is for Mill quite as truly inference as the first. All that he contends for is that the second is not indispensably necessary (as an intermediate step) to the first, though he admits that the circuitous route—viâ the universal—to the particular conclusion is of the utmost importance and advantage in many cases, and that it is absolutely indispensable to the testing of the validity of the inference.

We may take it, then, that the substance of Mill's view of Inference is given in the contention that it proceeds from particulars to particulars, and *may* always move from particular data to a particular conclusion *without the intervention of a general proposition.*

It is against the idea expressed in these italicized words that our criticism must be directed. It is necessary to maintain that *without such intervention* there can be no logical reasoning from particulars to particulars, that an indispensable part of the logical process is the disengaging of some universal from the particulars, through the help of a number of particular instances.

The instances which Mill chooses for justifying the process of inference from particulars to particulars, without the intervention of any universal proposition, are almost exclusively selected from the reasonings of children, dogs, savages, and ignorant people. This is no oversight on Mill's part. He is quite aware that the inference which proceeds without generalization is necessarily of a primitive type, or, to use his own expression, 'the rudest and most spontaneous form' of mental operation.* Generalization, he

* J. S. Mill, 'A System of Logic Ratiocinative and Inductive,' Book II., ch. iii., § 3.

insists, is 'the most important of all helps' in reasoning, and not to generalize 'is a defect and often a source of errors.' Mill is, indeed, so conscious that to reason without the help of universal propositions represents a low intellectual level that he makes admissions which clearly show that the reasoning has no *logical* character at all. Thus, when speaking of the man who reasons from particulars to particulars without using intermediate general propositions, he says of him: 'Though he may conclude rightly, he never, properly speaking, knows whether he has done so or not.'* But the truth is that, in all the cases adduced by Mill in support of his thesis, the reasoning, if even it rises at all above the level of mere association, is certainly not logical—*i.e.*, it is not actuated by any logical ideal consciously held, it is not reasoning conscious of the grounds that justify it. The mental process which keeps the once burnt child from allowing itself to be burnt again is probably the result of mere association. Inference is reasoning in its *logical* aspect, reasoning conscious of the grounds upon which it ventures its conclusions, so that it is not illuminating to have instances of 'inductive instinct,' the tendency to learn somehow by experience, offered to us in place of instances of 'inductive inference.' This inference of one particular fact from another, says Mill, is a case of induction; and he adds that it is of this sort of induction that brutes are capable—'Not only the burnt child, but the burnt dog, dreads the fire.'†

If Mill had considered what really happens when we argue, on the basis of real evidence, from one particular concrete fact to another, he would have seen, as he practically *did* see in writing his later chapters, that we are logically justified in so doing only when we have first discovered some general law, or laws—preferably some causal law—that shall bind the two instances together.

Consider the argument, 'All men who have so far lived have also died. Therefore So-and-so, now living, will also die in his turn.' This conclusion can be logically reached only by a process which, through observation and analysis, passes from the observed instances to some general law indissolubly connecting mortality with the conditions of human existence. The process of explanation (as we should prefer to name it) moves, therefore, not from one particular to another, but from particular instances to a systematic network of general laws, and back again from this network of laws to other particular instances.

It may not be amiss to close this discussion concerning Mill's view of Inference with a brief statement, retrospective and prospective, of our own view.

Inference, according to our definition, is the valid disimplication of the meaning of the premisses according to the Laws of

* J. S. Mill, 'A System of Logic Ratiocinative and Inductive,' Book II., ch. iii., § 8.

† *Id.*, *ib.*, § 3.

Thought—of Identity and of Non-Contradiction. Mill, as we have seen, persuaded that the conclusion so reached is not radically novel, denies that this is Inference at all. But the essential requirement of Inference (in our sense of the word) is not novelty, but logical irresistibility. For the really novel and fruitful, we must turn to Explanation. Nor should we forget that the inability of an inferred conclusion to trespass beyond the accepted data does not render it superfluous or tautological. Mill himself admits that the conclusion brings out explicitly what is only implied in the premisses.

We hold, then, that the disengaging of the universal from its expression in particular instances, the disengaging which issues in a generalization or the formulation of a hypothesis, is not *Inference*, but only the *first stage* in scientific Explanation of facts. It is certainly not Inference in the strictly logical sense in which we have defined the term. Its principle, in fact, is to go *beyond* the evidence, from the known to the unknown. It is, therefore, not *Inference*. Nor can it be Explanation, for it includes no verification. It is simply generalization, tentative generalization, as yet undeveloped and untested.

CHAPTER XXXV.

X. (ii.) THE FUNCTION AND VALUE OF A FORMAL DISCIPLINE.

Formal Logic is the Logic of Validity as distinguished from the Logic of Truth, and its central concern is the proper interpretation and correct application, in Opposition, Eduction, and Syllogism, of the fundamental principles of thought.

As a study in mere Validity, Formal Logic has definite positive value. But it is more than a study in correct thinking. It is the indispensable propædeutic for the study of scientific Method. For the deductive reasoning which enters so vitally into the attempt to interpret Fact through Hypothesis, though it is there exercised in the service of material truth, can render this service effective only through respecting the requirements of Validity. A hypothesis is of little use to Science unless its consequences can be inferred with logical precision. And such precision can be adequately learnt only in that school of Validity, the domain of Formal Logic.

Further, though framed in the interests of Validity, the Logic of Form sets us at least on our way toward the Logic of Science. Thus Syllogism, properly understood, introduces us, in the simplest and most instructive way, to the fundamental ideas of scientific theory and system.

The rules of the Syllogism, for instance, form a 'science' in miniature. Dependent as they are on the single principle of logical Validity, which they express in a variety of forms, they foreshadow to the student the principles of Science and the coherent systems of laws into which these principles can be developed. They are, moreover, *collectively* applied in the solution of difficulties, and in this way bring home the important truth that in solving any difficulty in any special science all the systematic resources of the science are available, and may be requisite, for the solution.

At the same time it cannot be too strongly urged that the distinction between the procedure of Formal Logic (of Syllogism in particular) and that of the Logic of Science or Methodology is radical.

This will be clear if we consider for a moment the Enthymeme as the point of departure of the two systems of logical treatment. A man says 'This plant has milky juice; therefore it is poisonous.' Formal logicians, with their interest centred solely in Validity, 'complete' the reasoning as follows:

> 'All plants with milky juice are poisonous plants.
> 'This plant is a plant with milky juice.
> '∴ This plant is a poisonous plant.'

The Logic of Science, or Methodology, completes the reasoning by *analysing* the data of the situation. Has the observation been correctly made? And is the milky appearance of the juice of a plant a trustworthy index to its poisonous character? In a word, whilst a Formal treatment seeks to express the general conditions upon which the *validity* of the reasoning depends, whatever the particular circumstances may be, scientific treatment analyses the conditions upon which depends the *truth* of the conclusion.

It has sometimes been stated that Formal Logic is useless, since people can reason without its help. Let us briefly consider the justice of the statement.

It is, of course, true that Logic does not teach us to reason. As Prof. Carveth Read puts it, 'We learn to reason, as we learn to walk and talk, by the natural growth of our powers, with some assistance from friends and neighbours. But, to be frank,' he adds, 'few of us walk, talk, or reason remarkably well.'* It is the business of Logic to train us into reasoning *well*.

But it may be said that Mathematics, and indeed any study that depends on close reasoning for its evidence, is a discipline in good reasoning. This is true enough; but in none of these sciences are we explicitly taught the *principles* upon which sound reasoning depends. Apart from logical theory we can give no final or conclusive justification of our reasoning. We may fall back, as in

* 'Logic Deductive and Inductive,' third edition, ch. i., p. 6.

Mathematics, on Axioms and Postulates and Definitions; but it is only Logic that concerns itself with studying the nature and value of these first principles.

W. S. Jevons, in the Preface (p. x) to his 'Studies in Deductive Logic,' lays stress on the fact that 'a mathematical education requires to be corrected and completed . . . by a logical education.' The conditions of quantitative reasoning are, after all, *special* conditions, and the pure mathematician is apt to confuse these with the conditions of argumentation in general. Thus, having proved that, if a triangle have two sides equal to one another, the opposite angles will also be equal, a mathematician might be tempted to give a separate proof of the proposition 'If a triangle has two unequal angles, the sides subtended by these angles will also be unequal,' not recognizing that the second statement is an immediate inference from the first (*cf. ibid.*, p. ix).

CHAPTER XXXVI.

X. (iii.) TRUTH-INFERENCE, FORMAL AND REAL.*

WHERE the process of Inference is associated, as it habitually is both in Science and in ordinary life, with an interest in knowing whether the evidence from which a conclusion is validly drawn is itself true or false, the new interest, which is essentially an interest in *verification* and *proof*, tends at once to introduce a radical change into the character of the inference. It is not, however, the *form* of the argument which is thus essentially changed. So long as the logical process consists simply in disimplicating the meaning of given premisses, the process of inference remains structurally the same, whether we are or are not interested in our premisses as items of knowledge. In either case the start is made with given premisses, and the goal is the conclusion *drawn with logical necessity* from these same premisses. What is changed is the function. With the expansion from the validity-interest to the truth-interest, Inference ceases to be an end in itself, and becomes an episode in the inquiry through which we seek the truth of material fact.

There are two well-marked stages in the evolution of the truth-interest, and, corresponding to these, two main varieties of

* The reader will find an interesting and suggestive estimate of the historical connexion between the so-called Traditional Logic and the Logic of Induction in Mr. Joseph's 'Introduction to Logic,' ch. xvii., particularly pp. 344-349. *Cf.* also Prof. Minto's important Introduction to the *second* book of his Logic.

truth-inference. The interest in Fact and in the truth of it may be either *formal* or *real*.

The potential significance of a given range of fact may be conventionally limited in the service of some definite human interest, or it may be regarded, as in the various departments of Natural Science, with a disinterestedness that has no limit except its respect for a reign of law. In the former case, the Truth-inference will be formal ; in the latter, real.

Of the formal types of truth-inference, we may specify three as of special importance. The world of fact within which the truth-interest is restricted may be a world of make-believe, the make-believe world of a child, or the world which is real for legend or romance. Again, it may be a world of closed beliefs, in which authority supplies the facts and the premisses of inference. The truth-interest of the scholastic Logic moved within a world of this kind. Truth, it was held, had already been found. It lay bound up in the creed of the Church and the wisdom of Aristotle. All that was needed was consistently to unravel the implications of these fundamental dogmas and systematically apply them to the needs of life.

Finally, the truth-interest may be strictly limited in the interest of discussion. The hold on Reality in this case may be very feeble indeed. In the game of Question and Answer with which Professor Minto associates the origin of the Aristotelian Logic,* the dominating interest seems still to have been the interest in Validity. Here the questioner, in his endeavour to entangle the respondent in self-contradiction, was limited by this one consideration, that he could not go beyond the admissions of the respondent. His data consisted simply in the respondent's explicit admissions. These admissions it was the questioner's function not to criticize, but to disimplicate.

The interests of discussion may, however, be far more closely bound up with those of truth than was the case in this playful dialectic, or in the formalities of the scholastic discussion-class. They may even be integrally allied with that impartial pursuit of truth which marks the procedure of the Natural Sciences. Scientists may thrash a matter out in the laboratory, with Nature herself as referee, and such discussions would be essentially 'real.' But it is customarily and naturally held that the requirements of discussion, as compared with those of investigation, are relatively subjective and conventional. Disputants must be ready to accept each the other's point of view, and argue *ad hominem* (*vide* p. 288) rather than *ad rem*.

The distinction between a *formal* and a *real* reference to Reality, between formal and real types of evidence, is, within certain limits, a relative distinction. In playing with Reality—and whenever

* W. Minto, 'Logic,' pp. 3 *et seq.*

we content ourselves with formal evidence our contact with Reality may not unjustly be termed playful—one form of 'play' may be relatively real as compared with another. 'Within the domain of Chess, Chess Problems are (relatively) formal, while Games are relatively real.

'In the problems, we start with isolated positions arbitrarily constructed, and considered without any reference to their occurrence as stages in the development of a game. Indeed, many of them could not occur in a game.

'In a Game we proceed according to the same rules of operation as in Problems, but we start with that initial arrangement of pieces which involves all possible chess combinations—the whole system of chess-reality. The subsequent development is within this system, and is throughout controlled by it' (Professor Stout).

There is, however, a point at which the relativity of the distinction gives way, and that is where we pass from Reality as regulated by conventional rules to Reality as governed by natural law. The appeal to Fact as an embodiment of natural law sharply discriminates between a real and a formal attitude towards Reality. Where the reference to Reality stops short of a reference to Reality-as-under-natural-law—*i.e.*, to a general system of Reality—the reference is definitely formal in the logical sense of the word. The first logical conversion from a formal to a real grasp of Reality takes place when material evidence is handled in the interest of the scientific belief in natural law. 'To return to Chess, both games and problems are formal processes from the point of view of Logic. A Game at Chess is logically a formal process, because its total datum—the arrangement of the pieces and the nature of their movements—is not regarded as a part or phase of a general system, but as an ultimate assumption, neither requiring nor admitting any further explanation. And the series of inferences involved in playing a game are quite isolated; they lead to nothing further; they do not even supply a point of departure for the next game' (Professor Stout).

We see, then, that the essential characteristic of the truth-interest, in the form in which it is active in Real Inference, is that the evidence upon which it ultimately relies is the potentially unlimited evidence of Fact unfettered by any convention that would prevent the fact from fully displaying its own peculiar nature as a fact under natural law. The appeal in scientific reasoning is always back to Nature and natural law. The facts of Nature, as facts under law, are here the ultimate repositories of evidence; and the conviction that this evidence remains perennially fresh and full of surprises, and that the fund of reality to be drawn upon is practically inexhaustible, is the characteristic belief of all the Natural Sciences.

Real Evidence, then, differs essentially from formal evidence by

its perpetual back-reference to still undiscovered or unexhausted Fact as its ultimate source of supply. It is essential to the effectiveness of disputational argument based on *formal* evidence that the evidence should be accepted by both disputants, otherwise the deductive process is abruptly checked; but it is quite unessential that the facts, as they exist outside the statements of these disputants, should confirm the evidence the disputants are prepared to accept. On the other hand, in talking with Nature the conditions are changed. Nature is a silent respondent, and even when Science, the questioner, wrings from her a provisional answer, it is only on the condition that the enforced disclosure be revisable in the light of what may be subsequently elicited.

This back-reference to an inexhaustible source of further evidence is the distinguishing and controlling feature of Inference based on Real Evidence. The conclusions from Real Evidence can never be considered closed, except in so far as one can claim complete control over the source of supply, in the sense of understanding all the possible disturbing elements and the conditions of their effective appearance. In mathematical Science alone do we possess over real evidence a control of this kind, and consequently 'closed' conclusions. In formal evidence, on the other hand, the reality referred to may be so limited as to make closed conclusions inevitable. Thus every conclusion that necessarily follows from premisses accepted by all the interested disputants as true bases of argument is considered closed, so long as the consent originally given to these same premisses is not withdrawn.

Evidence, then, may be either formal or real. But in either case the meaning of the terms in which the evidence is presented must be unambiguously understood. There is, however, a difference; for, in order to fix this meaning, we fall back in formal evidence upon formal definitions, whereas in real evidence we fall back upon real or scientific definitions. Again, formal definition itself implies a formal reference to Fact, and real definition a real reference to Fact; so that ultimately it is always the difference in the respective references to Reality that is the essential difference between the two types of evidence. In the one case the reference is fragmentary and subjectively conventional; in the other, it is systematized, objective, and methodical.

We cannot leave the question of evidence in its relation to Inference without recalling the reader's attention to the important fact to which reference has already been made in the Introduction (p. 7) and elsewhere—to the fact, namely, that in drawing an inference the interest may be entirely transferred from the nature of the evidence to the logical nature of the reasoning. The interest in the distinction between truth and falsity may entirely disappear, and the strictly Formal or abstract interest in *Validity* take its place. In such event, the reference to Reality is no longer even playful;

for the interest in the material truth or falsity of our statements* has completely vanished. There is no further make-believe, or conventional acceptance of stated premisses as true. In the strictly Formal treatment of Logic, the treatment whose sole ideal is Validity, the premisses may be true or they may be false, but their truth or falsity is logically irrelevant.

* *Vide* Note, pp. 9, 10.

XI.

INDUCTION AND THE INDUCTIVE PRINCIPLE.

(i.) General Theory of Induction (ch. xxxvii.).
A. The Pure Inductive Method.
B. The Essentials of Induction.
C. Induction and 'Inductive Inference.'
(ii.) Hypothesis (ch. xxxviii.).
(iii.) Generalization (ch. xxxix.).

CHAPTER XXXVII.

XI. (i.) GENERAL THEORY OF INDUCTION.

A. The Pure Inductive Method.

The history of Logic, from the time of Bacon and Galileo downwards, has been mainly determined by the desire to divert Logic from the word-chopping tendencies of formal discussion to the requirements of natural *fact*. The movement in which this desire found a systematized expression is commonly known as the Inductive movement. It aimed at substituting for the consistent elaboration of dogma the true explanation of Nature. Its fundamental working principle was embodied in the general requirement of Fidelity to Fact.

The first tendency of the so-called Inductive logicians was to keep to the letter this new principle of investigation, and to uphold the necessity of allowing the facts to speak for themselves, the mind maintaining with regard to them a purely receptive attitude.

Hence arose the pure Inductive Method, in the stricter, clearer sense of the term—the method which aims at avoiding Hypothesis, for the simple reason that to make a hypothesis is, temporarily, to go beyond the strict evidence of the facts. According to this method 'No hypotheses or guesses are to be made; but we must wait till our tabulations of the particular phenomena reveal the general "form" or principle which belongs to them' (Prof. Creighton, 'Logic,' p. 29).

The natural goal of Science based on this radically empirical method is to become a Science of Statistics so compiled and arranged as to force upon the methodical collector of observations the laws which the facts require to explain them. In this way, laborious method takes the place of the scientific imagination and the happy idea.

Much may be said in favour of this primitive conception of the Inductive Principle. The tendency to avoid the use of Hypothesis *in the beginnings of Inductive Science* is, as a matter of fact, justified by the very nature of the conditions under which alone Hypothesis can be usefully employed. To be fruitful and not barren, a hypothesis must be rooted in a scientific system; where there is as yet no scientific system, we cannot expect any fruitful application of Hypothesis.

Moreover, we are bound to recognize the great value of that impartial and unprejudiced collection of facts upon which this early empiricism laid so much stress. And yet it is imperative to add that the mind cannot collect facts methodically unless it also *selects* them. Not only is there a natural tendency to observe in the light and under the selective guidance of a thought, but, further, it is essential to the interests of scientific progress that the mind should go to meet the facts armed with ideas. It is not enough to be faithful to fact. Science must be faithful to *its own* facts, to the facts *relevant* to its own ideas. What Induction needs as its guiding principle is not a vague Fidelity to Fact, but Fidelity to Relevant Fact.*

Speaking from the standpoint of a developed Science, it follows (1) that, prior to any collecting, the mind should know towards what end and for what purpose the collection is to be made; and (2) that during the collection of material it must be ready to seize on any indication of a law embodied in the material, to make a hypothesis of it, and to test it either by experiment, or, if experiment is impossible, by further collection of material.

In illustration of the form which the Baconian method would thus assume, we quote the following from Darwin's 'Autobiography': 'After my return to England, it appeared to me that, by following the example of Lyell in Geology, and by collecting all facts which bore in any way on the variation of animals and plants under domestication and nature, some light might perhaps be thrown on the whole subject. . . . I worked on true Baconian principles, and without any theory collected facts on a wholesale scale, more especially with respect to domesticated productions, by printed enquiries, by conversation with skilful breeders and gardeners, and by extensive reading. . . . I soon perceived that selection was the keystone of man's success in making useful races of animals and plants. But how selection could be applied to organisms living in a state of nature remained for some time a mystery to me. In October, 1838—that is, fifteen months after I had begun my systematic enquiry—I happened to read for amusement Malthus on *Population*, and being well prepared to appreciate the struggle for existence which everywhere goes on from long-continued observation of the habits of animals and plants, it at once struck me that, under these circumstances, favourable variations would tend to be preserved, and unfavourable ones to be destroyed. The result of this would be the formation of new species. Here, then, I had at last got a theory by which to work.'†

This adoption of the Baconian method seems to have come

* *Vide* Introduction, pp .1-8; also the concluding chapter on the Inductive Postulate.

† 'Charles Darwin: his Life told in an Autobiographical Chapter, and in a Selected Series of his Published Letters. Edited by his son, Francis Darwin,' ch. ii., pp. 39, 40.

naturally to Darwin through his being a born 'collector,' as witness the following extracts :

At eight years of age 'my taste for natural history, and more especially for collecting, was well-developed. I tried to make out the names of plants, and collected all sorts of things—shells, seals, franks, coins, and minerals. The passion for collecting which leads a man to be a systematic naturalist, a virtuoso, or a miser was very strong in me, and was clearly innate, as none of my sisters or brother ever had this taste.'*

'No pursuit at Cambridge was followed with nearly so much eagerness, or gave me so much pleasure, as collecting beetles.'†

During the voyage of the *Beagle* 'another of my occupations was collecting animals of all classses, briefly describing and roughly dissecting many of the marine ones.'†

It is therefore not surprising to read the following : 'My mind seems to have become [at seventy-two years of age] a kind of machine for grinding general laws out of large collections of facts.'

On the other hand, this Baconian method was not adopted to the exclusion of the 'Newtonian.' 'I have steadily endeavoured,' he says, 'to keep my mind free, so as to give up any hypothesis, however much beloved (and I cannot resist forming one on every subject) as soon as facts are shown to be opposed to it. Indeed, I have had no choice but to act in this manner, for, with the exception of the Coral Reefs, I cannot remember a single first-formed hypothesis which had not after a time to be given up or greatly modified.'‡ With this we may compare a passage from his son's 'Reminiscences.' Speaking of his father, the writer remarks :

'He often said that no one could be a good observer unless he was an active theorizer. This brings me back to what I said about his instinct for arresting exceptions ; it was as though he were charged with theorizing power ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into importance.'§

We conclude, then, that, whatever stress we lay on the value of collecting instances with open mind and letting the facts speak for themselves, we cannot dispense with Hypothesis. Hypothesis is needed to give meaning to Fact.

The attitude taken by the radical empiricists of the early Inductive period may be explained, though not, indeed, justified, as a reaction against the apotheosis of dogma in the days of Scholasticism. The principle of investigation here was not fidelity to fact, but

* 'Charles Darwin : his Life told in an Autobiographical Chapter, and in a Selected Series of his Published Letters. Edited by his son, Francis Darwin,' ch. ii., p. 6.

† *Ibid.*, p. 20.

‡ *Ibid.*, p. 52.

§ 'Life and Letters of Charles Darwin, edited by his son, Francis Darwin,' vol. i., ch. iii., p. 149.

loyalty to dogma. The dicta of Aristotle and of Church Theology were treated as beyond the reach of criticism, and the aim of Science was construed as that of reconciling Nature with Dogma, and of proving herself the dutiful handmaid of Philosophy and Theology.

Thus the discovery of Copernicus was opposed by reasoning such as the following :

> 'Theology teaches that the sun was made to give light to the earth.
>
> 'Now, when we wish to light our houses, we do not move the house about the torch. On the contrary, we move the torch about the house.
>
> 'Therefore it is the sun that moves about the earth, not the earth about the sun.'

An old scholastic professor, when his pupil one day brought to his notice Galileo's discovery of sun-spots, made the following characteristic remark :

'My friend, I have read Aristotle twice from beginning to end, and I know that there cannot be spots on the sun. Just wipe your glasses a little more carefully. If the spots are not in the telescope itself, they must be in your own eyes.'*

B. The Essentials of Induction.†

(1) *Principle and Method.*

The principle upon which the early empiricism proceeded—namely, that of Fidelity to Fact—was sound at heart, and, as applied by the empiricists, contained implicitly the tendencies requisite to its own correction. Busied as they were with fact, they were equally concerned about Method; and fidelity to fact implied, as its precondition, fidelity to the method, instrument, or organon, through which the facts were to be approached and studied. For them, fidelity to fact really meant fidelity to fact along the lines of pure inductive method. Relevancy to scientific purpose, though not explicitly recognized, was still unquestionably

* *Vide* Ernest Naville, 'La Logique de l'Hypothèse,' p. 17.

† More correctly 'Scientific Induction,' for inductive procedure may be either formal or real. It is formal in the service of such restricted interests as that which dominated the Socratic method of finding definitions, or that which prompts the guessing of a riddle. It is real in the service of Science. In the chapters that follow, however, we shall treat 'Induction' and 'Real or Scientific Induction' as synonymous terms. We should add that we regard the processes of Definition and Division, whether formal or real, as operations subsidiary to Induction, rather than as Inductive processes proper. The Principle of Fidelity to Relevant Fact is naturally regulative, not only of Induction itself, but of all operations subsidiary to it.

implied in their whole attitude towards Nature. Thus the task of the later Induction has not been that of substituting a fresh principle and a fresh method, but of reconstructing both. *Fidelity to Relevant Fact* may be accepted as an adequate principle for modern inductive research, but it is simply a specification of the vaguer principle of Fidelity to Fact. Still, the specification is of the first importance. In explicitly introducing, through the use of the term 'relevant,' a reference to scientific purpose, it draws attention to the fundamental truth that fact out of relation to idea is meaningless, and that *it is only in so far as fact is relevant to idea that idea can be true to fact.*

Closely connected with this fundamental improvement in the conception of the principle, we have an equally radical improvement in the method of its application. The method was improved by giving Hypothesis a central place in inductive procedure, whilst the empirical ideal was adequately guaranteed by insistence on *Verification.* Through the Verification-test subjective prejudices are warded off quite as effectively as though they had never been allowed a chance of expressing themselves, and Science is the gainer by enlisting the powerful assistance of the scientific Imagination. The due recognition that the limitations characteristic of the earlier empirical method were not only uncalled for in the interests of fact, but prejudicial to such interests, coincides with the first clear perception of what we really mean by that 'Fidelity to Relevant Fact' which is the fundamental guiding idea of Scientific Explanation.

Nor must it be supposed that this jealous care for *Fact* implies any opposition to the interests of *System.* On the contrary, it is precisely the loyalty to Fact that explains the organization of Science into System. A perpetual willingness to be lessoned by fact is, indeed, the characteristic of all Science that has ever succeeded in systematizing itself. And this is very significant; for it shows that the fact which Science calls Nature is itself systematically structured, and that the systematic character of successful Science is forced upon it by the very nature of the facts it endeavours to interpret. And that the facts can only be interpreted systematically means just this—that Nature, being itself systematic, can only be understood in the light of a system. To interpret facts is to systematize them.

The Principle of Induction, then, cannot be understood in any sense that implies disparagement of System. To be controlled by the facts—the relevant facts—just means to be shaped into systematic coherency through the essential coherency of fact itself. In refusing to go beyond the evidence, the scientific spirit is assimilating the principles of an objective order, making them its own, and approximating more and more to the ideal of systematic unity in interpretation.

And yet it would be misleading to say that Science *aims at* its own systematization. For the true lesson of Science is that systematic coherency can be gained only indirectly. To aim directly at being systematically coherent is to forfeit the *true* objective system to which *Fact* is the one and only key, and to win a partial systematization that is hopelessly subjective, and doomed on that account to eventual barrenness.

Hence the supremacy, for Science, of Fact over System.* The true interest of System is secured by subordinating the desire for systematic coherency to the determination to be at all costs faithful to the facts. Fact dominates System because it is, in its true nature, itself the ideal and standard system.

(2) *Hypothesis.*

We have seen that, in the application of the new Inductive Method, Hypothesis is a central and indispensable factor. It is the essential medium of contact between the scientific system, which is being applied to the explanation of facts, and the facts to which it is being applied. As consistent with the system of which it has been provisionally enrolled a corporate member, a hypothesis belongs to the system. As the direct agency through which the facts are to be explained, it belongs to the facts. Consistency is required of it from the one point of view; truth is required of it from the other.

The hypothesis is not artificially grafted on to the scientific system in question, but is its product, and presupposes it. The law of gravitation, as conceived by physical Science, presupposes a whole theory of dynamics, and in the formulation of it the requirements of this theory of dynamics were all along kept rigidly in view (*cf.* Newton's treatment of Descartes' 'Theory of Vortices'). It is the scientific system which gives to the hypothesis its explanatory *resources*, its deductive vitality. Consider how forlorn and resourceless a theological or philosophical hypothesis is when grafted on to a scientific system, and *vice versa.*

(3) *Deductive Inference.*

We may distinguish three main functions of deductive inference:

1. The development of a hypothesis.
2. The application of a hypothesis.
3. The proof of a proposition from axiomatic premisses.

* We do not say 'of Fact over *Idea*,' for Fact is nothing out of relation to Idea. Our point is simply that Science attaches primary importance to being true to Nature in so far as Nature is relevant to its inductive idea. Its own organization as Science is a subsidiary matter.

1. *Deductive Inference as the Process through which a Hypothesis is developed.*—We define an important aspect of deductive inference when we refer to it as the process by which some hypothesis, or theory, is, through its connexion with the resources of a system, developed into such necessary consequences as admit of being directly tested by the facts.

The inference here (in the simplest form of the process) consists in the drawing of a conclusion from two premisses, of which one is the statement of the hypothesis, and the other some statement representing the system. Thus, the hypothesis may be that 'No micro-organisms arise by "spontaneous generation."' Looking at this proposition in the light of our systematized knowledge concerning micro-organisms and the conditions under which their life can be maintained, we are able to connect it with the second proposition, 'Germs produced in sealed sterilized infusions arise by "spontaneous generation,"' and thence we can infer that 'No micro-organisms are produced in infusions so treated.'

> All germs produced in sealed sterilized solutions arise by abiogenesis (premiss supplied by the system).
> No micro-organisms arise by abiogenesis (undeveloped hypothesis).
> ∴ No micro-organisms are produced in sealed sterilized solutions (developed hypothesis).

The development of a hypothesis, or verificandum, will, as a rule, be less simple than in the case of the illustration just given. The development may take the form of a Sorites :

$$\begin{array}{ll} \text{SaM}_1 & \text{(undeveloped hypothesis)} \\ \left.\begin{array}{l} \text{M}_1\text{aM}_2 \\ \text{M}_2\text{aM}_3 \\ \quad\cdots\cdots \\ \quad\cdots\cdots \\ \underline{\text{M}_{n-1}\text{aP}} \end{array}\right\} & \text{(system)} \\ \therefore\ \underline{\underline{\text{SaP}}} & \text{(developed hypothesis).} \end{array}$$

Or, in the equivalent pure-hypothetical form :

$$\begin{array}{rl} \text{If S, then M}_1 & \text{(undeveloped hypothesis)} \\ \left.\begin{array}{r} \text{But if M}_1\text{, then M}_2 \\ \text{If M}_2\text{, then M}_3 \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \underline{\text{If M}_{n-1}\text{, then P}} \end{array}\right\} & \text{(system)} \\ \therefore\ \underline{\underline{\text{If S, then P}}} & \text{(developed hypothesis).} \end{array}$$

Thus we may wish to test the suggestion 'If cats abound, red clover is also abundant.' The development will then take the form of the following Sorites :

If cats abound, red clover is also abundant (undeveloped hypothesis).
If red clover abounds, humble-bees (which alone pollinate this clover) must also abound.
If humble-bees abound, then their inveterate foes, the field-mice, must be few in number. (system)
∴ If cats abound, field-mice are few (developed hypothesis).

No one can deny that, through the development of the hypothesis, the problem of verification has been considerably simplified. So far we have dealt only with that form of development which proceeds by analysing the predicate-concept of a categorical proposition, or the consequent of a hypothetical proposition. The development—though it here assumes a different form—may equally well take place through the analysis of the subject of the categorical, or the antecedent of the hypothetical, proposition. Thus the proposition to be tested may be 'S is P,' and analysis of S may show us that S is M. We then substitute for the original hypothesis, not the developed hypothesis S is M, but a *new* hypothesis, M. is P. For if we can show that M is P, then, since S is M, we shall have shown that S is P. To recur to the illustration just cited, we may wish to justify the statement that, 'If cats abound, red clover also abounds.' We may then argue as follows :

If cats abound, field-mice are few.
If field-mice are few, humble-bees abound.
∴ If cats abound, humble-bees abound.

Now, if we can justify the proposition, 'If humble-bees abound, red clover also abounds,' we shall also have justified the hypothesis we started with, for we can then reason as follows :

If cats abound, humble-bees abound.
If humble-bees abound, red clover also abounds.
∴ If cats abound, red clover also abounds.

2. *Deductive Inference as the Process through which an Hypothesis is applied.*—When once the hypothesis has been suitably developed, the application to particular cases follows at once, so soon as the case can be brought under the hypothesis through the discovery of a suitable middle term. Thus we apply the hypothesis that 'No micro-organisms arise by "spontaneous generation"' to the case of *Bacillus subtilis* by bringing this species under the more general concept of 'micro-organism':

No micro-organisms are produced by abiogenesis.
Bacillus subtilis is a micro-organism.
∴ *Bacillus subtilis* is not produced by abiogenesis.

So, again, we might have occasion to apply the œcological hypothesis that the Whortleberry belongs to the Heather Association of plants. Formulating our hypothesis in the proposition, 'All districts in which Heather is plentiful are districts in which the Whortleberry is found,' we might apply it by the help of an inference such as the following :

All heather districts are whortleberry districts.
Dartmoor is a heather district.
∴ Dartmoor is a whortleberry district.

We see, then, that in the transition from a Formal Syllogistic to Scientific Explanation, the central function of logical Inference, the drawing of conclusions with logical necessity from stated premisses, loses none of its importance. It is, in fact, only when transported into this new inductive setting that logical Inference is seen in its true light as the process through which the implications of *Knowledge* are unfolded. In its Formal setting, Inference did not develop what had been or was to be adequately verified, but only what had been accepted as data in the interests of the Implication-problem. Formal Inference aims only at validly developing accepted premisses ; scientific Inference aims primarily at developing the *truth*. Validity is here a secondary, though still an essential requirement.

Formal Inference gives way, then, in Scientific Method to an ideal of Deductive Inference, to valid inference *from real grounds*—inference as logically necessary as was Formal Inference, but differing essentially from the latter in its function, which, in last resort, is that of furthering the work of scientific explanation. It is through Deductive Inference that the principle of logical necessity is brought into relation with the investigation of fact : whilst retaining a strictly logical character, Inference now becomes an integral factor in the progress of Science (*vide* Chapter XXXVI. on 'Truth-Inference, formal and real ').

Deductive Inference should not be confused with *Deduction*. Deduction is the wider term. It includes not only the strict inference, but also, as in the proof of Euclid's theorems, for instance, the spatial and other imaginative constructions, and those ingenious combinations of the parts of a whole treatment which provide the strictly novel and progressive element in the whole procedure. The systematic construction of complex trains of reasoning involves, to use Mill's phrases, a great deal of ' scientific dexterity ' and ' artful combination.'* The use of Deduction, in the large sense of the word, is always likely to involve this vital, creative element, this native tact or ingenuity, this inventive imagination, which is guided by glimpses, suspicions, analogies, rather than by method.

* 'A System of Logic,' Bk. II., ch. iv., § 4.

In brief, Deduction includes extra-logical elements. Deductive Inference, on the other hand, is a strictly logical process. It can only make the implied explicit. The inexhaustible fruitfulness of mathematical reasoning, though bound up with its deductive character, is in no sense a product of mere deductive *inference*.

Let us consider, in this connexion, the part played by construction in the deductions whereby Euclid proves his theorems. Ostensibly the proofs rest upon the security of definitions, postulates, and axioms. But these in their turn refer us back to Euclidean Space—*i.e.*, to an intuitive basis which is as rich a storehouse of geometrical fact as the living organism is fruitful of data for the biologist. Thus the properties of a circle cannot be deductively inferred from its definition, excepting so far as the definition is that of 'a circle *in* Euclidean Space,' and is supported by actual constructions. To prove that the angle in a semicircle is a right angle, we must construct a circle in Space (as idealized to meet the purpose of our traditional geometry), and draw special lines within the circle. We thus avail ourselves of the inexhaustible fertility of intuited space when penetrated by the invisible network of geometrical ideas. To realize intuitively the continuity and tridimensionality of space, and its susceptibility of being determined into forms through inclosure of its parts, is to be master of properties that admit of endless combination and development through the medium of construction.

This ultimate reference to a space-intuition is concealed in Euclid's Geometry. Thus, as Professor Latta has pointed out,* (1) the definitions and postulates presuppose this system of space 'without showing how the figures described in the definitions, or the right to demand these postulates, follow from the very nature of the space itself'; and (2) the proofs are stated as though they were deduced solely from these definitions, postulates and axioms, whereas 'the proof of each proposition requires a "construction" of some kind to be made, such as the producing of lines or the superposition of figures. . . . If you produce two sides of a triangle in order to prove something about its angles, you implicitly recognize that the triangle is not a self-complete system, the properties of which may be directly deduced from its definition, but that it is an element in a surface, and that its internal properties are logically dependent on its external relations, or, at least, are in the most intimate connexion with them.'

3. *Deductive Inference as the Process through which a Proposition is* PROVED.—In the endeavour to *prove* or *demonstrate* the truth of a proposition, the aim must be to discover a basis of proof which does not itself stand in need of proof, and from which the proposition may be inferred with logical necessity.

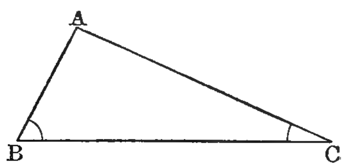
* 'On the Relation between the Philosophy of Spinoza and that of Leibniz,' *Mind*, New Series, vol. viii., 1899, p. 335.

There are three main ways in which we may thus transform our demonstrandum into the conclusion of a deductive inference of which the premisses are either axiomatic or can themselves be demonstrated as necessary conclusions from axiomatic premisses.* The deductive inference through which the proof is in last resort effected may be categorical or disjunctive or hypothetical.

Thus the demonstrandum 'All S's are P's' may be accepted as proved, provided we can accept 'All M's are P's' and 'All S's are M's' as axiomatic propositions or as propositions already proved. The proof will then take the categorical form :

'All M's are P's' is true.
'All S's are M's' is true.
∴ 'All S's are P's' is also true.

Where the inference is disjunctive, the proof proceeds by a Method of Exclusion, and may be called Proof by Exclusion† or Proof by Exhaustion.‡ Suppose we wish to prove the proposition 'S_1 is P_1,' and are in a position to set down the proposition 'Either S_1 is P_1 or S_2 is P_2 or S_3 is P_3' as axiomatic or as demonstrable from axiomatic grounds. We have then only to show that '$\overline{S_2 \text{ is } P_2}$' and '$\overline{S_3 \text{ is } P_3}$' are true in order to demonstrate the truth of 'S_1 is P_1.' Thus, to borrow the example cited by Mr. Milnes, suppose our thesis to be that of Euclid I, 19 : 'If, in any triangle, one of two angles be greater than the other, the side subtending the greater



angle is greater than that subtending the less. This we may prove by arguing as follows :

Either the side AC is greater than the side AB, or else it is less than the side AB, or else it is equal to it (axiomatic). But AC cannot be less than AB, nor can it be equal to it. Therefore AC is greater than AB—*i.e.*, the side subtending the greater angle is greater than that which subtends the less.

* In the discussion that follows, we assume that such axiomatic or self-evident propositions are obtainable. For a brief consideration of this point, see Chapter XLV. There is, however, no attempt made in the present volume to deal adequately with the theory of Mathematical Knowledge, or with the problem of self-evidence.

† See Dr. Christoph Sigwart, 'Logik,' vol. ii., ch. iii., § 81, 10. English Translation by Helen Dendy, p. 201.

‡ *Cf.* Alfred Milnes, 'Elementary Notions of Logic,' second edition, p. 85.

The exclusions through which, in the application of this method, the proof of the non-excluded alternative is effected are frequently justified by a form of proof usually known as *Indirect Proof*. The proposition ' S_2 is P_2 ' is assumed to be true, and it is then shown that on this assumption the proposition ' S_n is P_n ' must also be accepted as true. Hence, if the proposition ' S_n is P_n ' contradicts a proposition already proved to be true, if follows by the Modus Tollens that ' S_2 is P_2 ' cannot be true. Thus the main argument in Indirect Proof runs as follows :

If S_2 is P_2, then S_n is P_n.
But $\overline{S_n \text{ is } P_n}$.
$\therefore$ $\overline{S_2 \text{ is } P_2}$.

The argument through which an indirect proof is effected is known as a *reductio ad absurdum*, the absurdity to which the argument reduces us being that of self-contradiction.

The *Reductio ad Absurdum* has been utilized, under the name of Indirect Reduction or *Reductio per impossibile*, to test the correctness of inferences drawn in the ' imperfect ' figures of the Syllogism (Figs. II., III., and IV.), on the assumption that inferences drawn in the first or ' perfect ' figure, according to the *Dictum de Omni et Nullo*, could be accepted as correct. The process, however, is applicable to syllogistic reasoning only so far as this is treated as a truth-inference, so that the premisses and conclusion can legitimately be characterized as true or false. Indirect Reduction, then, may be defined as a proof, effected by means of a syllogism in the first figure, that the truth of certain conclusions drawn in the ' imperfect ' figures follows with logical necessity upon the truth of their premisses, because, if those premisses are true, the contradictories of those conclusions are necessarily false.

The traditional Logic, following Aristotle,* who in Direct Reduction made use of Conversion only and did not recognize Obversion,† singled out *Baroco* and *Bocardo* as the most suitable forms of Syllogism for illustrating this process of Indirect Reduction. We may therefore take one of these—*Baroco*—and use it for the illustration of the method.

If it is suggested that the inference in *Baroco* is not correct—so the method argues—then in that case, though we assume the premisses to be true, the truth of the conclusion ' Some S's are-not P's ' will not necessarily follow. That is, granted the truth of the premisses ' All P's are M's ' and ' Some S's are-not M's,' still ' Some S's are-not P's ' may be false. But since the denial of a proposition is logically equivalent to the affirmation of its contradictory, it follows that the proposition ' All S's are P's ' may be true. Let us, for the sake of the argument, assume that it *is* true. Taking

* *Vide* An., Pr., A., c. 45, p. 51*b*, 1, 2. † *Cf.* above, pp. 189–192.

this proposition, 'All S's are P's,' as the minor premiss of a syllogism, and combining it with our original major premiss, 'All P's are M's,' we can at once draw the conclusion 'All S's are M's' in the standard or 'perfect' figure. But this conclusion contradicts our original minor premiss 'Some S's are-not M's.' Now the premisses of the original syllogism are both, *ex hypothesi*, true. The statement 'All S's are M's' is therefore false. Hence, since the form of reasoning in Fig. I. is admittedly valid, one at least of the two premisses which necessitated this conclusion must be false ; for, if both were true, then, by the Principle of Identity, the conclusion 'All S's are M's' would also be true. But of these two premisses, 'All P's are M's,' being our original major premiss, is, *ex hypothesi*, true. Hence the *assumed* statement 'All S's are P's' must be false, and its contradictory, 'Some S's are-not P's,' must be true. But this is the conclusion of our original syllogism in *Baroco*. Thus we see that in *Baroco*, granted the truth of the premisses, the truth of the conclusion necessarily follows. We have thus shown that *Baroco* is a valid form of Syllogism.

Any other of the valid forms may be justified in a precisely similar way.

The Method of Proof assumes a peculiar form whenever the premisses from which the demonstrandum is deducible are reversible or simply convertible. In this case these requisite premisses may be discovered by means of a process of regressive analysis based upon the assumption that the demonstrandum is true. We may show that, on this assumption, certain consequences necessarily follow which are already known to be true. We may then reason back, in Sorites form, from these known truths to the demonstrandum. Thus, let D be the demonstrandum. The regressive analysis will then take some such form as this :

If D is true, C is true.
If C is true, B is true.
If B is true, A is true.
But *A is true*.

Now, if the premisses are of such a character as permits us to argue from the truth of the consequent to that of the antecedent—*i.e.*, if the premisses of the sorites are reversible—we can present the proof as follows, in the form of a series of hypothetical syllogisms in the Modus Ponens :

A is true.
But if A is true, B is true.
∴ B is true.
But if B is true, C is true.
∴ C is true.
But if C is true, D is true.
∴ *D is true*.

C. Induction and 'Inductive Inference.'

(1) *The Meaning of Induction.**

It is essential to note the wide sense in which we are proposing to use the term 'Induction.' It is not unusual to identify Induction with the *first* stage in the whole process of Scientific Explanation, the stage which starts with the observation of facts and terminates in the *formulation* of some hypothesis. Were this nomenclature adopted, Induction, Deduction, Verification would be the three successive stages in a complete scientific explanation. But if we do adopt this nomenclature, we must cease to talk of *Inductive* Logic, and must speak, instead, of the Logic of Scientific Explanation. For it is Scientific Explanation, *qua* completed process, which *alone* is governed by the fundamental principle of Fidelity to Relevant Fact. So far as the goal of a reasoning-process is the mere formulation of an undeveloped, unverified hypothesis, it is surely unreasonable to contend that it aims at not transgressing the evidence of fact. Does it not embody a tendency to go beyond the facts rather than *not* to go beyond them ?

Assuming, then, that we accept 'Fidelity to Relevant Fact' as the Inductive Principle, the use of the word *Induction* in its narrower sense of a tentative passage from particulars to universals is, strictly speaking, illegitimate. We cannot be faithful to fact by reposing on untested generalizations from experience and dispensing with the test of Verification. In the wider and legitimate sense of the term, 'Induction' covers the whole process of Scientific Explanation, the formulation and verification of Hypothesis, a process which ends only when, through its various methods, it has made sure that its tentative explanation does not go beyond the evidence of the facts.

(2) *The So-called 'Inductive Inference.'*

No account of Induction, however introductory in character, can dispense with an allusion to the much-abused term 'Inductive Inference.' We hold, for our part, to the simple conviction that there is only one fundamental type of logical Inference—that, namely, which consists in rendering explicit what is implied in a system of given premisses, through the sole help of the principle of logical Validity. We consequently view the term 'Inductive Inference' as a misnomer. Inference may be Formal or Deductive ; may be drawn, that is, from Formal premisses in the light of a strictly abstract validity-interest, or else from material grounds in the light of a genuine truth-interest. But in either case it is a strictly

* See also footnote, p. 316.

logical process governed exclusively by the Law of Identity and the Law of Non-Contradiction. Now, in inductive procedure, the only inferential stage of this kind is that of the deductive development and application of Hypothesis. Hence 'Inductive Inference' is either a misnomer for Deductive Inference, or it is the name for a type of thinking which is not exclusively governed by the Law of Logical Validity—that is, by the above-named Laws of Thought.

It is in this latter sense, of a heuristic, tentative suggestion or supposition, that the term is customarily used. Thus, having observed that a large number of instances of a certain class have the mark x, we are said to *infer* (inductively) that *all* the instances of that class may be found to possess the mark x. So again, in the case of what is known as 'Analogical Inference,' there is a precisely similar use of the word 'Inference' in the sense of a tentative, though it may be a well-grounded, suggestion. We are accustomed to say that, since A resembles B in many important respects, we *infer* (by Analogy) that it will resemble it in some further respect also.

The meaning we have given to the term 'Inference' prevents us, however, from making use of it to designate any tentative form of argument, whether enumerative, analogical, or of any other kind. We therefore, somewhat reluctantly, renounce the use of the familiar and time-honoured term 'Inductive Inference,' and with it the use of such cognate expressions as 'probable' and 'analogical' inference. The Theory of Induction, as we conceive it, is Induction without Inductive Inference. As a general substitute for 'inference' in this sense, we propose to use the term 'conjecture.' Thus, on the ground that certain S's are P's, we *conjecture* that all S's will be found to be P's ; and, on the basis of the many important resemblances between the Earth and Mars, we *conjecture* that Mars will resemble the Earth in being inhabited also. In this way we hope to avoid confusion, though we cannot hope either to satisfy the ear or to uproot the inbred prejudice in favour of drawing 'inferences' from grounds which do not imply but only suggest them.*

* It may be worth while, at this point, to draw attention to the ambiguity attaching to the present use of the more natural term 'Inference.' As currently used, it denotes now a Formal inference, now a deductive inference, now a deduction, now a complete induction, now some form of tentative guess-work culminating in a hypothesis. It also denotes now a process, now a product. Thus, the conclusion of a syllogism is frequently spoken of as an inference from the premisses, whilst the process of disimplicating the conclusion from the premisses is also referred to as an inference. Our own use of the term 'Inference' is intended to refer exclusively to *processes* of strictly *valid* reasoning—that is, reasoning in which the conclusion follows from the premiss or premisses with logical necessity.

CHAPTER XXXVIII.

XI. (ii.) HYPOTHESIS.

J. S. MILL defines a Hypothesis as follows : 'An hypothesis is any supposition which we make (either without actual evidence, or on evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real.'* Not every supposition, therefore, can rank as a Hypothesis. A Hypothesis is a supposition made in view of a truth-interest. It is a supposition which (1) admits of being developed into its consequences, and (2) requires and admits of verification.†

What Mill thus defines is, in fact, the *legitimate* Scientific Hypothesis. To be legitimate a Hypothesis has one essential condition to satisfy : it must be verifiable.‡ But to be verifiable it must be adequately developable.

A legitimate hypothesis, again, is identical with a *working hypothesis* in the widest sense of that term. For a working hypothesis (*e.g.*, 'Electricity behaves as though it were a fluid'; 'Vegetable mould is due to the action of earthworms') is a hypothesis that works—works, that is, by attempting to explain the facts.

A successful working hypothesis is a hypothesis that works well. To work well, a hypothesis must be both resourceful and fruitful. To be resourceful, it must be rooted directly (or indirectly through the medium of a general *working idea*) in a reasoned system or science. To be fruitful, it must be capable of continually extending its sphere of verification, and of bringing more and more facts under scientific control.

A working hypothesis is, as a rule, closely allied with what we may suitably call a *working idea*. This is the germinal conception out of which the true working hypothesis is shaped. The working hypothesis is developed out of the working idea not by being deduced from it with logical necessity (the 'idea' would in that case be only a more fundamental working hypothesis), but by processes of imaginative construction of a purely tentative kind. The working idea stands to the working hypothesis thus developed from it in a relation somewhat analogous to that in which the subject of discourse stands to the particular proposition through which it is at any moment being developed.§ It stands for the relatively

* 'A System of Logic,' Book III., ch. xiv., § 4.

† The term 'Verification' is, in its current use, ambiguous. Ordinarily, as here, it means a 'test'—a test that may result in disproving the hypothesis. In its stricter use, 'Verification' is the process which *pro tanto* confirms the truth of a hypothesis. Only *some* hypotheses would, in this sense of the term, admit of being verified. Similar remarks apply to the use of the term 'verifiable.'

‡ For a more radical criterion of the legitimacy of a hypothesis, see the chapter on the Inductive Postulate.

§ *Cf.* pp. 118, 119.

indeterminate guiding conception which may subsist unchallenged through the successive failures of a great many working hypotheses which have been constructed along the general lines marked out by it.

ILLUSTRATION OF THE IMPORTANCE AND SIGNIFICANCE OF THE WORKING IDEA FROM THE HISTORY OF ASTRONOMICAL SCIENCE.

Problem : To formulate a theory or system that shall exhaustively account for all the varied movements of the heavenly bodies.

1. *Ptolemy's System.*

In the second century of our era, the Alexandrian astronomer Claudius Ptolemæus brought all the observed movements of the heavenly bodies into one system. The main feature of this system is that *the Earth at rest* is taken as the centre of the universe, while sun, moon, planets, and stars revolve in various circles around it.

The main working ideas* of this system were :

1. The Earth, as the one fixed centre of the universe, is certainly at rest, and the celestial movements we see are the *real* movements. The opposite suggestion was characterized by Ptolemy as 'the height of absurdity.'
2. The heavenly bodies are divine and incorruptible, and must therefore move in circles (the circle being considered by the ancients as the most perfect geometrical figure).

On the basis of these working ideas the system was built up. Now it had long been noticed that the apparent paths of the planets were anything but simple. A planet watched night after night would be seen first to move from west to east with a varying velocity, then to stop, then to adopt a retrograde movement, then to stop again, then to go forward again, and so on.

The problem was to analyse these movements into a system of circular movements, *the Earth at rest* being the ultimate centre of the whole system.

From the nature of these observed movements it was plain that the simple device of making each planet go in a circle round the Earth as centre would not do. It was therefore necessary to invent a system of *epicycles*, an epicycle being a smaller circle whose centre moves along the circumference of a greater. Each planet was then supposed to move uniformly along the circumference of the epicycle, whilst the centre of the epicycle itself moved uniformly in a circular orbit round the Earth. Thus the system of epicycles was an attempt to explain the movements of the planets ; but the attempt was not

* That Ptolemy himself should have treated these as axiomatic does not affect their true logical character as 'working ideas.'

altogether successful. It was found necessary to supplement the system of *epicycles* with a system of *excentrics*.

This 'excentric' device consisted essentially in allowing the planet's cycle (not its epicycle) to move about a point outside the Earth, this point itself being supposed to revolve about the Earth as centre, so that in last resort the sanctity of the first idea should not be violated. The sanctity of the second was preserved inviolate through the care taken to make all the orbits *circular*.

Such a system was Ptolemy's. Now, as observations became more accurate, it was found continually necessary to modify this system of wheels within wheels. New circles had continually to be added (for instance, an epi-epicycle revolving on an epicycle), until in Copernicus's day seventy-nine of these circles were found necessary in order to represent, even with the roughest approximation to accuracy, the movements of the heavenly bodies.

2. *The System of Copernicus.*

The main advance made by the system of Copernicus on the system of Ptolemy lay in its completely shifting the point of view and shattering the first of Ptolemy's dogmatic Ideas. It showed by a clear mathematical treatment (which took thirty years to develop) that the observed movements of the planets were *apparent*, not *real*; that the Sun was the centre of the system of planets; and that the Earth was only one planet like the others. Copernicus showed that no amount of mere modification could justify the Ptolemaic conception of things; that its fundamental assumption, being entirely incorrect, must be given up in favour of the Heliocentric Theory.

The great work of Copernicus, then, was the shattering of this first Idea. But he was still himself a slave to the second—the idea that the circle, as the only perfect figure, could alone be worthy to represent the path of a planet. The fundamental change which transferred the centre of motion of the solar system from the Earth to the Sun had made perfectly easy the explanation of the stationary points and retrograde (apparent) movements of the planets; but there were still certain minor irregularities which Copernicus tried to solve by means of a new system of epicycles and excentrics.

Moreover, he had not completely emancipated himself from the first Idea. The Sun took the place of the Earth, not only as the centre of the solar system, but as the centre of the *universe*, so that the Ideas of Copernicus—propositions which were to him axiomatic —were—

1. There is a centre of all things, the Sun.
2. The circle is the only perfect figure; therefore all heavenly bodies move directly or indirectly in circles.

The stars were conceived by Copernicus as absolutely fixed in a great all-embracing sphere. They did not shine by their own light—*i.e.*, they were not suns themselves—but, like the moon, reflected the light of the Sun, the source of all the light as well as the centre of all the movement of the universe.

3. *Kepler.*

It was left for *Kepler* to remove once and for all the second Idea of the Copernican System; and he also made a great step towards the removal of the first. This latter step he took when he showed that the Sun itself is only a star, and that the stars are suns made star-like by distance. He showed that the stars must be at least 2,000 times as far away as Saturn, the planet which at that time was regarded as the outside warder of the solar system. But he still imagined that the stars were all of a piece, all on one sphere at the same distance from the Sun. Further, it was a long time before Kepler removed the hoary prejudice involved in Idea No. 2. It was only because the fundamental Idea on which he himself worked was seen eventually to require its suppression that he was able at length to lay aside the prejudice of 2,000 years.

Kepler's one leading Idea was that the Creator must have been a geometer. The idea is Pythagorean. It was adopted by Plato, and borrowed from Plato by Kepler. It was no longer 'The Creator must have arranged the orbits of the heavenly bodies on a *circular* pattern,' but 'He must have arranged them on a *geometrical* pattern.' At the same time Kepler was at first quite unconscious of this distinction, and all his first efforts, directed mainly towards explaining the movements of Mars on the basis of Tycho-Brahé's observations, were spent on making hypotheses of the old circular kind. But try as he would, there still remained a large unexplained error of about eight minutes of arc, about one-eighth of a degree, as compared with Tycho's observations.

He then said boldly that it was impossible that so good an observer as Tycho could be wrong by eight minutes, and added: 'Out of these eight minutes we will construct a new theory that will explain the motions of all the planets.' He then proceeded to work out the theory of motion in ellipses. For he had at length found to his great satisfaction that when the sun was placed not at the centre of the ellipse, but at its focus, and the planet was supposed to move in such a way as to describe equal areas in equal times, all the irregularities were adequately explained. The extension of this discovery to the movements of all the other planets followed very easily; and, instead of the old cumbrous system of epicycles and excentrics, Kepler produced a system of the greatest simplicity, which had the paramount merit of explaining to a close degree of approximation the various movements of all the planets.

We have, then, two main laws of planetary motion already established. They are known as Kepler's first and second Laws of Planetary Motion.

1. Every planet revolves around the sun in an elliptic path, the sun being at one of the foci.
2. Every planet moves round the sun with such a velocity at every point that a straight line drawn from it to the sun passes over equal areas in equal times.

To these Kepler added a third :

3. The squares of the periodic times are proportional to the cubes of the mean distances from the sun (the periodic time being the time which the planet requires for the completion of its orbit)—

$$I.e., \frac{T^2}{D^3} \text{ is constant for all the planets.}$$

In these three laws we have the ripe expression of the great movement of scientific thought which had its root in the Working Ideas of the Ptolemaic system. But the culminating point is reached when Newton passes beyond these three laws to the single principle of Gravitation, which at once explains and transcends them.

4. *Newton.*

In 'The System of the World' Newton explains his own position very clearly in the light of a historical retrospect. The ancients satisfied the instinctive desire they felt for a causal explanation of the planetary movements by their theory of crystal orbs or spheres. These orbs served to keep the planets in their places, and held them as it were by a support which, though invisible, was still material. It was the comets that first broke up this old theory.* 'Above all things, the phenomena of comets can by no means consist with the notion of solid orbs,' for, he adds,† 'as it was the unavoidable consequence of the hypothesis of solid orbs, while it prevailed, that the comets should be thrust down below the moon, so no sooner had the later observations of astronomers restored the comets to their ancient places in the higher heavens, but these celestial places were at once cleared of the incumbrance of solid orbs, which by these observations were broke into pieces and discarded for ever.'

But with these orbs disappeared also the supporting forces that explained the apparently unsustained movements of the planets through the heavens. Other forces had to take their place. Descartes, in his vortex theory, made a distinction between at least

* Newton's 'Works,' American edition, p. 511. † *Ibid.*, p. 512.

two kinds of matter, one very tenuous, the other heavier, the matter of the planets. This heavy planetary matter was whirled round the sun in a vortex of the tenuous matter. This tenuous matter, a mid-way form between ether and ponderable matter, filled the whole of space, and was endowed with great velocity, forming by its rotatory movements the eddies in which the planets were borne round.

This Cartesian vortex theory was very popular. It was adopted by Fermat, Huygens, Bernoulli, Leibniz. It was a very sane and ingenious idea. All storms move in vortices. It is very probable that the great disturbances on the face of the sun in connexion with sun-spots and faculæ are cyclonic storms of fiery gas (*cf.* Lord Kelvin's 'Vortex Theory of Matter'). But it will not work mathematically.

Now Newton, while justifying the instinct that compelled astronomers to adopt the working idea of Mechanical Force, and to see force at the root of movement, stated very clearly that the only essential property of this force that was of any use or interest to astronomers was its mathematical law.

Thus, on the one hand we read* 'From the laws of motion it is most certain that these effects must proceed from the action of some force or other,' and then, 'but our purpose is only to trace out the quantity and properties of this force in a mathematical way, so as to avoid all questions about its nature or quality.' Compare also the following passage in the 'Principia,' p. 506 (conclusion): 'Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the causes of this power. . . . But hitherto I have not been able to discover the cause of these properties of gravity from phenomena, and I frame no hypotheses (*hypotheses non fingo*). . . . And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.'

The force by which Newton sought to explain the movements of the heavenly bodies he called 'attraction,' and its mathematical law, with which he was solely concerned, was the following: 'That all bodies tend to attract each other mutually with a force that varies directly as the product of their masses and inversely as the square of the distance between them.'†

In his attempt, then, to explain causally the movements of the heavenly bodies, Newton adopts as his Working Idea not the notion of mere *force*, but rather that of the *law* according to which gravitative force is operative. In modern Science the idea of Force has given way to that of Energy or Capacity for Work, and its fundamental Working Idea is the Law of the Conservation of Energy. It is essential not to forget that this great law is after all only a 'Working Idea.'

* Newton's 'Works,' American Edition, p. 512.

† *Ibid.*, p. 385.

The Problem of Verification.

1. *Negative Aspect.*—If the Working Idea has only a relative permanency, the working hypothesis is emphatically and essentially a provisional explanation, to be laid aside at once if it fails to account for the facts it professes to account for. If any fact absolutely refutes it, the hypothesis must give way before the fact. But we must be quite sure that the opposing fact is genuine. Thus it is a natural primitive objection to the Law of Gravitation that some bodies—balloons, for instance—tend to move away from the earth. But this is not a genuine objection, since the balloon really tends to fall, but is thwarted by the upward pressure due to the gravitation of the air. If we were to remove the air, and leave the balloon *in vacuo*, it would fall like a stone.

But even if compelled to admit the objection, we need only correct or modify the hypothesis to the extent required for the removal of the discrepancy. In discussing the character of the successful experimentalist, W. S. Jevons shrewdly points out ('Principles of Science,' vol. ii., p. 232) that 'Readiness to reject a false theory may be combined with a peculiar pertinacity and courage in maintaining an hypothesis as long as its falsity is not actually apparent;' and he quotes Leslie's remark concerning his own experimental investigations into the nature of heat: 'In the course of investigation I have found myself compelled to relinquish some preconceived notions; but I have not abandoned them hastily, nor till, after a warm and obstinate defence, I was driven from every post (*ibid.*, p. 234). There are circumstances which will justify a scientist in clinging to a theory through everything.

An investigator may say: Given A, B will follow; and if B is not observed to follow under the conditions symbolized by A, the result is said to be negative. But this negative result need not be accepted as conclusive, and this for three reasons:—

(1) The result, though genuine, may disprove not the working *idea*, but only a certain determinate form of it, the specific working hypothesis under investigation.

(2) The result may not disprove even the working hypothesis. It may only show that the conditions of the actual experiment were not satisfactory. The effect may have been produced, but in too slight a form to be detected, or the arrangements of an experiment may not have been suitable; just as Œrsted could not detect electro-magnetism so long as his wire was perpendicular to the plane of motion of his needle (*ibid.*, p. 239).

(3) It may be that the fact itself has not been properly observed. The fault in this case lies, not with the hypothesis, nor with the apparatus, but with the observation.

Jevons reminds us that Faraday, convinced on general grounds that some mutual relation must exist between Light and Magnetism, struggled against negative evidence for forty years in his attempt to prove their kinship. His conviction was that the various forms of energy have a common origin, and are so directly related as to be mutually convertible. In this case his courage was justified when a happily devised experiment showed that magnetic force had the power of twisting the plane of polarization of a ray of light. The whole theory was subsequently developed mathematically by Clerk-Maxwell, and is now known as the electro-magnetic theory of Light. Here struggle against negative evidence meant partly a revision of working hypotheses, partly a further elaboration and perfection of apparatus. Meanwhile the working idea remained as the germinal, controlling conception which directed the whole procedure.

Again, experiments devised by Faraday to prove the connexion between gravity and electricity gave only negative results. 'The experiments,' he wrote, 'were well made, but the results are negative;' and yet he adds: 'I cannot accept them as conclusive' (Jevons, *ibid.*, pp. 238, 239). Here Faraday's meaning, logically expressed, amounted to this: that he did not consider that he had tested *all* the working hypotheses by which his working idea could be verified.

We see, then, that the refusal to accept a negative result as conclusive does not mean refusing to take the result as a trustworthy indication that something is wrong either in the working hypothesis, or in the apparatus, or in the use made of one's own senses. It *does* mean this. But a *negative* result only says that *something* is wrong; it can give no indication as to what precisely is wrong. If a chain breaks, any link may have given way; and the negative result is just the signal for revision.

2. *Positive Aspect.*—The verification of hypothesis naturally consists, in the first instance, in showing that the hypothesis in question accounts for known facts. Thus, in the case of the Undulatory Theory of Light, it had, of course, to be shown that, when developed into its consequences, it could adequately account for the known facts of reflection and refraction. But a more striking form of verification occurs when the consequence is a novelty, and experiment is required to attest or disprove its presence. This derivation of a previously unobserved fact as a necessary consequence from a given hypothesis is what is known as Prediction.

'Prediction' means stating what the facts are before they have been observed. The time-element in prediction in no way affects its logical nature. To predict 'in advance,' or to predict 'backwards,' is, so far as the thought-process is concerned, essentially one with prediction that has no time-reference at all. Thus to predict eclipses in time, whether backwards or forwards, is just the same process as predicting from the known laws of motion that

eclipses, in general, must take place under certain conditions ; or if it differs, it differs only as regards precision of statement.

It is sometimes supposed that power to predict is an infallible proof of the truth of a hypothesis. This is a perfectly ungrounded statement. A *successful* prediction is only an interesting, striking form of verification.

A hypothesis that may afterwards prove to be erroneous may furnish verified predictions. The one general test of the truth of a hypothesis is *continued* conformity with fact. There may be developed two or more theories of which each sufficiently accounts for all the fundamental, perhaps all the known, facts. It is the agreement with *further* facts still untested or still to be discovered which must then decide between the two hypotheses. Sooner or later the *instantia crucis*—the crucial instance—presents itself, and the *experimentum crucis*—the experiment of the finger-post—decides between the rival claims. A crucial instance is a circumstance that is decisive between rival hypotheses, admitting of one explanation only ; a crucial experiment is a test so arranged that the result is bound to *ap*prove one hypothesis, and *dis*prove the other. Professor Minto ('Logic,' pp. 347, 348) gives the following crucial instance as proving decisive against the Cartesian Theory of Vortices : 'The fact that comets pass into and out of spaces where the vortices must be assumed to be in action without exhibiting any perturbation is an *instantia crucis* against the hypothesis.' As an instance of the *experimentum crucis* we may cite Foucault's experiment on the velocity of light in two different media, the denser water and the lighter air.* Foucault showed that light took longer to travel a certain distance through glass or water than it did to pass through the same distance in air or through a vacuum. This confirmed the Undulatory Theory of Light, and effectively demolished the Corpuscular Theory, which required that light should move more rapidly through the denser medium.

It should be noticed that the crucial instance, though it negatives the defeated theory, does not absolutely *prove* the one that is successful. It only serves greatly to increase the possibility of its being the true one. No inductive hypothesis can ever be regarded as the perfected, unimprovable expression of the truth. Verification in a word is not Completed Proof. We must distinguish the hypothesis as *verified* from the hypothesis as proved.†

Let us suppose that the hypothesis 'A is B' is to be tested, and that the consequence 'P is Q' is, with logical necessity, derived from it and from the scientific system in which it is rooted. Let us further suppose that the verification of 'P is Q' has been conclusive. This verification of 'P is Q' does not, of course, prove the truth of 'A is B.' To suppose that it did so would be to commit the fallacy

* *Cf*. Prof. Welton, 'A Manual of Logic,' vol. ii., bk. v., ch. iv., p. 103.

† For a discussion of the process of Proof proper, see above, pp. 322–325.

of 'affirming the consequent.' We have shown that the hypothesis, properly enunciated, enables us to explain the facts without transgressing the evidence. To *prove* the hypothesis, we have still to show that *no other* hypothesis will adequately account for the facts. The evidence afforded by the verification must be adequate to prove this, if we are to be justified in saying that the hypothesis is *proved*.

Let us consider from this point of view the status of a typically fundamental and fruitful hypothesis, that of gravitation.

Newton proved clearly by exact mathematical reasoning that, given the working idea of 'attraction' or 'gravitation,' there was only one admissible form of that idea—the law that the attraction varied inversely as the square of the distance, and directly as the product of the masses of the bodies concerned. He *proved* the working hypothesis *on the assumption that the working idea was sound.* He discovered once and for all the only form into which the working idea of gravitation could be developed so as to explain the facts. But the working idea itself still remains subject to revision. Thus Faraday's conception of the affinity between gravitation and electricity, if justified, would involve a revision of the working idea of gravitation.

Barren Hypothesis.—A barren hypothesis is a supposition from which, or from the functional substitute for which (*vide* p. 320), no verifiable conclusion can be drawn. It is the opposite of what is usually known as a 'permissible' or 'legitimate' hypothesis—*i.e.*, of a hypothesis which is workable, and workable because it suggests a mode of operation which is, at least to some extent, analogous to operations with which we are already familiar. A supposition such as 'This havoc has been wrought by a ghost,' or, again, 'Neptune's irregularities are the frolic of a demon,' would be a barren hypothesis. How can we aspire to verify either of these suggestions? We know nothing sufficiently definite about ghosts or demons to enable us to deduce from the hypothesis consequences which could be compared with facts. Referring to the 'catastrophic' or 'convulsion' theory in Geology, Professor Creighton points out that, in assuming its truth as a basis of investigation, we are assuming the operation of incalculable forces, the positing of which leads and can lead to nothing. 'Instead of these mysterious agencies, Lyell assumed that causes similar to those with which we are now acquainted had been acting uniformly for long ages. The nature of the causes at work being known, it became possible to calculate the nature of the effects, and thus to reduce the facts of Geology to order and system.'*

A hypothesis, however, may be barren at one stage of scientific culture and yet prove fruitful later on. When we know more about ghosts and their ways, the ghost-hypothesis may cease to be barren.

* 'Logic,' p. 243.

Three thousand years ago, as Professor Carveth Read points out,* the hypothesis that the Sun is the centre of our planetary system would have been a perfectly barren hypothesis. It needed the observations of the Greek and Alexandrian astronomers, and the discoveries of Galileo, Kepler, and Newton to make it really verifiable and fruitful.

False Hypothesis.—A barren hypothesis must be carefully distinguished from a false or erroneous hypothesis. A barren hypothesis is one that, under the given conditions of scientific knowledge,† cannot be verified, and *in this sense* is illegitimate (*vide* above p. 337), a hypothesis improperly so-called ; an erroneous hypothesis —*e.g.*, the Corpuscular Theory of Light—is a legitimate but unsuccessful hypothesis, a supposition which can be developed and tested, but does not happen to fit all the facts. It is an attempt at explanation shown to be inadequate by means of some crucial instance or experiment. At the same time an erroneous hypothesis is not necessarily worthless. Its inadequacy means no more than that the hypothesis breaks down *at a certain point*, and before breaking down it may have done considerable service to Science.

And yet it is essential to note that it is certainly not *qua* erroneous that the hypothesis is serviceable. The Ptolemaic hypothesis, with its geocentric principle and its assumption that all the movements of the planets were uniform in speed and circular in direction, gave a definite steadying-point for astronomical science. But the service here rendered was due to the systematic way in which the hypothesis was developed, and not in any way to what was false in the theory. Moreover, in so far as it merely assumed that the Earth was the fixed centre from which all astronomical *observations* must be made, it assumed what all other astronomical theories have been obliged to acquiesce in. Hence the quality of the *observations* made at that period was not influenced in any way by the specific character of the Ptolemaic theory. Whatever the astronomical theory may be, the Earth remains, for our perception, the *apparent* centre of the universe.

But in so far as the hypothesis that the Earth was the *real* centre of the universe was brought into play, and the apparent movements of the planets treated as real movements, the only utility of this error lay in its eventually refuting itself through the bewildering complexities to which it gave rise. Except as an object-lesson in the truth that the path of error is a path of growing complexity, it had no intrinsic utility. A theory cannot be said to have been

* 'Logic, Deductive and Inductive,' third edition, ch. xviii., pp. 250, 251.

† It is this limitation which justifies the concession of the title 'hypothesis' to these barren suppositions. Were they intrinsically unverifiable, they would not be hypotheses at all. They would be intrinsically, and not merely provisionally, illegitimate. The intrinsically illegitimate supposition, from the strictly scientific point of view, is a supposition which defies the requirements of the Inductive Postulate (*cf.* Chapter XLVII.).

useful in so far as it has stood in the way of the application of far truer theories whose work, since it began, has shown itself to be one of progressive simplification and progressive fruitfulness.

So, again, the Corpuscular Theory of Light was useful in so far as it made it possible to explain the phenomena of the reflection and refraction of light, and so provided a rallying-centre for the advance of optical theory. But, unfortunately, it had Newton's brain behind it, so that it received a development out of all proportion to its intrinsic merits, and a prestige which gave it an artificial lease of life and prevented the wave-theory, as developed by Young and others, from exercising its due effect on the mind of the time. The *non*-utility of a false hypothesis is thus evidenced by the fact that the more perfectly it is elaborated, the more surely does it prove a hindrance to the development of Science.

It is true that Bradley's explanation of the phenomenon of Aberration, according to which each fixed star appears, in the course of a year, to describe a small orbit about its true position, was developed on the basis of the Corpuscular Theory, and experts aver that the discovery could not so readily have been made had the wave theory been adopted, so that in this case an erroneous theory led to an important discovery.* At the same time it is hard to convince oneself that what was positively erroneous in the theory could have led to the discovery in question.

CHAPTER XXXIX.

XI. (iii.) GENERALIZATION.

GENERALIZATION is a process so distinctive of the very activity of thought that it is not easy to define it. To be observing a fact or 'particular' in the light of an idea or 'universal' is already to be breaking through its sense-isolation and winning it for thought. A fact is indeed meaningless except in so far as it is relevant to some interest, and rests upon some background, however indeterminate, of questioning mental activity. We start the generalization of a fact when we first realize it as a fact, when we first question its meaning for us and our interests, and so transform the sense-datum into a thought problem. We complete the generalization when the fact is, in all its relevant relations, adequately systematized. If the

* 'The curious inference may be drawn that, if the more correct modern notions of the nature of light had prevailed in Bradley's time, it must have been much more difficult, if not impracticable, for him to have thought of his explanation of the stellar motions which he was studying; and thus an erroneous theory led to a most important discovery' (Arthur Berry, 'A Short History of Astronomy,' p. 265).

process of generalization may be said to consist in the progressive idealization of fact, in the continuous revelation of fact as an ordered system, the guiding-idea of that process may be defined as the systematizing of fact in the simplest and most economical way, so as to bring the simplest thoughts to bear upon the widest range of facts.

The fundamental form which generalization takes is that of fixing the flux of facts, as immediately experienced, within the steadying and permanent form of a concept. This process is commonly referred to as the *generalization of facts under concepts or notions.* It consists in starting from the observation of concrete objects, and proceeding thence to a knowledge of classes. This is done by progressively *omitting* the attributes peculiar to this object or to that, and by *retaining* those which are attributes of all the objects considered. So viewed, *Generalization* involves more or less complex processes of Comparison and Abstraction, and culminates in Definition.

In those discussions on the nature of the virtues with which Socrates began the first European 'Philosophy of Morals,' his method was to proceed by critical comparison of a number of instances, to abstract the common or essential features, and then to formulate these in a definition.

'If you were in need of a dinner,' asked Socrates, 'would you apply to a shoemaker? No, but to a cook. Again, if you were bestormed, and wished to make for a harbour, would you resort to the soldiers on board? Not so, but to the pilot.' After a batch of such questions the learner would be in a position to see that cook, pilot, and the rest shared a common quality in virtue of which, on the respective occasions, application was made to them rather than to others—namely, the quality of being technically qualified.

The process of generalizing facts under concepts is beset by two main difficulties, to which Dr. Venn calls attention in his 'Empirical Logic':

1. There is, first, the difficulty of clearly detecting the common quality in a number of given instances of a certain class. Thus, to take Dr. Venn's instance, 'let A, B, C be Sheffield grinders, a familiar and well-marked class. It had long been known that they were sickly and short-lived people, but the person who first clearly recognized the character of their symptoms, so as to bring the disease'—a sort of lung disease—'under one concept, had no easy task to perform.'*

2. The quality in question may be obvious enough, but the individuals A, B, C, in which the quality is recognized, may never have been classed together. We have then to bring them under one class-concept, the right one for our purpose. This will often

* Dr. Venn, 'The Principles of Empirical or Inductive Logic,' ch. xiv., p. 347.

be a matter of great difficulty. Briefly, the detection of the property to be generalized and of the class over which this property is to be generalized is often the most difficult and important of the many operations involved in the establishment of a complete induction.

If the problem of generalizing facts under concepts connects itself with the problem of Definition, that of *generalizing concepts under concepts* gives rise to the problem of Classification, a problem with which the name of Aristotle is as closely associated as the name of Socrates with that of Definition. In Classification (*vide* Chapter V.) we have the natural development of the same generalizing tendencies which find their first resting-place in Definition. But the problem of Scientific Classification is far more complicated now than it was in Aristotle's day. Aristotle worked on the basis of certain simplifying assumptions, such as that of the fixity of species, which modern theories of causation and development have rendered more than problematic. As a consequence, the nature of the generalizing processes involved in Scientific Classification and Definition has been greatly modified in modern times. The simpler processes of comparing attributes and abstract points of agreement have given way more and more to the more complex processes of analysing relations and gathering variations around diagnostically defined types. And *pari passu* with this complication in the nature of the process itself we find that the whole problem of bringing concepts under concepts becomes ever more intimately connected with a further aspect of the generalization-problem. This concerns the bringing of facts under laws—laws of causal interaction and laws of development. The very variability of objects 'forces us beyond the statement of a fixed complex of perceptible characteristics, and obliges us to include causal relations or laws of development in our enumeration of the attributes by which one class of things is distinguished from all others. . . . Quicksilver seems to admit of a simple statement of characteristics by means of which its attributes are expressed in a combination which belongs to no other object; but it is only at an ordinary temperature that it is such an easily recognizable object: it evaporates in heat and becomes solid in cold, it combines with other metals to form amalgams, and with sulphur to form cinnabar, and not until we have included these transformations in our concept can we claim to have stated what quicksilver is' (Sigwart, 'Logik,' vol. ii., part iii., ch. ii., § 77, 6; English Translation, II., p. 163).

In *the generalization of facts under laws*, the first step is the formulation of what are commonly called 'empirical' laws. There are three essential marks which must be included in the definition of an Empirical Law. In the first place, it must have been gained through direct observation of facts. In the second place, it must not already have been explained as a particular case or specification of some law more fundamental than itself; it is a law, in fact,

which has not itself been systematized. Thus Kepler's Laws of Planetary Motion were empirical in this sense *until* Newton showed that they were necessary deductions from his own principle of universal gravitation. They then became specifications or expressions of the Law of Gravitation. In the third place, the Empirical Law is not an explanatory law. It is a law *descriptive of the behaviour of facts*, without at the same time being explanatory, or descriptive of the mode of behaviour of a *cause*. For an explanatory law, as Science understands it, can mean no more (and no less) than a law which necessarily implies a *reference* to a cause, force, energy, or tendency, though it makes no attempt to explain the nature of that cause beyond defining the law according to which it works.

We may illustrate the meaning of an Empirical Law by means of certain instances which we borrow from Dr. Sigwart (*ibid.*, § 96; E. T., pp. 362-366). We may take the law of falling bodies, which states that, whenever a body falls freely from rest, it describes spaces which are proportional to the squares of the times. This is a law in the sense that it describes the motion of a falling body by means of a formula ($2\,s = gt^2$), but it is not a law in the causal or explanatory sense: it states the 'how,' but not the 'why.' So, again, Kepler's first law tells us that planets move in ellipses about the Sun as focus, tells us how and not why they move as they do; whilst his second law—the law of equal areas—states in a formula the relation between the velocity of a planet and its distance from the sun, and is similarly descriptive and empirical. The law which states the connexion between changes in the height of the tide and the changing positions of sun and moon with respect to each other and the Earth is, again, 'only a descriptive law concerning the regular accompaniment of one change by the other, and it is essentially different from the causal explanation which deduces this connexion from the attraction of moon and sun upon the waters of the earth' (*ibid.*, E. T., II., p. 365). These descriptive statements are particularly frequent and important in Natural History. Thus the following are empirical uniformities, mere laws of sequence and coexistence:

A Dicotyledonous seed, when sown in suitable soil under favourable conditions of moisture, temperature, etc., will commonly germinate. The embryo which it contains will protrude its radicle. This will grow downward, and from it will be developed the primary root which fixes the developing seedling in the soil. This root may branch repeatedly, and ultimately form a complex root-system. The two cotyledons will (in many cases) emerge from the seed-coat and develop into green leaves. The stem will grow upward. If an embryonic plumule is present, this will develop its rudimentary leaves into the first foliage-leaves of the growing plant. The stem, like the root, may branch repeatedly, and will give rise to more

foliage-leaves, and ultimately to the reproductive shoots known as flowers. The latter will (in many genera) be bisexual, producing both ovules and pollen. If the pistil is pollinated, this will normally give rise to further processes which will lead to the maturing of the seeds. Each seed will contain an embryo capable of developing into a form similar to the parent-plant.

The essential limitation of the Empirical Law is that it cannot be relied on beyond the range of those facts of which it serves to register the behaviour. Thus Kepler's law of equal areas, prior to Newton's generalization of it, was a law of the movement of the planets round the sun. In Newton's hands it became a law of the movements of *any* rigid body revolving under the influence of *any* central force acting, as gravitation does, according to the inverse square of the distance. But such an extension of the empirical law was not possible in Kepler's day. It was only the generalization of Kepler's law under Newton's that made this extension intelligible. This further problem of the generalization of laws through their progressive development and simplification, of *the generalization of laws under laws*, is, however, substantially identical with that of the generalization of facts under laws through Analysis and Synthesis.

The essential means for passing from fact to law is *Experimental Analysis;* and the explanation which is based upon such analysis takes the form of Synthesis. Facts are, as a rule, complex, and the function of Inductive Explanation is, through analyses and synthetic reconstructions based upon these analyses, to transform the complexity we start from into a strictly relevant coherency. The resolution of the complex fact into its simple factors is the work of tentative analysis—which must try and test itself at every step—the culmination of the generalizing process being reached in the formulation of the laws according to which these more general factors operate. The systematic reconstruction of the fact, in idea, through deductive processes which, proceeding from the ascertained laws of operation of the factors, show how these factors co-operate in producing the fact, is the work of Synthesis, a synthesis which finds its climax in successful Verification.

The distinction between Analysis and Synthesis may be stated in a somewhat different form by reference to the purposive idea within which the whole process of inductive inquiry takes place. The aim of Analysis, we may say, is to disengage from a given complex situation such elements and combinations of elements as are *relevant* to its purposive reconstruction. From this point of view Analysis is essentially a process of purposive Elimination, Synthesis a process of purposive Elaboration. Through Analysis we eliminate the irrelevant elements in a total complex datum; through Synthesis we elaborate the relevant residue of Analysis in the light of the idea which has dominated and controlled the whole process.

The immediately preceding paragraphs contain in themselves a complete programme of Inductive Method, a programme which it will be the main business of the following chapters to develop. It may not be out of place, however, to add, in conclusion, an illustration or two of the general process through which facts are generalized, and, as a further consequence, systematized and explained.

Let us take as our complex fact a door with its framework somewhat out of order. We proceed to analyse the obstruction by resolving the vague complex idea of a door out of order into the more simple ideas of a latch, a hinge, or fitting out of order. We find eventually that the upper hinge is to blame, and that its looseness has caused the door to lean and graze the floor : it *presses* against the floor, and so *scrapes* against it when moved. The problem is now reduced to a question of *pressure* and *friction*, and we may now proceed to explain the difficulty by showing how, according to the laws of pressure and friction, the obstruction necessarily came about. By this *simplification* the fact is *generalized*. It is generalized as a particular case of the operation of the general laws of pressure and friction. It is also systematized, or at least potentially *correlated* with a host of other facts ; for, by connecting the phenomenon with all other phenomena of pressure and of friction, we have taken it out of its mere particularity and isolation.

Again—to take an illustration given by Dr. Venn*—if we are attempting to explain the slipperiness of ice, we at once simplify and generalize the problem before us by displaying the fact of slipperiness as a specific variety of the forward and backward reactions that always take place between our feet and the surface of the ground against which they press. 'We slip,' we say, 'because the horizontal reaction to the impulse of the feet has fallen below a certain minimal amount.'

Once more, if we desire to explain the succulent habit of some desert-plant, we generalize the fact of succulence by showing it to be a specific form of 'xerophilous' adaptation, and by regarding this again as a special kind of that adaptive modification, in response to the influence of environment, which, gradually perfected through the process of natural selection, ultimately fits each species of plant to the conditions of its own particular habitat.

* 'The Principles of Empirical or Inductive Logic,' ch. xxi., p. 498.

XII.

APPLICATION OF THE INDUCTIVE PRINCIPLE TO 'INDUCTIONS IMPROPERLY SO-CALLED' AND TO 'IMPERFECT INDUCTIONS.'

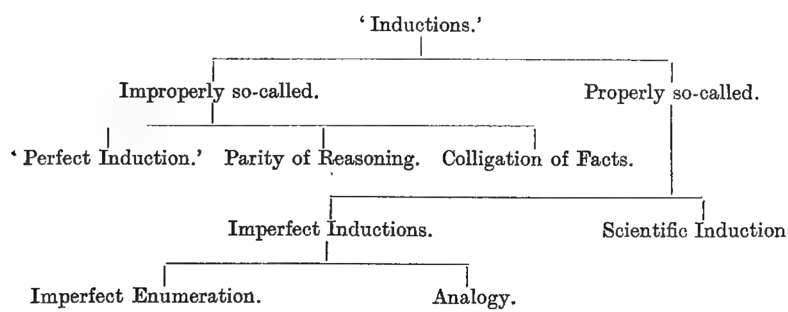
(i.) Inductive Inferences, improperly so-called (ch. xl.).
(ii.) The 'Imperfect Inductions':
  (A) Enumerative Induction (ch. xli.).
  (B) Argument from Analogy (ch. xlii.).

CHAPTER XL.

XII. (i.) INDUCTIVE INFERENCES, IMPROPERLY SO-CALLED.

The essential distinction between popular and scientific explanation has been expressed in the familiar saying that Science is just *organized* common sense. This organization implies two things—*principle* and *method*, the method being determined by the principle. And the principle whereby Science organizes common sense in the matter of the explanation of facts is just that of steadfast loyalty to the facts in so far as they are relevant to its scientific purpose. This principle gives to scientific investigation an ultimate standard or criterion—ultimate since the scientific aspiration and purpose does not extend beyond that of an adequate explanation of the facts of the sense-world. Hence, in inquiring whether any proposed method of dealing with the facts is scientifically adequate or not, we shall have to ask: In what sense does it provide an adequate Verification-test? This is the touchstone of Scientific Method, to which all else is subordinate.

For the purpose of carrying out this principle and applying this inductive standard in detail, we propose to adopt Mill's* Inductive Scheme, which may conveniently be laid out as follows:



Under the head of *Inductions improperly so-called* Mill reckons three types:

1. 'Perfect Induction.'
2. 'Induction by Parity of Reasoning.'
3. Colligation.

* *Vide* J. S. Mill, 'A System of Logic,' Book III.

1. '*Perfect Induction.*'

Suppose that we have certain knowledge that all the instances belonging to a given class have been considered by us; then, if we find that a certain attribute is possessed by *each* of these instances, it is an act of 'perfect induction' to *universalize—i.e.*, summarize— the discovery by stating that *all* the members of that class possess the attribute in question.

Thus:

'January, February . . . December have each twenty-eight days or more.
'January, February . . . December are *all* the months in the year.
'∴ All the months in the year have twenty-eight days or more.'

Now, is there anything in this reasoning which could allow us to consider it as a form of Scientific Explanation or Induction?

There is Scientific Explanation, as we have seen, only where a hypothetical conjecture is adequately verified. Two elements are here indispensable: the conjecture (going tentatively beyond the evidence), and the verification or the justifying of the conjecture.

Here, we might say, we have at least verification: the general proposition seems to admit of complete, final verification. But this confidence is illusory; for there has really been no conjecturing at all, and there is therefore nothing to verify. There has been no tentative supposition made, no uncertainty at any point of the reasoning, no temporary passage from the known to the unknown, which, when understood to mean a tentative passage from known facts to a hypothesis, is essential to all inductive procedure. In order that a scientific method may satisfy the criterion of Induction, not only must its verification-tests be adequate, but it must involve a verifiable conjecture. The legitimate hypothesis is the precondition of verification. 'Perfect Induction' lacks the legitimate hypothesis—lacks, indeed, the hypothetical element altogether. If it is something more than 'a mere short-hand registration of facts known,'* as Mill puts it, it is still essentially a self-contained deductive inference, and of the very simplest type. As such, it is no episode in a total process of Induction, and therefore not inductive in any sense of the word.'†

* J. S. Mill, 'A System of Logic,' Book III., ch. ii., § 1.

† For Jevons' defence of Perfect Induction, see 'Elementary Lessons in Logic,' p. 214. He appears to himself to be criticizing Mill, whereas he is only forcibly repeating Mill's own words. (*Cf.* 'A System of Logic,' *ibid.*: 'The operation may be very useful, as most forms of abridged notation are; but it is no part of the investigation of truth, though often bearing an important part in the preparation of the materials for that investigation.')

2. *Parity of Reasoning.*

When, after having shown that a certain fact is true of A, we argue that it must be true of B, not *because* it is true of A, but *for the same reason* as that which convinced us of its truth in A's case, we are said to proceed by Parity of Reasoning. Thus, 'having shown that the three angles of the triangle ABC are together equal to two right angles, we conclude that this is true of every other triangle, not because it is true of ABC, but for the same reason which proved it to be true of ABC.'*

But if this is the case, surely Parity of Reasoning is no more than Identity of Reasoning. And to reason identically is to take no further step in the reasoning at all, so that we have not only no induction, but no logical process whatever.

But the essence of the criticism which Mill brings to bear upon Parity of Reasoning, considered as an Induction, is that the conclusion drawn by it '*is not believed on the evidence of particular instances*,' and consequently that the process lacks the characteristic quality of Induction.

Instead of saying with Mill that an inductive inference is *grounded* in the instances whose sole function has been to *suggest* the generalization, we should rather say that an induction can be *grounded* only through a process of progressive verification exercised on the as yet unobserved or uncritically observed instances. Nor is the difference a mere matter of words. Mill repeatedly, though not invariably, argues as though the generalization were believed not on the evidence that *verifies* it, but on the evidence of the particular instances which have suggested it.

3. *Colligation of Facts.*†

The term is Whewell's, and means 'the act of bringing a number of facts actually observed under a general description,' as when Kepler, having made a large number of observations on the successive positions of Mars at different points of its orbit, brought them all together under the one collective conception of an ellipse. Mill and Whewell agree as to this definition of the term 'Colligation.' They disagree in this: that, whilst Whewell calls it 'Induction,' Mill denies that it is an induction at all.

Whewell, says Mill, is confounding a mere description of a set of observed phenomena with an induction from them. The description is an operation subsidiary to Induction, not itself an induction.

* J. S. Mill, 'A System of Logic,' Book III., ch. ii., § 2.

† *Ibid.*, Book III., ch. ii., § 3 *ff.*; Book III., ch. xvi.; Book IV., ch. i., ii. Whewell, 'Of Induction,' pp. 1-45. *Cf.* also G. F. Stout, 'Analytic Psychology,' ii., pp. 49-52; and J. Venn, 'The Principles of Empirical or Inductive Logic,' ch. xiv., pp. 353 *ff.*

It is essentially a mere act of comparison. Thus, what Kepler did in the case considered was to discover by careful comparisons the circumstance in which all the observed positions of the planet agreed. But to discover what is common to a series of observations is only to give a general character to these observations : it is not in any way an induction from them.

Whewell, in defence, contends that, besides giving a general character to the facts, Colligation introduces, as a principle of connexion, a conception of the mind not existing in the facts. The inductive act consists in a superinduction of a conception — the conception of the ellipse in Kepler's case—upon the facts. The statement about the elliptical orbit, he argues, 'was not the sum of the observations merely ; it was the sum of the observations *seen under a new point of view*, which point of view Kepler's mind supplied.' That the statement about the elliptical motion was not merely the *sum* of the different observations is plain from this, argues Whewell: that other persons, and Kepler himself before his discovery, did not find it by adding together the observations.*

We must further remember—a point not pressed by Whewell himself—that the orbit of Mars is *not* an actual ellipse, so that if Kepler had simply been summarizing the actual facts, he could not have arrived at the idea of the ellipse. 'What Kepler did was, from a finite number of observed positions to frame a rule for inferring all the intermediate unobserved positions, as well as those at any past or future time.'† Notwithstanding Mill's attempt to show that no generalization is involved ('Logic,' Book III., ch. ii., § 5), we must agree with Dr. Venn in this statement of the case. Kepler's procedure, we take it, was a true tentative generalization. To this extent we hold that Whewell is justified as against Mill. Colligation does, indeed, take us a first step beyond the observed facts by suggesting a tentative, descriptive generalization of them such as no mere observation could possibly have supplied, seeing that it refers to the unobserved as well as to observed instances. It certainly is not, as Mill maintained that it is, a mere summary of the facts thrust upon one by the facts themselves. But, from the point of view of the inductive criterion which we have adopted, Colligation is *not Induction*. It is an unverified Generalization. It has not been justified as a generalization true to the evidence of the facts.

* *Cf.* 'Of Induction,' p. 33.

† J. Venn, 'The Principles of Empirical or Inductive Logic,' ch. xiv., p. 354, footnote, *ad fin.*

CHAPTER XLI.

XII. (ii.) THE 'IMPERFECT INDUCTIONS.'

A. ENUMERATIVE INDUCTION.

THE term 'Enumerative Induction,' or the equivalent expression 'Induction by Simple Enumeration,' has been customarily used to represent the uncontrolled tendency of the mind 'to generalize its experience, provided this points all in one direction' (J. S. Mill, *ibid.*, Book III., ch. iii., § 2), consisting, as Mill puts it, 'in ascribing the character of general truths to all propositions which are true in every instance that we happen to know of.' It is a generalization based on the mere affirmation that we have never known an instance to the contrary.

This view of Enumerative Induction presupposes, however, a definition of Induction which is different from that adopted in the present volume. When Bacon and J. S. Mill speak of Induction by Simple Enumeration, they are identifying Induction with Generalization in a sense which excludes Verification. The tendency of the mind which moves from the known to the unknown by means of a generalization based on the counting of instances remains uncontrolled by the appeal to facts. It is liable, of course, to find its path blocked by an exception which pitilessly opposes this easy invasion of the universal, but it makes no attempt to search out exceptions, and never sees them unless they are thrust upon its notice.

But Induction, *as we have defined it*, is no mere conjecture from the known to the unknown, from facts to hypotheses, however probable these latter may appear to be. It is a process which, even in its most rudimentary form, must culminate in some kind of methodical attempt at verification. Its aim is the explanation of fact, and its means to this end is the Verification of Hypothesis.

We would therefore suggest the following scheme for the explanation of fact through a method of Enumerative Induction :

I. *Preliminary Observation.*

a, b, c, d, e [generalization instances] possess the property x.

II. *Generalization.*

(a) *First stage. Generalization proper* (Colligation).

a, b, c, d, e are instances that all belong to a class defined by the marks p, q.

(β) *Second stage. Universalization.*

$\therefore$ *perhaps all* instances that possess the marks p, q possess also the mark x.

III. *Deductive Development.*

If all instances of this kind possess the mark x, then a′, b′, c′, d′, e′ . . . (which all possess the marks p, q) must also possess the mark x.

IV. *Progressive Verification.*

a′, b′, c′, d′, e′ . . . [test-instances], which all possess the marks p, q, do (or do not) possess the property x.

Illustration :

I. These various individuals are suffering from this specific lung disease.

II. (a) They all belong to the class defined by the mark ‘Sheffield grinders.’

II. (β) ∴ perhaps *all* the individuals belonging to the *class* of Sheffield grinders, to which the observed individuals belong, also suffer from this disease.

III. But if *all* are thus afflicted, then Smith, Jones, Robinson, etc., who are also Sheffield grinders, should prove to be afflicted in this way.

IV. Attempt at progressive verification by further observation of Smith, Jones, Robinson, etc.

[NOTE.—In the generalization characteristic of Induction by Simple Enumeration, the two essential aspects of Generalization, the singling out of the class-marks and the universalizing element, take place separately and successively; and, since the class-generalization is taken for granted, it comes to pass that the second of these two aspects is identified as a rule with the *whole* process of generalization, as though Generalization were identical with Universalization.]

We must admit that *this* conception of Inductive Method as based upon enumeration of instances does answer, in outline at least, to the Inductive Idea as we conceive it. The essentials of inductive procedure are all present : Preliminary Observation of instances, Generalization of experience, deductive Development of hypothesis, attempt at progressive Verification through further observation of instances. It is therefore at least possible to *attempt* to solve the problem of Induction by the use of the Method of Simple Enumeration.

Our discussion of this simplest form of the Inductive Process will be concerned with two main questions :

1. What is the *value* of Enumeration in Inductive inquiry ?
2. What are the *limitations* of Enumerative Induction as an Inductive Method ?

1. *The Value of Enumeration in Inductive Inquiry.*

It is particularly valuable in those more complex Sciences, such as Meteorology and Sociology, in which the data from which generalizations are drawn can often be adequately given only in

the form of aggregates and averages. The Method of *Statistics*, though it includes an element of Analysis, is founded on Enumeration, or the counting of instances.

In every statistical investigation there are certain definitely stated phenomena to be counted and tabulated—as, *e.g.*, in a census, the number of married men or of bachelors, the number of widows and widowers, the proportion of those families that inhabit tenements with more than five rooms to those that inhabit tenements with five or less than five rooms, and so on. The method of counting such agreements and comparing the results constitutes the Method of Statistics.

'The first rule of a statistical investigation is: the phenomenon to be counted must be a countable fact that can serve as a unit.'* Thus, supposing it is required to count the number of houses in a town. What is a house? Any place inhabited by human beings? And of any size? Professor Scripture points out that a house census taken in India gave the greatest trouble in respect to this indefiniteness of the unit. We must therefore qualify the object to be counted so as to render it a suitable unit. Instead of counting up the 'hot' days in the year—to take a somewhat artificial instance—and contrasting these with the number of cold days, we should count up, say, all days in which the highest shade temperature exceeded 70° F., and contrast with the days in which it was less than 40° F.

The second main rule of a statistical investigation follows naturally from the first. It is that 'all things which are to be counted shall correspond completely and exactly to the stated definition of the counted object, and that nothing that does so correspond shall be omitted. This requires that all the properties of the thing counted shall be accurately determined before the count begins, and that they shall not be changed during the counting.'†

It is interesting to notice how the ancient Logic and Science disregarded the serviceability of these statistical, merely numerical, relations. The knowledge of the concept, so Aristotle argued, can gain nothing by our knowing how often it is realized. But in modern Science, where the desire to apprehend the Given fully and accurately, as an indispensable preface to understanding its laws, has been characteristically predominant, *number* has attained definite scientific value. 'Who can explain,' says Darwin, 'why one species ranges widely and is very numerous, and why another allied species has a narrow range and is rare? Yet these relations are of the highest importance, for they determine the present welfare, and, as I believe, the future success and modification of every inhabitant of this world. . . .'‡ 'As variations manifestly useful or pleasing to man appear only occasionally, the chance of

* E. W. Scripture, 'The New Psychology,' p. 16. † *Ibid.*, p. 18.
‡ 'The Origin of Species,' Introduction, p. 4.

their appearance will be much increased by a large number of individuals being kept. Hence number is of the highest importance for success. On this principle Marshall formerly remarked, with respect to the sheep of parts of Yorkshire, "As they generally belong to poor people, and are mostly *in small lots*, they never can be improved." On the other hand, nurserymen, from keeping large stocks of the same plant, are generally far more successful than amateurs in raising new and valuable varieties.'*

Illustration of the Value of Statistics.

(From Romanes' 'Darwin, and after Darwin.')

Statistics of the species of fauna found on islands (1) that have long been separated by great distances of sea from the mainland; (2) that have only recently been separated from the mainland.

As typical instances of (1) we take the Sandwich Islands, the Galapagos Islands, and St. Helena. We get the following results:

	Peculiar Species.					Non-peculiar Species.				
	Shells.	Insects.	Reptiles.	Land Birds.	Land Mammals.	Shells.	Insects.	Reptiles.	Land Birds.	Land Mammals.
Sandwich Islands	400	(?)	2	16	0	0	(?)	0	0	0
Galapagos	15	35	10	30	0	(?)	(?)	0	1	0
St. Helena	20	128	0	1	0	0	(?)	0	0	0
Totals	435	163	12	47	0	0	(?)	0	1	0

Thus, out of a total of 658 species of terrestrial fauna, *all* are peculiar, with the exception of a single land bird found in the Galapagos Islands and on the American Continent as well.

As a typical instance of (2) we take the Island of Great Britain.

Peculiar Species.						Non-peculiar Species.					
Plants.	Shells.	Insects.	Reptiles and Amphibians.	Land Birds.	Land Mammals.	Plants.	Shells.	Insects.	Reptiles and Amphibians.	Land Birds.	Land Mammals.
46	4	149	0	1	0	1,462	83	12,551	13	130	40

* 'The Origin of Species,' ch. i., p. 28.

The statistics furnished by this tabular analysis are eloquent, and point irresistibly towards the conclusion 'that *wherever* there is evidence of land-areas having been for a long time separated from other land-areas, there we meet with a more or less extraordinary profusion of unique or peculiar species, often running up into unique genera; and, in fact, so far as naturalists have hitherto been able to ascertain, there is no exception to this general law in any region of the globe. Moreover, there is everywhere a constant correlation between the *degree* of the peculiarity on the part of the fauna and flora and the time during which they have been isolated.'*

The complete subordination of statistical numeration to scientific explanation is made clear by the fact that, as soon as laws are actually established, statistical numeration ceases to be of interest. As Dr. Sigwart points out, the interest in counting how many eclipses of sun and moon occurred annually has completely vanished 'since the rule has been found according to which they occur, and can be calculated for centuries past and to come' ('Logic,' English translation, vol. ii., § 101, p. 483).

What, then, we ask in conclusion, is the value of *number of instances* in inductive inquiry? We may safely say that the strength of an induction is not necessarily proportional to the number of instances cited. The generalizations that lead to the most trustworthy hypotheses are based, not on the counting of the number of instances, but on the weighing of their quality and character. One crucial instance may be worth a hundred others. Again, strong *analogical* arguments can be based on the comparison of two instances only, whilst finally the most stringent inductions of all are those carried out by the Method of Difference, in which only one instance in two forms or, at most, two instances are needed.

We must add, however, that the work of true scientific analysis is often greatly assisted by the fact that the number of instances of a phenomenon or of the repetition of an event is large; for the larger the number of varied instances, the more easily can the unessential elements be detected, together with the form of the general law.

2. *Enumerative Induction in the Light of the Inductive Criterion.*

We have seen that the counting of instances may have definite scientific value during the preliminary stages of scientific observation.

We have now to consider to what extent a uniformity *suggested* by the counting of instances can be satisfactorily *verified* by the further counting of instances. This inquiry, which amounts to the

* 'The Origin of Species,' p. 235.

testing of Enumeration by the Inductive Criterion of Fidelity to Relevant Fact, will serve to define the extent to which it is adequate as an *Inductive Method*.

We shall see that insistence on the principle that facts (so far as relevant) must have their nature respected serves to deepen in every direction the significance of the processes which, in a crude germinal form, are present in Enumerative Induction itself, and to open out the way to true Scientific Induction.

How, then, does this Method stand in relation to the requirement of Verification ?

It is characteristic of Enumerative Induction that the process of Progressive Verification through which we attempt to justify the suggestion that all S's are P by verifying that S is P in each individual instance may culminate in a verification that is final and complete. The conditions may be such as to admit of a complete enumeration of instances, and on testing these instances individually it may be shown that they all possess the required mark.

This Complete Enumeration, as a genuine inductive process, should not be confused with the so-called 'Perfect Induction' already considered. The argument in an Induction by Complete Enumeration would run as follows :

I. *Preliminary Observation.*—a, b, c, d, e are all instances possessing the mark x.

II. *Generalization.*—(1) p, q are the marks common to all these instances.
(2) *Perhaps all* instances of this class (pq) possess the mark x.

III. *Deductive Development.*—If *all* these instances possess the mark x, then any individual instance must do so.

IV. *Verification.*—All the possible instances having been examined, we are able to assert categorically either that *all* instances of the class pq *are* accompanied by x, or that some instances lack the mark in question.

Thus in an Induction by Complete Enumeration the Verification may be complete. The question then naturally suggests itself : Have we not in Induction by Complete Enumeration an *ideal* form of Induction ? To this our answer must be that it does indeed realize *an* ideal, the ideal of Enumerative Induction ; but the further question remains : What are the intrinsic limitations of this kind of Induction ?

(i.) It can do no more than verify a 'that.' It cannot verify a 'how.' Given the observed fact that p, q are in a certain number of cases accompanied by x, the induction at best can only verify the suggested *fact* that p, q are in all possible instances accompanied by x. But this is not in any way a verified explanation of the connexion between (p, q) and x. *How* (p, q) and x are connected can

never be ascertained by this process, though it may, of course, be known *before the process begins.* Complete Enumeration then (when the results are wholly favourable) is just a verified conjecture from *one* fact (Some S's are P) to *another* fact (All S's are P). Enumerative Induction cannot, therefore, be used for establishing *laws* of connexion, whether of sequence or of coexistence, and so, in particular, cannot establish laws of causal connexion. For all scientific *laws* are uniformities which state *how* one fact is connected with another. To state *that* one fact is *always* connected with another is to state a fact of uniform connexion, but not the explanatory *law* of that connexion.

This limitation is fundamental.

(ii.) A *second* limitation is almost, if not quite, as fundamental. It is impossible through an Induction by Simple Enumeration, unsupported by analysis, to deal with what we may call 'apparent exceptions.' If an object possessing the marks p, q is found not to possess the mark x, it must, so far as this method is concerned, be put down as an exception, and with *one* exception the enumerative universal 'All pq is x' can no longer be completely verified.

The most obvious suggestion which this method supplies towards its own reconstruction is its indication of *Analysis* as the method required for meeting the difficulties with which it is itself unable to cope. Enumeration itself contains a germ of Analysis. Closer inspection, indeed, shows that, apart from that analysis which supplies the class-distinction and makes possible the generalization, enumeration could not come into play at all. Unless I know that I have to count instances characterized as pq, counting is impossible. Analysis, again, is the natural remedy for the deficiencies of mere Enumeration. To Enumeration Nature necessarily presents itself as a mere aggregate of instances. The systematic character of the given is by this method completely and inevitably ignored. But to do justice to that systematic character is the main function of the analytic method to which Enumeration by its own limitations points. Thus we see that Enumeration needs to be completed and transcended by the deeper method of Analysis.

A study of the Method's defects has made it clear that for the true interpretation of fact we need an experimental analysis of the systematic connexions of Nature. It would therefore seem appropriate to proceed at once to the consideration of that complete Method, the Method of Scientific Induction, which actually works on the lines thus indicated. But we must first consider the import and value of a Method which, though non-experimental, and but dimly foreshadowing the systematic character of real fact, is still a Method of Analysis, and, as such, takes us a full stage beyond the unanalytic Method of Simple Enumeration.

CHAPTER XLII.

XII. (ii.) THE 'IMPERFECT INDUCTIONS.'

B. ARGUMENT FROM ANALOGY.

THE Argument from Analogy is based upon resemblance. The form of the argument may be roughly stated as follows :

Two things (or classes), A and B, resemble each other in one or more properties, R_1, R_2. . . .
B possesses a certain other property, P.
We therefore conclude that A will resemble B in possessing this property also.

In arguing from Analogy we no longer count instances, as we do in Enumerative Induction, but we weigh properties.* In Enumerative Induction we lay stress solely on the number of corroborative instances—on the fact that, though the number of instances considered is great, no contrary instance has yet been met with. This unanalytic character of Simple Enumeration is its essential weakness. The argument by Enumeration gives way to an argument from Analogy so soon as attention is turned from an enumeration of observed instances to an analysis of their character.

Mill's view of the Argument from Analogy has sometimes been severely criticized, as though he had treated it as founded on a mere enumeration of resemblances. This, however, is hardly a fair criticism ; for Mill's process of Analogy undoubtedly begins with an analytic inquiry through which all those resemblances are eliminated which can be shown to be unconnected with the property in question ('A System of Logic,' Book III., ch. xx., § 2). Moreover, in his analogical argument about the habitability of the Moon, he shows how one single difference (the fact that the Moon apparently has no atmosphere) outweighs a large number of resemblances. Indeed, he shows that, viewed in the light of this one difference, 'all the resemblances which exist become presumptions against, not in favour of, the moon's being inhabited' (*ibid.*). And all through his discussion of False Analogies (Book V., ch. v., § 6) he implicitly insists on the importance of weighing rather than counting resemblances, showing that the important resemblances are those on which the suggested property *depends*.

The Argument from Analogy, then, as opposed to Enumerative Induction, proceeds by analysis of content. But if the analysis could be sufficiently thoroughgoing to disclose an invariable *causal* connexion between the property P and the properties R_1, R_2 . . .

* *Cf.* Professor Bosanquet, 'Logic,' vol. ii., ch. iii., p. 83.

which are common to both the objects A and B, the argument would lose all analogical significance. The gist of the argument would then be the scientific induction whereby the causal connexion had been discovered and verified. The remainder would be mere Parity of Reasoning. Thus :

> There is an invariable causal connexion (as witness the instance A) between the property P and the properties R_1, R_2, R_3. . . .
> Now in B we have the properties R_1, R_2, R_3. . . .
> Therefore in B we must also have the property P.

A conjecture, then, can be said to be based on Analogy only when the suggested property is *not* known to be causally or otherwise invariably connected with the properties of which the common possession by the two objects forms the basis of the Analogy.

On the Value of an Argument from Analogy.

The worth of an Argument by Analogy depends on the importance of the resemblances on which it is based, and the corresponding non-importance of the differences between the two objects concerned. The resemblances must be essential, the differences unessential. By this we mean that, if the argument is to be cogent, the properties which the two objects (or classes) A and B have in common must be closely related to the problematic property P, while their points of difference must be but loosely connected with it. We must not treat Analogy as though it were a question of Enumeration, and argue as if the strength of the analogical argument depended on the ratio of the *number* of points of resemblance to the number of points of difference. Mere *counting* of the points of resemblance and of difference is of little use.

For example, we might enumerate many points of external resemblance between the Whale and the Shark, and found upon them an analogical argument to the effect that the respiration-processes in the two animals must be similar. The whale, we might say, resembles the shark not only in all the common characters of Vertebrates, but also in its submarine habitat and in being (as regards many species) one of the very largest of marine animals. Like the shark, it is fish-like in external form, its fusiform body being well fitted for cleaving the water. Anteriorly its body passes into the head without any distinct neck, and posteriorly it is furnished with a swimming-tail into which the body gradually tapers. It has no hairy covering. Like the shark, again, it has a wide mouth, and it is of predaceous habit, feeding only on living animal nutriment. Therefore we may with great probability conclude that

its method of respiration is like that of the shark—*i.e.*, that it breathes the oxygen dissolved in the water, and has no need to be supplied with atmospheric air.

But this argument is unsound. The points that we ought to have observed are the characters *connected with the function of respiration.* The presence of gill-slits in the shark and their absence in the whale is a difference so essential to the inquiry that its observation would at once have been sufficient to make our analogy fall to the ground. And among the still more obvious external differences there is a single character which also should alone outweigh all the above-mentioned resemblances. The extremity of the shark's tail is expanded vertically; in the whale the flukes of the tail are placed horizontally. From many points of view this difference might be regarded as unimportant; but from the point of view of our analogical argument it is very important indeed, for it is intimately connected with the problematic point of resemblance that we are endeavouring to establish. For sea-creatures which, like the whale and the shark, spend their life in swimming freely through the water, the direction of movement is chiefly determined by the presence or absence of the air-breathing habit. Those creatures that have no need of atmospheric air move usually in straightforward and lateral directions, and for effecting such movements a vertically expanded caudal fin is admirably fitted. But creatures that need to rise frequently to the surface of the water for the purpose of respiration are constantly moving upward and downward. To movements of *this* kind a tail with horizontally expanded flukes is precisely adapted. Thus the whale's horizontally expanded tail affords a strong presumption in favour of the presence of the air-breathing habit; and this apparently trifling difference between the two creatures must be regarded as fatal to the cogency of the analogical argument.

We see, then, that it is the *important* difference which invalidates an argument from Analogy, this importance being purely relative to the problematic property (P) which the argument seeks to establish. So also it is only by resemblances that are in this same sense *important* that arguments from Analogy can be justified.

Thus, if we are arguing from the habitability of the Earth to that of Mars, the atmospheric and temperature conditions on the surface of Mars are the all-important points. Since these constitute marks of resemblance to the Earth rather than marks of difference, the analogical argument is proportionately strong.

It is frequently stated that, in estimating resemblances, a group of causally connected resemblances should only count as one. If the strength of an analogy depended on the *number* of *independent* points of resemblance, this would be a reasonable precaution to insist on. But in proportion as we cease to lay stress on number, this precaution loses its meaning. When points are weighed

instead of being counted, then the more complex and causally coherent any such relevant group of resemblances Σ (R_1) may be, the more likely is it that the property P will ultimately be brought within the same causal nexus.

The Logical Character of Analogical Argument.

The Analogical Argument finds its natural place as a stage in a complete inductive inquiry. It has genuine inductive value as a means of suggesting hypotheses and of sifting out, from an untested heap of mere guesses at truth, such as are 'soundest' and the most likely to repay the arduous work of development and verification: the true function of analogical reasoning cannot be other than that of recommending a hypothesis as worth the trouble of verification. To have any inductive value, the conclusion of an analogical argument must be verifiable. It is not essential that the verification should be immediately practicable. The argument may concern the conditions of life on Mars or on the Moon, and, in the present state of astronomical knowledge, it may not be possible for astronomers to put the conclusion to the test. In this case the conclusion should be treated as a suspended hypothesis—as a hypothesis of which the verification is deferred *sine die.* The analogical argument stands as a preliminary stage in an inductive inquiry that is not as yet completed.

The true place of Analogy is in the service of Scientific Induction. In relation to a complete scientific inquiry, its logical function is heuristic. It plays an important part in the Logic of Discovery, but has no place in the Logic of Verification.

The Value of Analogy in suggesting Scientific Hypotheses.

As Mill has said, it is when Analogy is used 'as a mere guide-post, pointing out the direction in which more rigorous investigations should be prosecuted,' that it has 'the highest scientific value.'* This point of view is, however, lightly passed over by Mill himself, concerning, as it does, the Logic of Discovery rather than the Logic of Evidence. Jevons, on the other hand, in his chapter on Analogy† in his 'Principles of Science,' emphasizes it almost exclusively. Quoting from Jeremy Bentham's 'Essay on Logic,' 'Discovery,' he asserts, 'is most frequently accomplished by following up hints received from analogy.' Thus, if a chemist is testing 'what he believes to be a new element,' and sees that 'in any one of its qualities the substance displays a resemblance to an alkaline metal . . . he will naturally proceed to try whether it possesses other properties common to the alkaline metals.'

* J. S. Mill, 'A System of Logic,' Book III., ch. xx., § 3.
† Vol. ii., pp. 283-305.

Jevons draws especial attention to the perfect example of Analogy presented by the analogy between Geometry and Algebra. So long as this analogy was not suspected, and the two sciences developed independently, as they practically did up to Descartes' day, they made but slow progress. When Descartes, in his system of Algebraic Geometry, had shown that the straight lines or co-ordinates 'required for fixing the position of a point' and the algebraical symbols x, y were fundamentally analogous, in such wise that a geometrical figure or curve could always be represented by an algebraical equation, the two sciences rapidly developed. Properties of curves were discovered by solving equations, and equations established by geometrical investigations of curves. The analogy between thing and symbol, when properly understood, is the most perfect and fruitful of analogies.

Another interesting instance of historically fruitful Analogy, cited by Jevons (*ibid.*, ii., p. 298), is the analogy between Jupiter and its moons on the one hand, and the Sun and its planets on the other. 'While the scientific world was divided in opinion between the Copernican and Ptolemaic systems, . . . Galileo discovered, by the use of his new telescope,' four of the 'small satellites which circulate round Jupiter, and make a miniature planetary world. These four Medicean Stars, as they were called, were plainly seen to revolve round Jupiter in various periods, but approximately in one plane, and astronomers irresistibly inferred that what might happen on the smaller scale might also be found true of the greater planetary system. The relation between Jupiter and its satellites is in many important respects analogous to the relation between the Sun and its planets. Therefore, since the satellites move round Jupiter, the planets should also move round the Sun.'

The so-called 'False Analogy.'

From the point of view of the Inductive Standard which we have adopted the tentative conclusion drawn through an analogical argument may be called *true* only when adequately verified by a subsequent process of scientific Experiment; *false* when disproved by the process.

Let the two instances A, B be symbolically analysed as follows:

$$A \equiv R_1, R_2, R_3, \ldots d_1, d_2, d_3, \ldots P.$$
$$B \equiv R_1, R_2, R_3, \ldots \delta_1, \delta_2, \delta_3, \ldots (P\ ?).$$

Here $R_1, R_2, R_3 \ldots$ are the 'points' of resemblance;
$d_1, d_2, d_3 \ldots$ 'points' possessed by A, but *not* by B;
$\delta_1, \delta_2, \delta_3 \ldots$ 'points' possessed by B, but *not* by A;
P a mark possessed by A;—the analogical argument consisting in concluding that B also possesses P.

A *true* conclusion from Analogy, then, would consist in a connexion between P and R_1, R_2, R_3 . . . founded upon resemblance, sustained by an analogical argument, and established by Scientific Induction.

A *false* conclusion from Analogy would be one that should break down under the criticism of a scientific verification, the suggested connexion between P and R_1, R_2, R_3 . . . proving to be unwarranted.

But so long as it is not practicable to proceed to this verification, the conclusion cannot be regarded as more than probable. 'True' and 'false,' as applied to it, are inappropriate epithets, and the place of this distinction is naturally taken by the distinction between '*sound*' and '*unsound*,' between well-grounded and ungrounded analogical conclusions. A sound or well-grounded analogy—using 'analogy' in the sense of a conclusion drawn through an analogical argument—would then be one in which the resemblances R_1, R_2, R_3 . . . were essential in relation to P, and the differences between A and B (again in relation to P) were unessential.

On the other hand, an argument from Analogy should be stamped as unsound or illegitimate when it can be shown that the differences in relation to P are essential and the resemblances unessential; or, in other words, when we can show that the suggested property P, far from being rooted in the known resemblances, is rooted in certain differences between A and B, in certain conditions which obtain in A, but not in B. Such an argument from Analogy could appropriately be called 'ungrounded.'

Illustrations of Illegitimate Analogy:

1. States must decay, as individuals do.*

In the case of the individual body, decay results from 'the natural progress of those very changes of structure which, in their earlier stages, constitute its growth to maturity,'—*i.e.*, from the conditions of old age, d_1, d_2, d_3—whereas in the case of the State these conditions are not present, for the properties δ_1, δ_2, δ_3, upon which its healthy existence at any stage depends, are such as point to an indefinite continuance of healthy growth. The decay of States is due, not to inevitable old age, but to disease which may be warded off. 'Bodies politic die,' says Mill, 'but it is of disease or violent death; they have no old age.'

2. A and B resemble each other in possessing a certain kind of skill. They can both make locks. *A* is further given to picking them with a view to robbery. Therefore *B* is also addicted to lock-picking with a view to robbery.†

* J. S. Mill, 'A System of Logic,' Book V., ch. v., § 6.

† *Cf.* Plato, 'Republic,' p. 334, A: 'What a man is clever at keeping he is clever at stealing too.'

Here A = skill to make locks implying skill to pick them + the will to pick them (d_1).

B = skill to make locks implying skill to pick them + a will about whose tendencies, one way or the other, nothing is known (δ_1).

The argument is that, as A actually *picks* locks, therefore *B* does too ; and the question is whether the argument is *sound.*

Now P, the addiction to robbery, is rooted in d_1, A's *will* to pick locks. Hence, as d_1 is not known to be a characteristic of B, there is no *ground* for connecting P with B.

Therefore the analogy is *unsound.*

So long as an analogical conclusion is not verified it cannot be called *true.* But, it may be urged, an Unsound Analogy may surely in the strictest sense be *false*—in the case, namely, in which P can be proved on the given data to be *incompatible* with certain differences, δ_1, δ_2, δ_3, possessed by B but not by A. In this case is not the argument from Analogy strictly disproved ? Certainly ; but the disproof would entail a Scientific Induction. It is as difficult to prove such incompatibility as it is to prove or disprove a causal connexion.

Thus, 'Is human life *incompatible* with absence of water ?' To answer this question satisfactorily we must analyse out the properties of Water (considered as a solvent, etc.) ; we must physiologically analyse the meaning of thirst, and the necessity of a supply of liquid ; we must consider in detail the whole question of possible substitutes for water. Not till these processes of analysis are completed can we venture on a downright use of the term 'incompatible.'

Incompatibility would thus seem not to be a matter that concerns Analogy. Analogy deals with points of difference, certainly, as with points of resemblance. But so soon as we can deepen the fact of difference into the fact of incompatibility, we seem almost as certainly to have gone beyond Analogy as we have when we have deepened the fact of resemblance into a fact of causal connexion.

XIII.

THE GOAL OF INDUCTION: CAUSAL EXPLANATION.

CHAPTER XLIII.

XIII. (i.) CAUSE AND CAUSAL LAW.

We have now to consider the form which Induction takes when it most adequately embodies the true interest and intention of Science. Fact, for Science, as we have seen, means Fact in the light of Law; but for Scientific Induction in particular Fact means 'Effect'—Effect that evidences the presence of Cause acting according to uniform Law. Conceived in an *explanatory* relation to effects, a law is known as a causal law or a law of causation.

The Meaning of the Term 'Law.'

Scientific Laws take shape and develop under the inspiration of the Inductive Principle of Fidelity to Relevant Fact—*i.e.*, to Fact in so far as it answers to the demand of the Inductive Postulate of Uniformity or Determinism (*vide* Chapter XLVII.). *Within the limits of this postulate* Fact is supreme. It is Fact that controls the tentative gropings of Hypothesis, and, in proportion as the latter conforms to its requirements, allows it to assume the status of Law. Scientific Laws are laws on sufferance, and they hold their office as interpreters of the world of real fact only so long as there are no other promising hypotheses to perform that function better.

This subordinate position of Law in Science is no exception to the general rule. Law, when healthily operative, is everywhere subordinate, be it to the constitutional authority of a nation's will, to the Principles of Reason and Conscience, or to the requirements of Relevant Fact. It is only the dead law of the Medes and Persians which altereth not.

When, in Science, we speak of a causal law, we in no way conceive it as causing or bringing about effects. The cause which is responsible for the effects is not itself the law through which those effects are interpreted. The law is a tentative, though approved, statement of uniformity; it is not a force. In no sense is it a power that can control the facts.

And yet, whilst submitting its laws to the sifting and refining control of relevant fact, Science still remains self-governed. Its

submission to Fact presupposes a fidelity to its own fundamental principles, for it is these alone which determine *how* Science shall study Fact. On the other hand, the fundamental assumption of Science, the Inductive Postulate, is not arbitrary. It has its root in the protest of the scientific spirit against the anthropomorphisms and animisms of pre-scientific ages (*vide* Chapter XLVII.), and testifies to the deep conviction of modern Science that Nature is the expression of *Natural* Law. It is out of this belief in the immanent laws of Nature that the ideal laws of Science have tentatively and gradually taken shape, bringing Nature at last, through man, into self-conscious possession of its own intrinsic orderliness.

Is all Inductive Explanation Causal ?

We may divide our answer to this question under two main heads :

1. *All complete Scientific Explanation is directly or indirectly 'causal' in the sense which this word properly bears in Science.*

It may be argued that, so long as we are simply inquiring how an object or fact is *constituted*, asking what are the simple elements or factors out of which it is constructed, the explanation is *substantial*, not causal. But this *mere* analysis of a fact into its factors is no explanation. The fact is explained only when we can show *how* it is the product of these factors. Indeed, Science, in last resort, reduces all questions concerning the coexistence of properties in a thing or substance to questions of causal connexion. So long as we are studying the connexions of what we take to be systematic in nature, we are studying causal connexions, whether these connexions be coexistences or sequences.

Uniformities of coexistence are the most obvious uniformities in Nature. Every natural classification, so far as it is natural, is a record of Nature's correlated facts—*i.e.*, of uniformities of coexistence. The name of a class is the sign by which we recognize the coexistence of a multitude of properties. For instance, by 'Gold' we understand a metal of high specific gravity, high melting-point, low chemical affinities, great ductility, yellow colour, etc. In the case of the higher divisions of a natural classification we are furnished with a similar clue to coexistent properties. For instance, 'Monocotyledon' is a sign whereby we are apprised of the whole list of the coexistences expressed in its definition. Again, the *inferences* that can be drawn from a natural classification are all expressive of the coexistence of correlated properties.

Correlations or coexistences—*e.g.*, that of two such properties as the ruminating habit and divided feet—in most cases can only be empirically formulated.

In the particular case of Geometry, however, we have a science of rationalized coexistences, the correlated properties being here all deducible from the essential properties of Space and Figure. All the properties of a circle are coexistent; and not only so, but exact reasons can be given to explain their coexistence. We have here a type of coexistence in which causal explanation passes into pure mathematical deduction. Explanation in Mathematics is non-causal. In abstracting from any reference to substance, it *eo ipso* abstracts from all causal considerations.

But when, as in all systems of Natural Classification, the properties are not correlated in reason only, the scientific belief that the connexions are not arbitrary, but must eventually admit of being reasoned out, leaves the scientist with the problem of discovering some common *cause* for all these coexistent effects. Precisely the same reasoning takes place where uniformities are sequence-uniformities, as in the sequences of day and night, summer and winter, etc. Here again we are driven to seek for some causal explanation, to find some cause of which the sequences are co-effects—though, of course, not simultaneous effects—or else to deepen the relation of sequence itself into one of causation.

All *complete* Scientific Explanation is, therefore, causal. If incomplete, as in Enumerative Induction and Analogy, where the 'that' is investigated, but not the 'how,' the explanation can at best be preliminary, or else subsidiary, to true Causal Explanation.

In what is called the *explanation of laws* we have a process of *systematization* which is not *directly* causal; but, though it does not deal with the relation of Cause and Effect, it does deal with the interrelations of uniformities which themselves must be regarded from the point of view of Causality, so that the procedure is still indirectly referable to Causal Explanation.

2. *If the word 'causal' be used in an ultimate sense, no Scientific Explanation is causal.*

In the scientifically causal explanation of a fact we look upon that fact as an effect consequent on certain ascertainable conditions. Here the first step in causal explanation consists in the ascertainment of the conditions on which the effect depends, and the main step is the discovery and verification of the laws according to which these conditions operate in bringing about the effect. In Scientific Explanation we are in the habit of contenting ourselves with the tracing of effects to the operation of conditions according to ascertained laws.

Against Scientific Explanation as thus conceived it has been urged that it is really no explanation at all, but mere description. Only a free agent, it is said, can be a cause; and these antecedent conditions are certainly not free agents, but must themselves be

explicable by previous antecedent conditions. A cause that is not a first cause is a cause only in the name. It is entirely determined by its antecedents, and is, therefore, a mere effect. So Berkeley asserts that 'The mechanical philosopher . . . inquires properly concerning the rules and modes of operation alone, and not concerning the cause; forasmuch as nothing mechanical is, or really can be, a cause.'*

The objection is fundamental, implying, as it does, that, far from every complete scientific explanation being causal, *none* is causal.

The objection is valid and useful, provided its bearing be properly understood; for it simply emphasizes the restricted sense in which the term 'cause' is used for scientific purposes. Science elucidates laws of connexion, but does not explain why the connexion should be as it is, and not otherwise. As Causal Explanation, in Science, is not causal in the true teleological sense, as it does not deal with final causes (which are the concern of Philosophy), so it does not deal with causes *qua* efficient forces, *though it is obliged to presuppose them.* It deals with causes solely as conditions, as conditions apart from which the event or effect in question would not take place.†

It remains to follow up these remarks by an attempt to analyse the meaning of 'Cause' as interpreted in Inductive Science.

The Meaning of 'Cause' for Inductive Science.

A cause is defined by Mill as the sum total of the conditions, which being realized, the consequent invariably and unconditionally follows.‡

Let us consider separately the two aspects of Cause suggested by this definition.

1. *Cause as the sum total of the conditions.*

By 'the sum total of the conditions' Mill can only mean the sum of all the conditions that are both relevant and sufficiently important. Apart from this limitation, as Venn points out, we should

* Dr. George Berkeley, 'Siris,' § 249, Fraser's edition, vol. ii., p. 457; 'Selections,' fifth edition, p. 293.

† The Cause, or Causal Antecedent, should be carefully distinguished from the merely logical antecedent as it occurs in a hypothetical judgment. The latter is the logical ground of a stated consequent, whereas the former is the existential ground of an effect, the ground apart from which the effect would not exist at all. Of course, the consequent *may* stand for an effect, and the antecedent which serves as logical ground for the consequent *may* stand for the cause, as when we say, 'If it rains, the grass will be wet.' Here the reason for our being able to infer that the grass will be wet is stated to be the presence of the cause. But there are other reasons besides those furnished by causes. Any *sign* may furnish a reason. Thus I may say: 'If the barometer falls, the grass will soon be wet.' Here the reason is *not* the cause.

‡ See 'A System of Logic,' Book III., ch. v., §§ 3, 6.

be quite unable 'to secure that repetition of occurrence which we require in order to apply the sequence we have noticed in the past to some new instance in the future.'* If a causal connexion is to be tested, the antecedents must be under control; must, therefore, be limited in number, and must be dealt with in abstraction from such individual characteristics as are not repeatable.

Among these conditions Mill distinguishes the positive from the negative. 'The negative conditions,' he says, 'may be all summed up under one head—namely, the absence of preventing or counteracting causes . . . one negative condition invariably understood, and the same in all instances (namely, the absence of counteracting causes) being sufficient, along with the sum of the positive conditions, to make up the whole set of circumstances on which the phenomenon is dependent.'†

Among positive conditions we may distinguish two classes, which may be called respectively Predisposing and Exciting or Initiating Causes. Predisposing Causes are the relatively permanent conditions whose presence may precede the effect for any length of time, but which, for lack of the Exciting Cause, remain inoperative. The Exciting Cause is an instantaneous change, a something which, by coming into play, brings all the conditions into effective action and precipitates the effect. When we say that small causes may produce great effects, we have initiating causes in mind—the spark that burns a city, and the speech that brings about a war. Here is a fictitious instance given by Boscovich, and quoted by Tyndall ('Heat a Mode of Motion,' ninth edition, Lecture III., pp. 66-68):

'Boscovich . . . pictures a high mountain rising out of the sea, with sides so steep that blocks of stone are just able to rest upon them without rolling down. He supposes such blocks, diminishing gradually in size, to be strewn over the mountain—large below, moderate at the middle height, and dwindling to sand-grains at the top. A small bird touches with its foot a grain on the summit; it moves, sets the next larger grains in motion; these again let loose the pebbles, these the larger stones, these the blocks; until finally the whole mountain-side rolls violently into the sea, there producing mighty waves. Here the foot of the little bird unlocked the energy, the rest of the work being done by gravitation.'

Similarly a 'spark acts like the foot of the bird; it starts a process which is continued and vastly augmented by the molecular forces of the fuel. . . .' Tyndall further points out that 'The action of the nerves in unlocking the power of the muscles also falls in admirably with the conception of Boscovich here described.' Again, when a number of conditions combine to keep a body in a position of unstable equilibrium—*e.g.*, to keep an egg balanced on one end—an infinitesimal determining cause is all that is needed to

* Dr. John Venn, 'The Principles of Empirical or Inductive Logic,' ch. ii., p. 57.
† 'A System of Logic,' Book III., ch. v., § 3.

decide the fate of the system. Hence the importance of the determining or exciting cause even when, considered in isolation, it might seem to be of a very trifling nature.

The predisposing causes constitute what Professor Bain calls a Collocation. In considering the causes of a summer thunderstorm we may regard the presence of aqueous vapour, the high temperature of the earth's surface, and the ascending currents of air as constituting, together with difference of electric potential, the Collocation. The downward rush of cold dry air, which (in conjunction with the ascensional movements of warm moist air rising from the heated ground) causes a sudden condensation of great quantities of aqueous vapour, would be here the determining cause.

In Pathology the distinction between predisposing and determining causes is often clearly marked. If we are investigating the causes of a case of 'nervous break-down,' we may have to reckon among predisposing conditions an inherited neuropathic tendency, insufficient or improper nourishment, bad air, insufficient exercise, late hours, over-excitement, over-work, long-continued anxiety; while the exciting cause may be a sudden shock or a single sleepless night.

But these distinctions are not absolute. We might have regarded a sudden rise of temperature as the exciting cause of the 'heat thunderstorm,' and all the other conditions as constituting the Collocation; nor had the descending masses of cold dry air any better right than the ascending columns of warm moist air to be called the determining cause. So again a long-continued succession of nervous shocks and an established habit of insomnia might be regarded as predisposing causes of a neurasthenic 'break-down,' while the exciting cause might be a day's fasting or an hour's worry. The important point is that a change through which a total cause produces its effect can take place only when *all* the *essential* conditions are present. If n be the number of essential conditions, then, if *any* $(n-1)$ of these are present, they are present as predisposing causes until matters are precipitated by the arrival of the n^{th}. It is always the *last* straw that breaks the camel's back, no matter in what order they are successively placed in position.

We have said enough, perhaps, sufficiently to emphasize the importance of not taking the antecedent conditions in an undiscriminated heap. Clearly we must at least insist on the distinction between the collateral or predisposing and the initiating or exciting conditions.

There are other distinctions which may at any time assume importance, and require to be emphasized. Thus, wherever the agency of human beings is presupposed, there is the practically important distinction between the controllable conditions and those beyond human control. Take the case of a carelessly hung

picture which, on the clapping of a door, suddenly falls. Here the conditions are :

(1) Weight of picture.
(2) Rotten cord or loose nail.
(3) Slamming of door (the precipitating cause).

Here the important distinction lies between (2), on the one hand, and (1) and (3) on the other, (2) being the eminently controllable condition. In hanging pictures we may see to it that the cords are sound and the nails well driven in. We cannot so easily be responsible for the non-slamming of doors or for the weight of the pictures.

When we are considering causal influences of a persistent character, a commonly recognized distinction is that between periodic and non-periodic causes. With this is associated, in somewhat complex relations, the distinction between cumulative and non-cumulative effects.

A periodic cause might be defined as the rhythmic recurrence of identical or similar conditions in a regular order of succession. Thus the successive systole and diastole of the heart are a periodic cause with regard to the circulation, and the rhythmic succession of muscular contractions and relaxations which alternately expand and diminish the thoracic cavity is a periodic cause of respiration.

A non-periodic cause is the continuous persistence (or irregular recurrence) of identical or similar conditions tending in one and the same causal direction. Whatever action there is is non-rhythmic. Thus the gravitation of bodies at the surface of the earth is a non-periodic cause, and so is any steadily maintained pressure like that exerted by the mainspring of a watch or by the steady pull of the hand in drawing out a cork with a corkscrew.

But modern Science tends to multiply periodic causes, and if we include among them all vibratory and undulatory motions it may be that the Science of the future will regard the action of every physical force as in some sense periodic.

An effect is said to be cumulative when it persistently increases in one and the same sense. Such effects may be produced either by non-periodic causes or by causes of which one at least is periodic.

(*a*) *Cumulative effects produced by Periodic Causes.*

Rhythmic circulation of the blood may result in growth, and daily exercise in continually increasing strength. The beat of a bird's wings results in continuous flight, and the alternate movements of the legs may result in locomotion to an ever-increasing distance from the starting-point.

Tyndall gives the following instance of a cumulative effect produced by a total cause of which one main factor was periodic. A sheet of lead, covering the sloping choir-roof of Bristol Cathedral

in two years moved bodily down through a space of eighteen inches. The lead was exposed to a periodically varying temperature, higher in the day-time, lower at night. Under the action of gravity it expanded more freely downwards than upwards, and, in contracting, 'its upper edge was drawn more easily downwards than its lower edge upwards. Its motion was therefore that of a common earth-worm; it pushed its lower edge forward during the day, and drew its upper edge after it during the night, and thus by degrees it crawled through a space of eighteen inches in two years.'*

We take another typical example from Sir Robert Ball's 'Time and Tide,' p. 101. 'You see,' he says, 'a heavy weight hanging by a string, and in my hand I hold a little slip of wood no heavier than a common pencil; ordinarily speaking, I might strike that heavy weight with this slip of wood, and no [perceptible] effect is produced; but if I take care to time the little blows that I give so that they shall harmonize with the vibrations which the weight is naturally disposed to make, then the effect of many small blows will be cumulative, so much so that after a short time the weight begins to respond to my efforts, and now you see it has acquired a swing of very considerable amplitude.'

Mr. Edmund Catchpool gives similar instances: 'The regular tramp of soldiers crossing a bridge will break the bridge down if its period of oscillation agrees with the interval between the steps; and the vibrations of the air caused by an organ (it is said even by the voice) will break a pane of glass in a window if the pane is of such a size that it vibrates' with a frequency corresponding to the successive impacts of the sound-waves.†

Again, the periodic movements of the tides have a cumulative effect in the retardation of the Earth's rotation on its axis. Consequently the day is always lengthening and the moon retreating, and there seems to be no counteracting agency anywhere discernible.

(*b*) *Cumulative effects produced by Non-periodic Causes.*

As an example of an effect of this kind we have the results of the conduction of heat from the Earth's interior to its surface, and its loss thence by radiation into space. This process is extremely tardy; but, going on incessantly and always in the same direction, it produces gigantic effects. Other examples are the changes produced in rocks through the age-long pressure of the superincumbent strata, and—in the biological sphere—the gradual adaptation, to a new habitat, of a race of animals or plants.

The effects produced by both types of cause may be at least partially non-cumulative. Thus, as a non-cumulative effect due to

* John Tyndall, 'Heat a Mode of Motion,' ninth edition, Lecture IV., p. 95.

† 'A Text-book of Sound,' second edition, ch. v., p. 76.

the action of a total cause in which a large factor is periodic, we have what Sir Robert Ball has called 'that stupendous Annus Magnus of hundreds of thousands of years during which the Earth's orbit itself breathes in and out in response to the attraction of the planets.'*

As a non-cumulative effect due to the action of non-periodic causes, we may instance the periodic undulations maintained on the surface of the sea by the continuous pressure of a storm-wind; and all cases of the maintenance of equilibrium by a system of mechanical forces might be taken as examples of this kind of effect if we were to regard as negligible the *progressive* character of the results of internal strain and pressure.

Composition of Causes and Intermixture of Effects.

Mill follows up his discussion of Cause as the totality of the conditions by a more specific consideration of the case (in his view 'almost universal') in which 'two [or more] different agents, operating jointly, are followed, under a certain set of collateral conditions, by a given effect' (Book III., ch. vi., § 1). The main interest attaching to this more complex conception of Causation lies in the nature of the effect which results from the co-operation of the different agents.

We must first notice that the effects of these several agents may intermix or not. Where there is no Intermixture of Effects, the effect of each of the separate causes is clearly distinguishable from those of all the others, and can readily be disengaged from the total effect. Two writers, possessing completely different styles, may co-operate in producing a book, and the handiwork of each may be of so characteristic a quality as to be easily distinguishable within the total effect due to the combined work of both.

The operating causes may, however, interfere with each other's work, so that the result is Intermixture of Effects. But here Mill makes an important distinction. The result of Intermixture may be, with regard to the causal agencies which produce it, either *homogeneous* or *heterogeneous*. It is homogeneous when the concurrent causes, whilst more or less modifying, or even counteracting, each other's effects, still exert their full efficacy, each according to its own law—its law as a separate agent. It is heterogeneous or heteropathic when 'the agencies which are brought together cease entirely, and a totally different set of phenomena arise: as in the experiment of two liquids which, when mixed in certain proportions, instantly become, not a larger amount of liquid, but a solid mass' (Book III., ch. vi., § 1).

Where the Intermixture of Effects produces a homogeneous result, the co-operation of the causes which thus combine their

* 'Time and Tide,' p. 72.

individual effects is called by Mill a Composition. 'I shall give the name of the Composition of Causes,' he says, 'to the principle which is exemplified in all cases in which the joint effect of several causes is identical with the sum of their separate effects' (*ibid.*).

In illustration of the Intermixture of Effects due to Composition of Causes we may cite the movement of the Earth in space relatively to the Sun. The distinguishable causes which contribute in producing this single total effect are many in number. There is the gravitative action of the Sun and of the Moon and of the various planets, each of these bodies contributing definite items to the total effect; and there is also the Earth's own momentum in the direction of the tangent to its orbit.

A more general illustration is that of the resultant effect produced by the composition of any forces acting according to dynamical laws. For instance, two forces, P, Q, acting on a particle, and represented, in magnitude and direction, by the adjacent sides AB, AC of a parallelogram ABDC, will cause that particle to move in the direction of the diagonal AD; and the position to which the particle is eventually transported through the simultaneous action of these two forces, operating during a given time T, will be precisely the same as that to which it would have been brought had it been acted upon in succession first by the one force and then by the other, each of them acting during a time T.

The mutual interference of homogeneous effects may, as a limiting case, amount to complete Counteraction. Thus, if the two forces P, Q are equal in magnitude, but act upon a particle in opposite directions, the particle will remain at rest. Mill gives the following illustration: 'A stream running into a reservoir at one end tends to fill it higher and higher, while a drain at the other extremity tends to empty it'; and he adds that 'even if the two causes which are in joint action exactly annul one another, still the laws of both are fulfilled: the effect is the same as if the drain had been open for half an hour first, and the stream had flowed in for as long afterwards' (Book III., ch. vi., § 1).

In reference to this phenomenon of Counteraction, Mill makes the important remark that laws are not broken even when the causes whose mode of operation they attempt to explain, through counteraction of their natural effects, apparently fail to act in accordance with these laws. He refers to 'the popular prejudice that all general truths have exceptions,' and to the fallacy underlying this prejudice when it is made to bear on laws of Causation. 'There are not a law and an exception to that law, the law acting in ninety-nine cases, and the exception in one. There are two laws, each possibly acting in the whole hundred cases.' And again: 'What is thought to be an exception to a principle . . . is always some other and distinct principle cutting into the former' (Book III., ch. x., § 5).

Mill proposes to remedy the confusion caused by this misinterpretation of the meaning of scientific law by an appropriate use of the expression 'tendency.' 'All laws of causation, in consequence of their liability to be counteracted, require to be stated in words affirmative of tendencies only, and not of actual results. . . . Thus, if it were stated to be a law of Nature that all heavy bodies fall to the ground, it would probably be said that the resistance of the atmosphere, which prevents a balloon from falling, constitutes the balloon an exception to that pretended law of Nature. But the real law is that all heavy bodies *tend* to fall; and to this there is no exception, not even the sun and moon; for even they, as every astronomer knows, tend towards the earth, with a force exactly equal to that with which the earth tends towards them' (Book III., ch. x., § 5).

2. *Cause as the 'concurrence of antecedents' on which an effect is 'invariably and unconditionally consequent.'*

Mill is very firm in insisting on the point that the theory of Induction requires no other notion of Cause than such as can be gained from experience; and by 'experience' Mill means 'sense-experience.'* He considers that our conviction of the invariability of causal consequences is obtained by Induction from experience so understood. It is important to notice that by 'invariable' Mill does not mean merely 'invariable so far as our experience has gone,' or even 'unvaried within the limits of human experience.' He saw quite clearly that invariability in this sense is not sufficient to constitute any causal connexion.

'The succession of day and night,' says Mill, is as much an invariable sequence as the alternate exposure of opposite sides of the earth to the sun. Yet day and night are not the causes of one another. Why? Because their sequence, though invariable in our experience, is not unconditionally so; those facts only succeed each other provided that the presence and absence of the sun succeed each other, and if this alternation were to cease, we might have either day or night unfollowed by one another. There are thus two kinds of uniformities of succession, the one unconditional, the other conditional on the first: laws of causation, and other successions dependent on those laws.'† To confuse these two is to commit the fallacy of 'Post hoc, ergo propter hoc.'

Thus the invariability of Mill's causal consequence is *unconditional* invariability.

In another passage Mill connects the idea of unconditionalness

* 'It may, therefore, safely be laid down as a truth both obvious in itself, and admitted by all whom it is at present necessary to take into consideration, that of the outward world we know, and can know, absolutely nothing, except the sensations which we experience from it' (Book I., ch. iii., § 7).

† 'Auguste Comte and Positivism,' second edition, Part I., pp. 57, 58.

with that of necessity. 'If there be any meaning,' we read, 'which confessedly belongs to the term "necessity," it is *unconditionalness*. That which is necessary, that which *must* be, means that which will be, whatever supposition we may make in regard to all other things. The succession of day and night evidently is not necessary in this sense. It is conditional on the occurrence of other antecedents. That which will be followed by a given consequent when, and only when, some third circumstance also exists, is not the cause, even though no case should ever have occurred in which the phenomenon took place without it.'*

It follows at once from the passages quoted above that the antecedent upon which an effect follows 'unconditionally,' in Mill's sense of the word, must be the 'total' antecedent. The partial antecedent will always be dependent for its effectiveness on the conditions that are lacking—*i.e.*, on 'a third circumstance'—and therefore cannot be a 'cause' according to Mill's definition of the term. Hence the two requisites of totality and unconditionalness are essentially akin, and the question forcibly suggests itself whether the two aspects of 'Cause' that we have been considering—(1) Cause as the sum total of the conditions, and (2) Cause as the concurrence of antecedents on which an effect is *invariably and unconditionally* consequent—may not eventually prove to be but two renderings of one and the same conception.

With a view to determining the relation between these two aspects, let us once more consider what it is that Mill understands by the totality of the conditions. In the first place, having respect to Mill's explicit statement that the cause he is considering is only the 'physical' and not the 'efficient' cause, we must not press the meaning of the expression beyond its relevance for strictly scientific purposes. Again, the 'total' antecedent might be understood to include an endless chain of causes, for the immediate total antecedent is itself dependent on a preceding totality of conditions, and so backward *ad infinitum*; for a first and unconditional cause would never be reached. A completely sufficient reason, indeed, cannot be given by Inductive Science for any of the effects that arise within the endless chain of phenomena. But it is essential to add that Mill, in his causal theory, makes no pretence of exhausting the significance of the term 'Cause.' His sole aim is to interpret it in the way most purposive for the true interests of inductive inquiry. 'I make no research,' he says, 'into the ultimate or ontological cause of anything' (Book III., ch. v., § 2).

We can trace Mill's real meaning most clearly when we turn to the sections in his Logic (Book III., ch. v., §§ 2 and 3) in which he first states and develops his definitions of Cause. The idea which Mill initially connects with the law of Causation is that of *invariable sequence*. This conception, on Mill's view, is inductively reached as

* 'A System of Logic,' Book III., ch. v., § 6.

a generalization from experience—from observed facts of Nature. 'The only notion of a cause which the theory of induction requires,' he says, 'is such a notion as can be gained from experience;' and he adds: 'The Law of Causation, the recognition of which is the main pillar of inductive science, is but the familiar truth that invariability of succession is found by observation to obtain between every fact in nature and some other fact which has preceded it' (Book III., ch. v., § 2). We shall be concerned in a later chapter with the critical discussion of this view of the Law of Causation. At present we are only concerned to notice that the notion of *invariable sequence* is the idea which Mill holds to be most essential to the idea of the causal relation.

From this point Mill proceeds by an easy transition to connect the idea of invariable sequence with the conception of Cause as the total antecedent. 'It is seldom, if ever,' he writes, 'between a consequent and a single antecedent that this invariable sequence subsists. It is usually between a consequent and the sum of several antecedents' (*ibid.*, § 3); and towards the end of this same section we find him specifically defining the Cause as 'the sum total of the conditions positive and negative taken together; the whole of the contingencies of every description, which being realized, the consequent invariably follows.' Thus, 'if a person eats of a particular dish, and dies in consequence'—to take Mill's own illustration—'there certainly is, among the circumstances which took place, some combination or other on which death is invariably consequent: as, for instance, the act of eating of the dish, combined with a particular bodily constitution, a particular state of present health, and perhaps even a certain state of the atmosphere.' We see, then, that by the sum total of the conditions which make up the Cause Mill means the sum of just those antecedents with which the consequent can be connected by an invariable law. But beyond this there is an ideal meaning of causal connexion which forms no part of Mill's explicit doctrine. The causal connexions which inductive inquiry must seek to establish are *reversible* connexions. Not only must the antecedent be invariably followed by the consequent, but the consequent must be invariably preceded by this same antecedent. Mill's belief in what he calls 'plurality of causes' (*vide* p. 383), according to which doctrine 'one fact may be the consequent in several invariable sequences' (Book III., ch. x., § 1), for 'many causes may produce mechanical motion,' and 'many causes may produce death,' prevents him from introducing the idea of a 'reversible connexion' into his definition of the causal relation. But Mill's own expressions sometimes, though, as it seems, unintentionally, point us to this deeper conception of the causal relation. Thus, in the passage (Book III., ch. v., § 2) in which he is insisting on the fact that a cause must be invariably followed by the same consequent, he

writes: 'The invariable antecedent is termed the cause; the invariable consequent the effect'; and adds: 'Let the fact be what it may, if it has begun to exist, it was preceded by some fact or facts with which it is invariably connected.'

Mill's language here seems not to express his meaning quite accurately. Were it not misleading to press the literal meaning of the words, we might say that if a consequent is invariably connected with an antecedent which is itself invariable—*i.e.*, presumably, the invariable antecedent of the consequent—the connexion must be reversible, and the causal antecedent must precede the consequent, just as invariably as the consequent follows the antecedent.

But when once the causal connexion is held to be ideally reversible, the meaning of the words 'total' and 'unconditional' is correspondingly modified. The 'totality' of the relevant conditions must mean that totality which is relevant to the ideal of a reversible connexion. All other elements in the antecedent must be eliminated as accidental. And it is, in fact, in this very direction that Mill is feeling when, after pointing out that the sun, friction, percussion, electricity, chemical action may all be causes of heat, he adds that 'If, on further analysis, we can detect in these any common element, we may be able to ascend from them to some one cause which is the really operative circumstance in them all. Thus it is now thought that in the production of heat by friction, percussion, chemical action, etc. the ultimate source is one and the same' (Book III., ch. x., § 3). Again, in the light of the ideal of reversibility we may give a new meaning to the term 'unconditional.' For since antecedent and consequent in a reversible connexion belong to each other as closely as they belong to themselves, we may appropriately equate the two conceptions of reversibility and unconditionality, and account a connexion unconditionally valid when it is reversible. It should, however, be borne in mind that this unconditionality is not absolute, but relative to the inductive point of view. The discussion in Chapter XLVI. (below) should make this clear.

Again, in the light of the reversibility-ideal, we see more clearly the importance of that requirement of *immediacy* which Science is accustomed to attach to the connexion between Cause and Effect.

One main distinction between the popular and the scientific conceptions of 'Cause' lies in the indifference of popular usage to this requisite of immediacy. The ordinary man, looking forward rather than backward, is not interested in reversibility of causal connexion. For practical purposes he often requires some view ahead, and he is then opposed to any attempt to bring Cause and Effect so near together that this view ahead is lost. To screw up Cause and Effect into close juxtaposition (to use Dr. Venn's

expressive phrase*) would be to secure certainty and reversibility of connexion at the price of usefulness. Thus death is conceived as the effect of taking poison, ruined health as the effect of intemperance, despite the long uncertain interval between cause and effect in each case. We may grant that Dr. Venn is quite right in supposing that the popular consciousness is perfectly ready to allow any desired interval to elapse between cause and effect, provided this implies practical gain. The ordinary man adapts his phrase to his needs. But his interests are just as frequently furthered by noticing the more nearly immediate effect as by emphasizing that which is more remote. Thus the lighted match is causally connected in the popular mind with the smoking tobacco and the pleasant feeling that speedily ensues, the touching of hot iron with a feeling of pain, and so forth. The practical man seems indifferently to emphasize either the most conspicuous after-effect or such after-effect as happens at the moment specially to interest him. It still remains perfectly true, as Dr. Venn points out, that the 'plain man' will not screw cause and effect tightly together in order to ensure a degree of scientific certainty which is useless to him.

But the main interest of Science in the matter is to secure at all costs regularity and certainty, and, where possible, reversibility of causal connexion. This entails a bringing of the two components in the causal sequence, the causal antecedent and its consequent, into such closeness of contact with each other as will suffice for this purpose Now, ideal regularity, certainty, and reversibility cannot be attained so long as the mind cannot grasp and control every link in the chain that leads from antecedent to consequent. If we ignore these intermediate links, we labour under a disability very similar to that which we experience when we overlook essential elements in the sum or totality of the conditions. As Mr. A. Sidgwick well observes ('The Process of Argument,' ch. xi., p. 153), 'Intermediate links in a chain of causation are so many opportunities for counteraction, in the same way as the length of a piece of railway provides opportunities for an accident. They are intermediate *conditions*. The pull on the trigger will fire the shot if, and only if, the catch, the spring, the hammer, the cap, and so on, all act in the expected manner. Therefore our forgetfulness of intermediate links takes effect just in the same way as our forgetfulness of conditions generally ; it may give us a false security.' In the interests of vigorous induction it is essential to aim at bringing cause and effect sufficiently close to exclude all intermediate links of which we cannot render an intelligible account. It is in this sense that the inductive interest calls for something approaching to continuity between cause and effect. Thus it would be unsatisfactory to assign Chlorine as the cause of the bleaching of vegetable dyes. It is true that, popularly speaking, Chlorine, added to

* 'The Principles of Empirical or Inductive Logic,' ch. ii., p. 56.

vegetable dyes, bleaches them; but it is not the Chlorine that is the bleaching agent. The Chlorine, by uniting with the Hydrogen of the water, liberates Oxygen; and it is this nascent Oxygen which, reacting upon the colouring matter of the dye, forms a colourless compound, and so effects the bleaching.

The requirement of immediacy can be pressed only in so far as it is called for in the interests of ensuring the unconditionality and reversibility of the causal sequence. In a chain of necessary connexions any link may be held to be causally connected with any other link, though many intermediate links may intervene between the two. If, instead of being considered as a mere means for ensuring unconditionalness and reversibility, the requisite of immediacy were treated as an end in itself, the only logical terminus of such a view would be an insistence on the absolute simultaneity or identity of cause and effect; and indeed, as Mill points out, '*Cessante causâ cessat et effectus* has been a dogma of the schools' (Book III., ch. v., § 7).

With regard to this maxim, we would point out that, if it were strictly true, there would never be any chain of causation at all; and that, though it is undoubtedly true that every cause expresses itself in producing *an* effect, still, the direct effect, which is partly contemporaneous with the action of the cause, need not be the effect relevant to the scientific interest. All that Science requires is that the connexion between the action of a certain cause and an element in the succession of after-effects shall be regular and certain, and, if possible, reversible. Any element, in the total 'after-effect,' that is so connected with the cause can in the truest scientific sense be called an effect of the cause. As a matter of fact it is usually very close to the cause itself.

Where the direct causal action lasts an appreciable time, it is customary to distinguish in the causal process three clearly defined time-stages:

(*a*) The time prior to the causal action (antecedent);
(*b*) The time when the cause is actually producing an effect (synchronous);
(*c*) The time following on the direct causal action (subsequent).

Example.—When A stabs B, the direct causal action, strictly so called, is limited to the time in which the dagger is penetrating the body and tearing the tissues. From this should be distinguished that which precedes and that which follows the actual wounding: the seizing of the dagger, the movement of the arm, etc., on the one hand; the opening of the bloodvessels, the loss of blood, and still remoter effects, possibly death itself, on the other.*

* Dr. Christoph Sigwart, 'Logik,' vol. ii., part iii., ch. i., § 73: English Translation by Helen Dendy, vol. ii., p. 106.

We have already drawn attention to the logical relation between the 'immediate' and the 'unconditional'; and we have also seen that the 'unconditional' antecedent must be identified with the 'total' antecedent—*i.e.*, with the sum of the relevant conditions. The practical consciousness attaches as little value to the causal requisite of *totality* as it does to the requisite of immediacy. It is characteristic of the popular conception of Causation that it is content to identify any single condition with the whole causal antecedent, provided that condition is a sufficiently striking one. In ordinary life we look out for causes, not primarily to gratify intellectual curiosity, but chiefly in order to produce some particular desired effect. We are anxious to produce rather than to explain. Hence the discovery of any obvious antecedent condition suffices us. As against this view, Science, as we have seen, insists on the identification of the Cause with the totality of the relevant conditions.

We have now to point out how the further insistence on the part of Science that the *effect* can be justly estimated only in so far as it is adequately analysed enables it to keep clear, at least theoretically, of the popular pitfall known as 'Plurality of Causes.'

Plurality of Causes.

By 'Plurality of Causes' we do *not* mean that a number of conditions or partial causes *conspire* to produce a certain effect. We mean that the same effect may at different times be produced by different *total* causes. Plurality of causes means plurality of *possible* causes, of which only one is actual in any given instance.

The doctrine of a plurality of causes asserts that the relation between Cause and Effect is not necessarily reciprocal. It requires invariability of connexion between Cause and Effect only in the one sense, not in the other. The same cause may invariably produce the same effects, but, given the effect, we cannot with certainty argue back to the cause.

It has been maintained, and with justice, that this doctrine is consistent with the popular, though not with the scientific, view of Cause and Effect. It is quite true that 'death' is an effect that may be due to drowning, strangling, overeating, etc.; but this 'death' is a *general* and not a particularized effect. The effect is, in fact, loosely defined, in accordance with the popular view, by some one or two salient characters—characters common to the effects respectively produced by a large number of possible causes; Drowning, strangling, overeating may all cause death; but (and this is what the practical consciousness commonly forgets to notice) the death which they cause in common is marked by different

symptoms, according as it is this cause or that which has been operative, the symptom being just part of the effect. 'There are many causes of death,' as Dr. Mellone concisely puts it, 'only because there are many kinds of death.'*

Professor Minto gives the following illustration: 'A man's body is found dead in water. It may be a question whether death came by drowning or by previous violence. He may have been suffocated and afterwards thrown into the water. But the circumstances will tell the true story. Death by drowning has distinctive symptoms. If drowning was the cause, water will be found in the stomach and froth in the trachea.'†

We see, then, that, given a certain very *general* effect and nothing further, it is impossible to tell to which of a given number of possible causes it is due; but that, if we study the effect more closely, and observe its more specific symptoms or marks, we are able to point out the cause (*cf.* Minto, *ibid.*, p. 343).

We conclude, then, that, if only the effect is given in all its detail, there will never be more than one possible cause to account for it. The relation between Cause and Effect is thus, *ideally*, reciprocal or reversible. Each additional line in the delineation of an effect may strike out one or more of the various conceivable causes, until, when the whole effect is analysed out, all possible causes except one are eliminated. It may happen that all known possible causes are thus eliminated. In that case the cause remains unknown.

But however exhaustively an effect be analysed and its remotest ascertainable symptoms studied, it cannot be traced back to non-essential antecedents. If these were to be reckoned as part of the antecedent of a causal sequence, then the causal relation would no longer be reciprocal, and the plurality of causes would still subsist. It is only when the total cause is regarded as consisting exclusively of essential or effective conditions that it is possible to specify the cause by diagnosis of the effect. *E.g.*, a nail is driven one inch into a piece of wood by a blow from a blue-coloured hammer made of steel. Now, it will be driven in just as far and in exactly the same way by a red-coloured hammer made of iron, provided the weight of the two hammers is the same, and each offers the same surface-resistance to the nail. Whenever the blue-coloured steel hammer strikes with a certain momentum a nail of a certain kind placed in a certain position, the nail will be driven one inch into the wood; but the colour, and, in a more restricted degree, the material, are non-effective conditions, and, as such, cannot enter into any reciprocal connexion with the effect.

* Dr. Sydney Herbert Mellone, 'An Introductory Text-Book of Logic,' first edition, ch. viii., p. 260.

† 'Logic Inductive and Deductive,' p. 342. *Cf.* also Professor Carveth Read, 'Logic Deductive and Inductive,' third edition, ch. xiv., pp. 179, 180.

The preceding discussion on the Plurality of Causes has an important bearing on the theory of Deductive Inference in its hypothetical form. For it shows that in proportion as our knowledge of Reality becomes more accurate and comprehensive, the less do the fallacies of affirming the consequent and denying the antecedent imply any real error. We are, indeed, led to see that when *both* Consequent and Antecedent are *precisely* defined—*i.e.*, defined with a precision that is relevantly ideal—they mutually imply each other, so that when we deny the antecedent or affirm the consequent no fallacy is committed. Thus, to quote from Professor Stout: 'I can legitimately infer from the proposition "If a closed figure is three-sided, it has angles" that if anything has not angles, it is not a closed figure with three sides. The denial of the consequent involves denial of the antecedent. But I cannot infer that "If anything is not a closed figure with three sides, it has not angles." I cannot do so because I have not precisely defined the consequent. But when I do this in the proposition "If a closed figure has three sides, it has three angles," the fallacy of denying the antecedent is no longer a fallacy.

'Take a different example. "If all vegetables were cabbages, the vegetable I am eating would be a cabbage." Even if some vegetables are not cabbages, the one I am eating may still be a cabbage; but this is only because I have included more in the antecedent than is necessary for it *qua* antecedent of the assigned consequent. It is presupposed in the unity and identity of the universe that the fact of the vegetable I am eating being a cabbage has some *precise* antecedent statable in the form "If vegetables with certain specific characters are cabbages." Given this *precise* antecedent, I can argue from the denial of the antecedent to that of the consequent.'

Quantitative Aspect of the Causal Relation.

For comprehensive generalizations, such as the principles of the Conservation of Energy and of Mass, Science has the greatest respect. But what is typical of scientific procedure is not that it bears towards breadth of generalization—so, alas! does ignorance—but that it consistently refuses to reach comprehensiveness at the cost of accuracy. Hence the achieved reconciliation of the 'general' and the 'accurate' which constitutes the essential triumph of Science. As Science has become more comprehensive both in the array of its data and in the scope of its generalizations, it has *pari passu* advanced in accuracy and precision. But it is not its comprehensiveness of spirit that has ensured the precision. On the contrary, the precision, and that slowness of advance which precision demands, have alone made possible the vast and fruitful comprehensiveness.

Precision is doubtless a fundamental requisite of all scientific investigation. It is as essential to the success of the biologist or the archæologist as it is to that of the physicist or the mathematician. But it is only within the strictly physical and mathematical sciences that the numerical precision required for the quantitative treatment of Causation can be said to be obligatory. This is frequently forgotten by the votaries of these abstracter sciences. Thus Sir John Herschel, the astronomer, writes as follows: 'Numerical precision . . . is the very soul of science; and its attainment affords the only criterion, or at least the best, of the truth of theories, and the correctness of experiments. . . . Indeed, it is a character of all the higher laws of nature to assume the form of precise *quantitative* statement.'* This statement is true of the physical sciences only.

The need for numerical accuracy, where such is obtainable, is excellently brought out by Dr. Sigwart in the following passage of his 'Logic': 'If we are to speak accurately,' he says, 'we cannot say that eating stills hunger and drinking thirst, for a mouthful or a sip is no good; nor can we say that arsenic kills or quinine reduces fever, for it depends upon the dose; it is inaccurate, again, to say that common salt is dissolved by water, for it is not true that any quantity of salt is dissolved by any quantity of water.'†

The quantitative aspect of Causation is regulated by certain Principles of Conservation, notably by THE PRINCIPLE OF THE CONSERVATION OF ENERGY. This principle has been defined by Clerk-Maxwell in the following terms:

'The total energy of any material system is a quantity which can neither be increased nor diminished by any action between the parts of the system, though it may be transformed into any of the forms of which energy is susceptible.'‡

It is often stated as an axiom that the amount of energy *in the material universe* is constant. This statement is far from being the record of an ascertained fact. We may safely aver that no physicist has ever established an equation between the whole energy of the Universe at any time, including the energies of all the stars of heaven and all the cells of all living bodies, and its energy at a subsequent moment of time. The equation of constancy—of the constancy, that is, of the energy of the Universe as a whole—is, in fact, a most unjustifiable extension *in indefinitum* of the empirically established equation of equivalence in a limited and closed system. The fallacy involved in this extension is picturesquely expressed by Dr. James Ward: 'Those who insist that the quantity of this energy in the universe *must* be constant seem to me,' says Dr. Ward,

* Discourse on Natural Philosophy, §§ 115, 116, quoted by Professor Welton, 'A Manual of Logic,' second impression, vol. ii., ch. vi., p. 161.

† 'Logik,' Part III., vol. ii., ch. v., § 95: English Translation, pp. 346, 347.

‡ Dr. J. Clerk-Maxwell, 'Matter and Motion,' ch. v., p. 60.

'in the same position as one who should maintain that the quantity of water in a vast lake must be constant merely because the surface was always level, though he could never reach its shores nor fathom its depth.'*

Moreover, this so-called principle of the constancy of the energy of the Universe has not even the limited relative necessity of a regulative principle of Physics. What guides the physicist in forming his energy-equations is not the idea of the constancy of energy within the *universe*, but that of the balance of energy about any given change as fulcrum. The energy-level must remain constantly the same; or, in other words, the energy distributed within the given system must remain, through all its redistributions, constant in amount. The 'constancy of energy,' as a working Idea of Physics, comes, indeed, to nothing more than this: Given a finite, *known* quantity of energy, then, if that energy be measured after any transformation, it must be precisely equivalent in amount to the original quantity. In other words, lost energy can always be found again, provided the precise amount lost is known. There is no attempt to deal with the whole amount of energy in the universe at any time, a perfectly indefinite, incalculable quantum. Just as the 'postulate' of the indestructibility of matter is really nothing more than the balance of weights before and after a chemical change, so that of the indestructibility of energy is nothing more than the mathematical balance between the *capacity* for work within a certain closed system, before a certain amount of *actual* work is done, and the *capacity* for work within the same closed system after the transformation has taken place.†

The numerical equivalency between Cause and Effect required by the principle of the Conservation of Energy can be ascertained only when the amount of each causal agency and the total effect can be measured in terms of one and the same unit. In the case of the physical sciences such a unit has been found. It has been found possible to state the amount contributed by each physical agent in an act of causation in the form of its mechanical equivalent expressed in terms of the amount of work done in lifting against gravity, from the sea-level, a certain specified weight to a certain specified height.

Let E be a certain effect measured in terms of the unit U, and represented, let us say, by 47 U. Let C_1, C_2, C_3 . . . be the causes or agents known to be involved in producing E. Let the amount of work contributed by each be measured, as before, in terms of U. Let $C_1 = +8$ U; $C_2 = +20$ U; $C_3 = +15$ U. Then $C_1 + C_2 + C_3 = +43$ U. We infer, therefore, on the ground of the principle of the Conservation of Energy, that we have not yet found all the causes;

* 'Naturalism and Agnosticism' (Gifford Lectures, 1896-1898), vol. ii., Lecture XIII., pp. 76, 77.

† *Cf.* 'Personal Idealism: Philosophical Essays,' edited by H. Sturt, pp. 151-155.

but that causes whose combined capacity for work is equivalent to + 4 U still remain to be discovered.

With regard to the principles of the Transformation and Conservation of Energy in their application to Causal Explanation, we may note the following considerations :

1. Our knowledge that energy is not lost, but transformed or redistributed, helps us in discovering the factors that make up cause or effect, as the case may be. It used to be thought—by Newton, for instance—that energy lost in friction was lost for good. Now it is known to be transformed into heat and sound. Moreover, as the following examples will show, a causal explanation will often consist in noting what *transformations* actually do take place.

EXAMPLES :

(i.) *Question.*—Why can a man endure the heat of an oven in which a beefsteak is cooked ?

Answer.—Because he perspires freely, and thus a large part of the heat which enters his body is used in the performance of a certain saving mechanical work, while only a fraction of the total amount is effective in raising the temperature. 'The excess of heat, instead of being applied to increase the temperature of the body, is applied to change its aggregation ; the heat prepares perspiration, forces it through the pores, and vaporizes it. Heat is thus consumed in work.'* This is the fact that makes a Turkish bath endurable.

(ii.) *Question.*—Why does the Safety-lamp used in coal-mines prevent explosions ?

Answer.—Because, though the explosive gas may ignite within the lamp, yet, when the combustion reaches the close-knit wire gauze, a large part of the energy of the burning gas is transformed into the vibrational energy of the gauze, and thence into the vibrational energy of the heavy metal of the lamp. The conducting power of the gauze lowers the temperature of the gas, thereby stopping the progress of combustion. Hence the flame is kept within the lamp, and does not come into contact with the explosive gas outside.†

* John Tyndall, 'Heat a Mode of Motion,' ninth edition, Lecture IX., p. 243.
† John Tyndall, *ibid.*, Lecture IX., pp. 261-263.

2. When once the factors in an effect are discovered in accordance with our knowledge of the transformation of energy, the quantum of effect contributed by each factor can be estimated in accordance with the theory of mechanical equivalence. In this way all the various quota are measured in terms of one and the same unit.

3. Finally, by applying the principle of the Conservation of Energy we are able to test the *adequacy* of our explanations. The capacity for work contained in the effect must be the same as the capacity for work contained in the cause, the work done consisting in every case of some transformation or redistribution of energy.

Transition to the Method of Causal Explanation.

In the present chapter, relying mainly on Mill's guidance, we have attempted to reach a concept of 'Cause' that would provide a secure basis for the application of methods of Causal Explanation. The outstanding conclusion is that the ideal after which we should aim in Causal Explanation is to reach reversible relations between Cause and Effect. It would, therefore, seem to follow that the method which the interest in Causal Explanation requires is one that will adequately fulfil this fundamental requirement. Further, since the ideal of Reversibility—or of Unideterminism, as it is sometimes called—remains unrealized so long as the defect of a plurality of causes still subsists, it must be the primary aim of a Causal Method through the precision of its procedure to eliminate this radical defect.

CHAPTER XLIV.

XIII. (ii.) THE PROCESS OF SCIENTIFIC OBSERVATION.

THE goal of Observation, we may say, is to see a fact as a fact-under-law. Now, a closer analysis of what is involved in such Observation brings into relief its essentially *experimental* character, and convinces us that, if the facts are to exercise their full natural effect upon the mind, they must be subjected to an *analysis* of an essentially experimental kind. Through the very pressure of its own interest Observation develops into a process of experimental Analysis that can rest satisfied only when its experimental tests have brought out the law which interprets the fact.

Observation, therefore, cannot be adequately conceived as a mere preliminary stage in inductive procedure; for the principle of Fidelity to Relevant Fact, which actuates and inspires all scientific observation, requires that the whole induction, *from first to last,*

shall be under the control of the relevant facts. When Induction is conceived as *explicitly* dominated by this Inductive Idea, we see the observational interest predominant all through. Observation becomes a *permanent factor* in inductive inquiry, and its interest—the interest in dealing faithfully with fact—the controlling interest in the whole process.

This view, of course, implies a deepened view of the meaning of Observation. It implies that Observation is the expression of the observational *Interest* and *Purpose.* Thus it is no mere inspection of the senses, but, as Descartes clearly saw, an inspection of the mind; it is not a process of mere *sensation,* but a process of purposive perception, in whose service are engaged all the constructive powers of scientific Thought and Imagination.

We are only expressing the same conviction in another form when we add that Scientific Observation is essentially an *active* process. It is not a mere passive submission of the mind to its object, but an active attempt to see that object in the light of some dominant interest, and to bring it into touch with the body of our previously acquired knowledge. Were we content to allow the object to stamp itself in upon our minds, we, as a writer has tersely put it, should simply be stamped out.

The characteristics of this activity of Scientific Observation are (1) its *purposiveness*, and (2) its *experimental* character.

(1) *True Scientific Observation is essentially Purposive.*

For Observation to be effective, we must first know what is the aim or purport of our observation, and we must steadfastly adhere to the lines marked out by this purpose. It is of no use to observe 'in general' or at random. We must *select* what is relevant to our interest. In a book entitled 'A Chapter of Science' (p. 19) Mr. Stuart has an illustration in point. 'Many people,' he says, 'took observations of the meteor display on November 13, 1866. Some thought that their colour was the important point, and noted the colour of each they could see. Others thought that the rate they seemed to move at was the important point, and noted that. Others thought that the points they disappeared at was the important point to note, and noted that. Others thought that the direction of their motion was most important, and noted that; and others—and these the best informed—thought that the most important thing to note was the points at which they made their appearance, and noted that.' The writer then goes on to remark that, though the observations of the radiant point were of greatest scientific value, seeing that they served to fix the position of the swarm—the result most especially desired—yet all the other sets of observation, restricted as each was to some one aspect of the whole phenomenon, were serviceable in a subordinate way. On

the other hand, he adds: 'It is very improbable that anything could be learned from the observations of a man who noted of one meteor what was its colour, of another the point at which it appeared, of a third that at which it disappeared, of a fourth, its size, and so on; from such a man's observations there is very little chance that we should discover anything common to all the meteors.'

In this connexion it is extremely important to distinguish between the *relevant* and the *favourable*. True selective interest fastens faithfully on the relevant instances, whether these be favourable to preconceived ideas or not. The failure to give the relevant priority over the favourable constitutes what has been called the fallacy of 'Non-observation,' or of 'overlooking what ought to be observed.' 'It is exceedingly rare,' remarks Jevons,* 'to find persons who can with perfect fairness estimate and register facts for and against their own peculiar views and theories. Among uncultivated observers the tendency to remark favourable and forget unfavourable events is so great that no reliance can be placed upon their supposed observations. Thence arises the enduring fallacy that the changes of the weather coincide in some way or other with the changes of the moon, although exact and impartial registers give no countenance to the fact. The whole race of prophets and quacks live upon the overwhelming effect of one success, compared with hundreds of failures which are unmentioned and forgotten. As Bacon says, "Men mark when they hit, and never mark when they miss." We should do well to bear in mind the ancient story, quoted by Bacon, of one who in Pagan times was shown a temple with a picture of all the persons who had been saved from shipwreck, after paying their vows. When asked whether he did not now acknowledge the power of the gods, "Aye," he answered, "but where are they painted that were drowned after their vows?"'

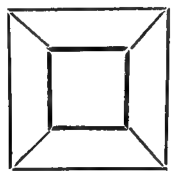
Among the rare persons of strict impartiality to whom Jevons here alludes must be included Charles Darwin. 'The success of the "Origin,"' he writes,† 'may, I think, be attributed in large part to my having long before written two condensed sketches, and to my having finally abstracted a much larger manuscript, which was itself an abstract. By this means I was enabled to select the more striking facts and conclusions. I had also, during many years, followed *a golden rule*—namely, whenever a published fact, a new observation or thought, came across me, which was *opposed to* my general results, to make a memorandum of it without fail and at once; for I had found by experience that such facts and thoughts were far more apt to escape from the memory than favourable ones. Owing to this habit, very few objections were raised against my views which I had not at least noticed and attempted to answer.'

* 'The Principles of Science,' vol. ii., p. 5.

† 'Autobiography,' p. 87.

It need not be deliberate bias which causes us to neglect these instances. Our non-observation of them may be due simply to the psychological law according to which we tend to perceive only such objects as fit in with the idea that is filling our mind. 'The only things,' says Professor James, 'which we commonly see are those which we pre-perceive;'* and it is a well-known fact that to have an idea of the object we wish to observe is the surest guarantee of our seeing it.

Example.—(1) Think of a square lamp-shade viewed from above, and then look at the accompanying figure.



(2) Think of looking down a long picture-gallery or ball-room, and then look again at the given figure.

It will probably be found that having idea (1) in mind prevents us from regarding the figure as a representation of (2), and that having idea (2) in mind prevents our regarding it as representing (1).

Purposive observation must be appropriately and adequately *analytic*. 'The observer,' says Mill, 'is not he who merely sees the thing which is before his eyes, but he who sees what parts that thing is composed of.'† So Lord Avebury reminds us that we have not really seen a mountain until we have seen through it. *How* an observed fact is thus mentally split up into parts or aspects will depend on the purpose of the investigation.

Observation includes not only perceptions, but also the interpretation of perceptions. But the observer must very carefully guard against confusing these two elements, more especially if the interpretation is merely conjectural.

An appeal to the testimony of the senses has logical value in proportion as, in observation, we distinguish between mere statement of fact as observed and conjectures drawn from the observed facts—*i.e.*, in proportion as we accurately and pointedly describe what actually appeared, keeping to essentials, and avoiding any mental confusion between what did happen, on the one hand, and what we feel *ought* to have happened, or what we should *like* to have happened, on the other. In reporting an experience we tend to relate not what we have actually seen or heard, but rather the impression which the events experienced have made upon us. We confuse fact, not with opinion only, but also with feeling.

* 'The Principles of Psychology,' vol. i., ch. xi., p. 444.
† J. S. Mill, 'A System of Logic,' Book III., ch. vii., § 1.

When the conjecture or interpretation is false, the fallacy takes the form of *Mal-observation*, the fallacy of *wrongly* interpreting what we immediately perceive through our senses.

In Mal-observation it is not the senses that are wrong. Thus, suppose we plunge our hands, one into ice-cold water, the other into very hot water, and then dip both into water that is tepid. The one hand will feel a sensation of heat, the other a sensation of cold. There is no mistaking the sensations. Mal-observation would come in were we to interpret these sensations by the statement that the water must be warmer at the one spot than at the other. Again, when our eyes seem to tell us that the sun rises, passes to its highest point, and sets, while our motor-sensations seem to assure us that the earth is at rest beneath us, we make a mal-observation if we interpret our sensations by saying: 'The earth is at rest, and the sun goes round it;' for it does not follow from *our* being at rest relatively to the earth that the earth itself is at rest. Hence by the fact that we *feel* no motion we are not justified in recording that the earth is at rest. It may be either at rest or in motion, for aught that observation can tell us. In order to avoid mal-observation in this particular instance, we must record our observations as follows: 'I *feel* that the earth is at rest, and I can *see* the sun in daily movement from East to West; but whether these perceptions are illusions or not only systematic Science can decide.' For interpretation of our perceptions we must appeal from Sense to Reason.*

(2) *True Scientific Observation is essentially Experimental.*

To see a fact scientifically is to see it in the light of an idea. Observation expresses a form of scientific endeavour which can be satisfied only when the fact is observed as fulfilling some law. The merely passive assimilation of fact is only the first stage in the complete process of Observation. Through such absorption of sense-data the mind develops suggestions which enable it to observe with new interest and efficacy. The facts, we say, are seen in a new light. We aim now at ascertaining whether the suggestion fits the fact. The centre of observational interest is transferred from passive watching to active testing, and the observation inevitably takes an experimental form.

Observation, we repeat, as animated by scientific purpose, is a process whose natural end remains unattained so long as the fact is not clearly seen as obeying law. Hence the experiments to which we subject facts *enter as an integral part* into every purposive observation. It is, of course, not mere manipulation of the object

* On Mal-observation, *cf.* Berkeley, the 'Third Dialogue between Hylas and Philonous,' pp. 455, 456; and for a good illustration of the same fallacy, see Professor Royce's 'Psychology,' p. 27.

that marks the difference between the passive and exerimpental stages in Observation. We may observe just as passively with our fingers as we can with our eyes or ears. Observation is passive just so long as it is purely assimilative. But once the manipulation passes into the service of an idea, and we proceed to put our object under such conditions as the idea requires for testing and verifying itself, our observation is no longer merely passive, but experimental. A definite, purposive interference of some sort with the production of the phenomenon to be noted is the *desideratum* of the experimental or testing stage.

Where this interference is impossible—*i.e.*, where we are unable to place our object in contexts that answer to the requirements of our *idea*—experimenting takes place under difficult conditions. We have to wait till Nature presents us our object under the desired conditions. But even then it is almost certain that Nature will only very roughly satisfy our requirements, and we may have to wait a long time. Nature, again, may present the right kind of instance, but in too strong a degree. For delicate electric investigations the electricity in a thunder-cloud will not do. Or it may be in too weak a degree, as in the production of heat by gentle friction, when a furnace-heat is required.

CONTROL OVER THE CONDITIONS under which the object shall present itself for study is the great desideratum, and the factor in experimental observation *which most influences the precision and certainty of the results obtained.* Such control enables us to vary the circumstances, and to repeat the observation as often as we please by simply reproducing the concurrence of conditions requisite for bringing into play the desired effect.

In certain sciences, as in Astronomy, such control is out of the question. The Astronomer must test his theories concerning the 'canals' in Mars or the belts of Jupiter whenever the conditions happen to be favourable, the planet in opposition, the weather fine. Occasionally, however, Nature contrives the very conditions that the idea requires for its verification. We then have a 'Natural Experiment.' As an excellent illustration of this, we may take that noticed by Dr. Thomas Fowler *—the observation of prominences during a total solar eclipse. Here, under normal conditions, the observer is dazzled by the excess of light, and the question as to the nature of the prominences cannot be answered; but when the moon appropriately obscures the sun's disc, we have 'experimental' conditions under which the answer is obtainable.

* 'The Elements of Inductive Logic,' sixth edition, pp. 50, 51.

CHAPTER XLV.

XIII. (iii.) THE METHOD OF CAUSAL EXPLANATION.

WHERE explanation is causal, fact, as we have seen, means *effect*, and the problem lies (1) in discovering the probable *causes* or conditions upon which the effect depends, and the *laws* according to which they are operative; (2) in testing the actual efficiency of these causes or conditions. We must be able to trace effects back to their determining conditions, and justify our surmises as to the nature of these conditions by working back with explanatory power from causes to effects.

The procedure in Causal Explanation is, in its general outline, dictated in advance by its inevitably inductive character. The Principle of Fidelity to Relevant Fact requires that, in our explanation, we shall pass from fact back again to fact; that we shall not be content tentatively to trace effects to causes, but that we shall also argue back from causes to effects. Preliminary observation of effects requires as its inductive complement experimentation upon causes. Hence, though it is customary to speak of *Methods* of Causal Enquiry, and to divide these into Methods of Observation and Methods of Experiment, yet, when we look at the matter synthetically, in the light of our inductive principle, we see that the Methods in question cannot be other than co-operative in the task of a complete Explanation, and that the so-called 'Observational Methods' cannot be more than preliminary to the Methods which test the hypotheses built upon these observations. Our main endeavour, therefore, will be to bring out the unity of the one Method of Causal Explanation and the interdependence of its various component stages.

A Method of PURE or SIMPLE OBSERVATION, in so far as it concerns the problem of Causal Explanation, may be diagnostically defined as a method which proceeds from effects to causes; and a Method of EXPERIMENT may be similarly defined as a method which proceeds from causes to effects.

I. PURELY OBSERVATIONAL METHODS.

The typical form of Purely Observational Method has been known since Mill's day as the Method of Agreement. He enunciates it somewhat formally, as follows:

'*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause* (*or effect*) *of the given phenomenon.*'*

* 'A System of Logic,' Book III., ch. viii., § 1.

In practical application, the term 'circumstance,' occurring in this enunciation of the Canon of the Method of Agreement, as given by Mill, must be replaced by the expression 'relevant and important circumstance.'

The requirement that the instances shall have *only* one relevant and important circumstance in common makes it imperative to ascertain that all the other relevant circumstances are different. Hence the method has been called the Method of *Single* Agreement. A method of mere agreement could be founded on simple enumeration only; a method of Single Agreement must be founded on elimination as well, for the singleness is established only through eliminating all the circumstances that are not common to all the instances in question.

By way of illustrating this purely Observational Method, it may be sufficient to quote the following example from Professor Read, who has it from Professor Bain:

'Let me begin by borrowing an example from Professor Bain ("Logic," Book III., ch. vi.). The North-East wind is generally detested in this country: as long as it blows, few people feel at their best. Occasional well-known causes of a wind being injurious are violence, excessive heat or cold, excessive dryness or moisture, electrical condition, the being laden with dust or exhalations. Let the hypothesis be that the last is the cause of the North-East wind's unwholesome quality; since we know it is a ground current setting from the pole toward the equator and bent westward by the rotation of the earth; so that, reaching us over thousands of miles of land, it may well be fraught with dust, effluvia, and microbes. Now, examining many cases of North-East wind, we find that this is the only circumstance in which all the instances agree: for it is sometimes cold, sometimes hot; generally dry, but sometimes wet; sometimes light, sometimes violent, and of all electrical conditions. Each of the other circumstances, then, can be omitted without the North-East wind ceasing to be noxious; but one circumstance is never absent—namely, that it is a ground current. That circumstance, therefore, is probably the Cause of its injuriousness.'*

The fact that this method is essentially a method of elimination explains the fact that the force of the method depends on the number and variety of the instances; for the greater the number of varied instances, the greater will be the chance of successfully eliminating the circumstances that are not common to *all* instances of the phenomenon under investigation. Thus Dr. Mellone (following the lead of Professor Henry Laurie,†) states Mill's First Canon as follows:

* Carveth Read, 'Logic Deductive and Inductive,' third edition, ch. xvi., p. 198.

† 'Methods of Inductive Inquiry,' *Mind*, New Series, vol. ii. (1893), p. 321.

'When observation shows that two events accompany one another (either simultaneously or in succession), it is probable that they are causally connected; and the probability increases with the number and variety of the instances.'*

Now, as Dr. Thomas Fowler has pointed out,† it is through this elimination (with its accompanying processes of analysis) that the Method of Single Agreement—which itself proceeds by enumeration of instances—essentially transcends the limitations of the unanalytic Method of Simple Enumeration. But this is not the only essential difference between the two methods as we regard them. The Method of Agreement is frankly and confessedly a merely heuristic method, proceeding tentatively from effects to causes, and is in no sense a Method of Verification, whereas the Method of Enumerative Induction (as we have formulated it) claims to be a complete Inductive Method which aims at the verification-ideal of a Complete Enumeration.

The essential limitation of the Method of Agreement, as Mill himself has clearly shown,‡ is the uncertainty due to the Plurality of Causes. We call it a limitation, and not a defect, since the method, being purely observational, cannot claim more than a heuristic function. It is frankly 'a mode of reconnoitring,' suggesting well-grounded starting-points for experimental inquiry, but making no pretence at causal explanation.

To illustrate the limitation in question, let us suppose that we have two instances which we may somewhat artificially, but still conveniently, symbolize as follows:

$$\text{Antecedents.}\left\{\begin{matrix}\text{A, B, C} & \ldots\ldots & \text{x, y, z.} \\ \text{A, B}', \text{C}' & \ldots\ldots & \text{x, y}', \text{z}'.\end{matrix}\right\}\text{Consequents.}$$

As Mill points out, it is not certain that A, the circumstance in which the antecedents agree, is the cause of the phenomenon x. For x—so long as the limitation due to Plurality of Causes holds good—is not necessarily produced on each occasion by the same cause; for all we can say, it may be due to B in the first instance, and to B′ in the second. Thus, if we were to give to each of half a dozen persons a glass of water to drink, having poured into each glass a different poison, it could hardly be said to be the *common* element—namely, the water—which caused the death of those six persons.

It is true that the uncertainty of the method due to this limitation is greatly reduced by increasing the number and variety of the instances; but the limitation is intrinsic to the method, being rooted in its merely observational and heuristic character.

If it is objected that the difficulties due to Plurality of Causes

* S. H. Mellone, 'An Introductory Text-Book of Logic,' ch. ix., p. 271.

† 'The Elements of Inductive Logic,' sixth edition, ch. iv., p. 222.

‡ 'A System of Logic,' Book III., ch. x., § 2.

may be met by making the observations less enumerative and more analytic in character, with a view to describing the nature of an effect with the greatest possible precision (*vide* p. 384), the reply must be that the results of such precise analysis can be utilized for the purpose of eliminating plurality of causes only when an adequate knowledge of causal connexions has already been gained through precise applications of Experimental Methods. Suppose that E is the effect in question, and that it is described with all desirable precision. The suggestion is then made that E may be the effect either of Cause C_1, or of Cause C_2, or of some other cause. This suggestion that E may have a plurality of causes can be met only in so far as we are able to show that C_1 cannot be the cause of E, because C_1 has a known constant effect, E_1 ; and, again, that C_2 cannot be the cause of E, because C_2 has a known constant effect, E_2, and so forth. But these assertions presuppose an accurate knowledge of the causal action of C_1, C_2, etc. It may be perfectly true that a Method of Agreement in conjunction with Experimental Methods may, by a sufficiently precise delineation of the effects from which it starts, get beyond any practical annoyance from plurality of causes ; but the Method of Single Agreement, by itself, as a Method of Pure Observation, remains permanently at the mercy of a possible plurality of causes. For the overcoming of this limitation we must look to the experimental stage of the complete Causal Inquiry.

But before we pass on to the consideration of the methods distinctive of this later stage, we must notice an important modification of the Method of Agreement, which, while it strengthens the Method, does not deprive it of its purely observational character. The Method in question has had various names given to it, but the 'Method of Double Agreement' expresses its real nature most simply and aptly ; for, as Mill himself points out, its use involves a double employment of the Method of Agreement. We start, in this method, from two sets of instances : (1) from instances in which the phenomenon E, to be causally explained, is present ; (2) from instances in which the same phenomenon is absent. Applying the Method of Agreement to the first set of instances, we look out for that relevant circumstance, C, which alone is present in all the instances we start from ; and then, applying the Method once again to the instances in which the phenomenon in question is absent, we examine whether, in all these cases of the absence of E, C also is absent, and is the only relevant circumstance invariably absent. If we find that it is so, the suggestion furnished by the first application of the Method, that C is the cause of E, is very considerably strengthened.

In illustration of the Method of Double Agreement, we adapt the following from Professor Carveth Read's 'Logic' :*

* Third edition, ch. xvi., pp. 204, 205.

Elaborating an illustration suggested by Dr. Fowler,* Professor Read supposes the case of a man who, having a taste for cucumber, attributes his chronic indigestion now to the salmon, now to the cheese, now to the pastry—no one of which is an invariable feature of his evening meal—but never to the cucumber, which he takes every evening. However, after having dined without cucumber on several consecutive evenings, whilst taking salmon one evening, pastry the next, and so on through the whole list of suspicious dishes, he cannot but notice that on no one of these evenings did any indigestion occur, and is thus brought to confess that the cucumber, after all, must have been the offending cause.

Let us briefly analyse this example, so as to see precisely how this double application of the method excels the single application in logical strength. We see that, so far as the single application is concerned, the man cannot logically be silenced when, having declared that on the evening when he partook of salmon as well as cucumber the cause of his indigestion was the salmon, he goes on to affirm that, were he to take salmon every evening, he would always get indigestion from so doing. There is, in fact, nothing to disprove the suggestion that salmon, pastry, etc., are each and all causes which, when present in a certain organism, invariably produce indigestion. It is this suggestion that they are constant causes of the phenomenon in question which is more or less convincingly refuted by the second application of the method. The salmon is taken without the cucumber ; so is the pastry ; so also is the cheese ; and in none of these three cases does indigestion occur. It is true that on these three occasions the tendency of the suspected dishes to produce indigestion *may* have been diverted into some other form of harmful influence, or counteracted by some other article of diet, such as pineapple, which is supposed to aid digestion ; the conjecture that salmon, pastry, and cheese constantly *tend*, when eaten, to cause indigestion may still remain unrefuted. But it has been clearly shown that there are occasions on which they do not actually cause it ; and in this way, if the instances of absence are sufficiently varied, the presumption that, in the case of the patient in question, the only constant cause of indigestion is the eating of cucumber may become very strong indeed.

It is, however, important to notice what is *not* proved through this second application of the method. It is not proved that the salmon may not really have at least helped to cause the indigestion on the first evening, nor that the pastry may not have caused, or helped to cause, it on the second. The indigestion still remains an effect that *on any given occasion* may be due to any one or more of a plurality of causes. Hence, in principle, the defect of Plurality of Causes is not overcome through this double application of the Method of Agreement. And the reason is that the effect to be

* 'The Elements of Inductive Logic,' ch. iii., pp. 138, 164.

accounted for is as unprecisely conceived in the second application as in the first. Nothing can obviate this defect, in our supposed inquiry, except an adequate specification of the variety of indigestion suffered on each successive evening—a specification sufficiently detailed to cancel all suggested causes save one. But this analytic process, as we have already seen, presupposes, for its success, the use of strictly experimental methods.

II. The Experimental Stage in Causal Inquiry.

A. *Positive and Negative Instances.*

Let E stand for an effect for the explanation of which a cause, C, is suggested. On inquiry, the suggestion, let us say, is found to be justified. The supposition of C's operation can be shown adequately to account for E. We have, then, shown that, given C, E will be its effect. But we have not yet shown that when E is the given effect, C must have been the cause. There may possibly be other causes which also have E as their natural effect. There may be a 'plurality of causes.'

But the interests of scientific precision cannot be satisfied by the conclusion that a given effect is due either to this cause or to that, or to some third or fourth cause. The ideal of Causal Explanation remains unrealized so long as the causal connexion between C and E is not proved to be reciprocally unideterminate. We must find a cause which satisfies the two conditions :

Whenever C, then E ;
Whenever E, then C.

Expressing the latter condition in the equivalent 'contrapositive' form

Whenever not C, then not E,

we have the requirement that C shall satisfy the two conditions :

(1) Whenever C, then E ;
(2) Whenever not C, then not E.

Experimental instances which illustrate (1) are named *positive* instances. We might also call them instances of the co-presence of cause and effect. Experimental instances which illustrate (2) are named *negative* instances. These we might call instances of the co-absence of cause and effect. For the adequate verification of a causal hypothesis instances of both types are essential.

Negative Instances should not be confused with Exceptions. The former are instances which corroborate a suggested causal

connexion; the latter are instances which disprove it. The form of argument from a negative instance may be stated as follows:

Here C is absent, and so also is E. (Negative instance.)
∴, so far as this evidence goes, C may be causally connected with E.

The form of argument in which an exception is being urged may take the following form:

Here C is present, but E is absent. (Exception.)
∴, so far as this evidence goes, C is not causally connected with E.

Again, under exceptions—or what appear to be exceptions—we should distinguish such results as are merely negative (*vide* pp. 334, 335) from such as are positively incompatible with the hypothesis we are seeking to prove. Negative results, in particular, should be carefully distinguished from negative instances.

In the light of the foregoing, we may state in a more specific form the distinction between Purely Observational Methods on the one hand, and Experimental Methods on the other. On p. 395 we described a Purely Observational Method as a Method that proceeds from effects to causes; an Experimental Method as one that proceeds from causes to effects. We may now state the distinction more precisely:

Where the start is made from effects—*i.e.*, from facts or events viewed in the light of a causal interest—and causal explanations are tentatively formulated, without any attempt to verify them by appeal to positive and negative instances, the method is Purely Observational.

Where, on the other hand, both positive and negative instances are used, with a view to the verification of suggested causal connexions, the method is Experimental. Where the control over positive and negative instances is adequately exhaustive, the connexion between cause and effect can, by this method, be shown to be reciprocal, and the ideal of Causal Explanation is realized.

B. *The Method of Scientific Experiment.*

The differentiæ of a strictly experimental method are (1) that the conditions of observation are under the observer's control; (2) that there is a well-controlled interference with the object to be observed. Thus, mere dissection of an animal or plant is not experiment. In dissection our aim is precisely *not* to interfere with the nerve, or the member of a flower-whorl, or any other structure that we desire to examine—except, indeed, so far as interference

is necessary as a mere aid to observation. Absolute non-interference would preclude the possibility of all but the most superficial observation. In order to make our object accessible to simple observation and study, we must often in one sense interfere with it. We must inflate collapsible structures, or inject with some highly coloured stain the finer ramifications of a vascular system; or we must cut sections with the microtome and make microscopic preparations, rendering differences of tissue conspicuous by means of careful staining. But all such interferences as these are aids to observation only. When we go further than this—when, for instance, instead of merely laying a nerve bare, and perhaps treating it with acetic acid to make it more conspicuous and easy to trace, we apply an electric stimulus to a living nerve and watch the effect—then we have at once a genuine experiment. In vivisection we have a constant application of the experimental method. For example, a carefully localized cortical point in the brain of an anæsthetized animal is stimulated, and the ensuing movements are accurately observed. In both dissection and vivisection there is control over the conditions; but in the latter there is also a modifying or constructive interference with the phenomenon to be observed, while in the former there is only so much interference as is necessary to aid observation of the object as it already exists.

Further, in a scientific experiment, properly so called, there is made a deliberate attempt to obtain precise and unambiguous results. This implies that, before experimentation, the conditions have been carefully analysed. Thus, the Method of Scientific Experiment, in the strict sense of the term, must be a method which presupposes control over conditions, implies a directly modifying or constructive interference with the phenomenon to be observed, and aims at precision both in procedure and in result.

The typical Method of Scientific Experiment, so understood, is discussed by Mill under the title of 'the Method of Difference.' Its essential import is very simply and concisely presented by Dr. H. S. Mellone in the following words:

'*When the addition of an agent is followed by the appearance, or its subtraction by the disappearance, of a certain event, other circumstances remaining the same, that agent is the cause of the event.*' 'When the suspected agent is present,' continues Dr. Mellone, 'we have the *positive instance;* when it is absent, the *negative instance.* What cannot be eliminated without doing away with the event is causally connected with it.'*

It will be gathered from this passage, taken together with the rest of Dr. Mellone's exposition, that his definitions of 'positive and negative instances' are different from ours. The 'positive instance,' as we have defined it, implies not only the presence of the suggested cause, but also the co-presence of the effect. It is an instance which

* 'An Introductory Text-Book of Logic,' ch. ix., p. 274.

is always *corroborative* of a suggested causal connexion. Similarly the 'negative instance,' as we understand it, implies not only the absence of the suggested cause, but also the co-absence of the effect, and, like the positive instance, is always corroborative. According to Dr. Mellone's definitions, the positive and negative instances do not, as such, necessarily *confirm* some suggested causal connexion; they simply *test* it. They are not necessarily instances of a causal law, but they are instances by means of which the claim of a suggested connexion to the title of causal law is challenged and sifted. In order to distinguish these 'positive and negative instances' (in Dr. Mellone's sense of the words) from the positive and negative instances of our own definition (which are all instances of the law, and serve to establish it), we propose to call the former 'test-instances.' Thus, when the suspected agent is present, we have the positive test-instance; when it is absent, the negative test-instance, whether the expected effect is present or not.

Following Dr. Mellone, we may illustrate the application of the Method of Difference by the familiar experiment of 'the coin and feather.' The question to be decided is, Why is it that, when a coin and a feather are simultaneously let fall from the same height, the feather does not reach the ground so soon as the coin?

We will suppose that Analytic Observation suggests that the retardation of the motion of the feather (the phenomenon or effect to be explained) is caused by the resistance of the air (the suspected agent).

We proceed to test this hypothesis by a systematic application of the Method of Difference.

(*a*) *Positive Test-instance—i.e., the instance in which, under experimental conditions, the suspected agent is present:*

The two objects 'are dropped simultaneously in the receiver of an air-pump, the air being left in.'

Result.—'The feather flutters to the ground after the coin.'

(*b*) *Negative Test-instance—i.e., the instance in which, under experimental conditions, the suspected agent is absent.*

The air is, as far as practicable, pumped out of the receiver, and the coin and feather are simultaneously dropped in the receiver as before.

Result.—They reach the bottom approximately at the same instant.

For the ideal success of the Method of Difference it is essential that the positive and negative test-instances should differ only in *one* relevant and important circumstance. As in the coin and feather experiment, so in other applications of the Method, the difficulty of adequately ensuring this requisite is, as a rule, overcome by using in both cases what is practically one and the same instance, modified, in the second case, in some one single point.

This is explained by Mill in the 'Logic' (Book III., ch. viii., § 3), and is emphasized by Dr. Mellone's above-quoted restatement of Mill's Second Canon.

Precautions to be Observed in Applying the Method of Difference.

The fundamental requirement for the successful application of the Method of Difference is that only one condition shall be varied at a time. Upon the observance of this rule depends the scientific precision of the method. There are two main reasons for observing it :

(i.) If we vary two conditions at a time, and find some effect, we cannot tell whether this effect is due to one, or to the other, or jointly to both.

(ii.) If no effect ensues, we cannot safely conclude that either of the two changes was ineffective ; for the effect of one may have neutralized the effect of the other.

Mill accordingly points out that, in using this method, the change must be introduced as rapidly as possible, and the whole process must not last a long time. For, whenever the introduced change takes a considerable time to make itself felt, and opportunity is thereby given for its effect to be fused with other unintended and unobserved changes, no satisfactory conclusion can be drawn from the experiment. We must beware of Intermixture of Effects.

The impossibility of complying with this requirement in the case of investigations belonging to the province of such a science as Geology is the main reason why in a science of this kind, which concerns itself with slow and age-long processes, the Method of Agreement is inapplicable. In the investigation of geological causation it is inconceivable that we should ever have control over all the relevant conditions ; and even if we had, and could proceed to introduce among those conditions that change which we have previously called C, there would still remain the difficulty that the causal change thereby initiated might take many centuries to produce any effect of geological significance. During this time there would creep in countless other agencies, and the 'effect,' when at length perceived by the geologists of a later generation, would probably be in great measure due not to our own cause, C, but to those other agencies at work without our sanction.

Limitations of the Method of Difference.

1. Mill* points out that the Method of Difference cannot cope with the difficulty of *permanent causes*—causes, that is, whose agency can never be excluded ;—for the very essence of the method lies in its producing an instance in which the cause in question is

* 'A System of Logic,' Book III., ch. viii., § 6.

absent. But he goes on to show that, though the permanent cause can never be other than a 'coexisting fact,' it may be prevented from operating as an 'influencing agent.' In illustration of this, he supposes that, in experimenting with a pendulum, we find that its oscillation is affected by the vicinity of a mountain. Now, though we cannot possibly remove the mountain, so as to apply the method of Difference directly, yet we can remove the pendulum to such a distance from the mountain that the disturbing attractive influence becomes practically inappreciable; and this amounts, indirectly, to a genuine application of the Method of Difference. But, again, suppose we wish to estimate experimentally the effect of the attractive power of the earth on the motion of the pendulum. Here, as Mill says, 'we cannot take away the earth from the pendulum, nor the pendulum from the earth.' The Method of Difference, strictly so called, is here unavailable. All we can do is, as Mill expresses it, to *modify* what we cannot *exclude*, the modification of a condition meaning 'change in it not amounting to its total removal.' It is a methodical modification of this kind which constitutes *the Method of Concomitant Variations;* and the most striking applications of this method, as Mill points out, take place in the cases in which the Method of Difference is quite inapplicable —*e.g.*, in establishing laws of heat, gravitation, friction. We here make a series of partial experiments, in which we proceed by a gradual quantitative modification of the constituent which cannot be wholly withdrawn. The following is an abbreviation of Mill's Canon of the Method of Concomitant Variations: *Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner is causally connected with that phenomenon.* Thus, if in a tropical country, as the rainfall of each year is more or less, the rice-crop is observed to be greater or less, or to fail, correspondingly, the two phenomena are at once connected together, and we proceed to determine according to what quantitative law the one varies with the other. In the same way, as Mr. Carveth Read points out,* the use of the thermometer illustrates this method. The rise and fall of the mercury is connected by the observer with the increase and decrease of the amount of heat, per unit of volume, in the atmospheric air in contact with the glass, and at once suggests a quantitative comparison of these two varying facts.

As a simple and typical example of this Method of Concomitant Variations, let us suppose that an electric bell is placed ringing under the receiver of an air-pump. The air is now gradually exhausted, and it is noticed that, *pari passu* with the exhaustion of the air, the sound of the bell grows fainter and fainter, until a point is reached at which it is no longer heard at all. The air is now allowed to pass back gradually into the receiver, and, as it does

* 'Logic Deductive and Inductive,' third edition, ch. xvi., p. 216.

so, the sound of the bell is heard, at first faintly, then more and more loudly, until the clearness of the original note is reproduced.*

2. The Method of Difference cannot establish between cause and effect any precise *quantitative* relation.

Here again, in order to supplement the Method of Difference, we must have recourse to the Method of Concomitant Variations. When, by the Method of Difference, it has first been ascertained that a certain cause produces a certain effect, the Method of Concomitant Variations may usefully be called in to determine according to what quantitative law the effect follows the cause.

But the application of the Method of Concomitant Variations is not without its dangers. When, within a limited range of variation, a continuous change of a phenomenon, C, in one direction is found to be always accompanied by a continuous change of another phenomenon, E, also in one direction, there is a tendency to take for granted that this correspondence will always hold even beyond the limits within which our investigation has been conducted, and we are inclined forthwith to frame a universal law to that effect. But there is such a thing as discontinuous variation. Change produced by a varying cause is often continuous only between two critical points, of which each marks a sudden change in the law of variation. Thus, as the temperature of water at the sea-level continuously increases from 0° C. to 100° C., its density increases between the temperatures of 0° C. and 4° C., but after 4° C. is reached it decreases. In order to meet the requirements of such cases as this the experiments by which concomitance of variations is established should extend over a wide range.

Another danger which peculiarly, though not exclusively, affects the application of this method is that of supposing, whenever we find two series of phenomena varying concomitantly, that one of the series is causally responsible for the other. But this by no means follows. It is quite as likely that the two series are co-effects of one and the same cause. By the Method of Concomitant Variations alone it is, in fact, never possible to ascertain which of the two suppositions is true.

3. A single application of the Method of Difference is not, as a rule, sufficient to verify the reciprocal relation between a suggested cause and its supposed effect. To reach this ideal of Causal Explanation we must have exhausted the series of the relevant representative negative test-instances,† and in each case established afresh the truth of the proposition

If not C, then not E.

* For further illustration of this method, see Dr. W. Stanley Jevons, 'Elementary Lessons in Logic,' Lesson xxix., pp. 249-251.

† We could not attempt to deal with more than a series of typical or representative instances, and even that series would not be exhaustively representative, but would be constituted by a relevant selection.

To this development of the Method of Single Difference—a development first suggested by Hermann Lotze—Professor Laurie gave the name of 'The Double Method of Difference,' and he formulated it, after Mill's manner, in an Inductive Canon. The title does not seem to be very appropriate, as a multiple application of a method is hardly the same as a double method; but the enunciation of this developed method in an additional canon is a useful step to have taken. Professor Laurie's enunciation runs as follows:

'When, by the Method of Difference, we have established a causal law connecting certain conditions with the production of a phenomenon, and when, further, we have failed to discover any case in which the phenomenon occurs without these conditions, there is a probability, increasing with the extent and variety of our negative instances, that the phenomenon can be produced in no other way.'*

We have now discussed at sufficient length the limitations of the typical method of Scientific Experiment, the Method of Difference. We have shown, with the single exception of the limitation due to the presence of an intermixture of effects, that remedies may be found in the use of supplementary methods, such as the Method of Concomitant Variations, or else in a repeated and varied application of the Method of Difference itself—an application so contrived as to exhaust the relevant and representative negative test-instances. There still remains the question whether the difficulties due to Intermixture of Effects may not be met in similar ways.

They may, in fact, be met by strengthening the application of the method by means of certain purposive analyses and deductions.

If the effect to be accounted for is 'intermixed'—*e.g.*, a heat effect—Analysis is required in order to break up the effect into as many constituents as there are causes which contributed to produce it, and to separate out the various lines of agency—*e.g.*, combustion, compression, electric action—which converge to produce the effect. Having thus passed, through Analysis, from effects to causal agencies, we may discover the laws according to which these agencies operate, through the application of Experimental Methods. But when the Experimental Methods have done their utmost, there will still remain the task of gathering up the various threads which these methods have singly and separately laid bare, and this can be done only through Deduction or Deductive Synthesis. The whole process represented by the methods hitherto discussed, comprising the purely observational and the experimental stages, is thus made preliminary to processes of deductive synthesis through which the final Verification is alone made possible. And yet not wholly preliminary, for a further appeal to the Experimental

* 'Methods of Inductive Inquiry,' *Mind*, New Series, vol. ii. (1893), p. 333.

Methods is needed in order to carry out satisfactorily these final processes of Verification. Referring to this very difficulty of Intermixture of Effects, in connexion with the application of his 'four Experimental Methods,' Mill writes: 'The instrument of Deduction alone is adequate to unravel the complexities proceeding from this source; and the four methods have little more in their power than to supply premisses for, and a verification of, our deductions' (Book III., ch. x., § 3).

In the light of this conviction, Mill proceeds to elaborate a strengthened Method of Causal Explanation, a Method especially devised to deal with the difficulties arising from Intermixture of Effects; and he gives it the name of the Deductive Method (see Book III., ch. xi.; also Book III., ch. xiv., § 4).

The Deductive Method, as Mill conceives it, is a Method in three stages:

(i.) *Induction.**—Starting with the given complex phenomenon, we aim first at discovering, through Analysis and Experiment, the simple antecedents to whose combined operation it is due, and the laws according to which these antecedents severally act.

(ii.) *Ratiocination* (or Deduction).—We calculate what would be the joint effect of the operation of these antecedents, each acting according to its own law.

(iii.) *Verification.*—'To warrant reliance on the general conclusions arrived at by deduction, these conclusions must be found, on careful comparison, to accord with the results of direct observation wherever it can be had' (Book III., ch. xi., § 3).

In illustration of the application of this Deductive Method, we can hardly do better than quote the instance given by Mill himself—'the deduction,' namely, 'which proves the identity of gravity with the central force of the solar system.'

'First, it is proved from the moon's motions that the earth attracts her with a force varying as the inverse square of the distance. This (though partly dependent on prior deductions) corresponds to the first or purely inductive step, the ascertainment of the law of the cause. Secondly, from this law, and from the knowledge previously obtained of the moon's mean distance from the earth, and of the actual amount of her deflection from the tangent, it is ascertained with what rapidity the earth's attraction would cause the moon to fall, if she were no farther off and no more acted upon by extraneous forces than terrestrial bodies are. That is the second step, the ratiocination. Finally, this calculated velocity

* 'In many particular investigations,' says Mill (Book III., ch. xi., § 1), 'the place of the induction may be supplied by a prior deduction; but the premises of this prior deduction must have been derived from induction.'

being compared with the observed velocity with which all heavy bodies fall, by mere gravity, towards the surface of the earth (sixteen feet in the first second, forty-eight in the second, and so forth, in the ratio of the odd numbers, 1, 3, 5, etc.), the two quantities are found to agree' (Book III., ch. xiv., § 4).

To the Deductive Method, with its three stages of Induction, Ratiocination, and Verification, Mill attaches the greatest importance. 'The human mind is indebted to it,' he writes, 'for its most conspicuous triumphs in the investigation of nature. To it we owe all the theories by which vast and complicated phenomena are embraced under a few simple laws, which, considered as the laws of those great phenomena, could never have been detected by their direct study' (Book III., ch. xi., § 3). It is, moreover, of supreme value in Sociological inquiry. It is, indeed, only when Mill comes to treat of the Logic of the Moral Sciences that he develops the full resources of the Method. In the first place, a distinction is drawn between an abstract and a concrete application of the Method. Where the simple procedure of Geometry is taken as the model, the application—so far, at least, as the requirements of Sociology are concerned—is abstract; where the deductions, and the inductions which they presuppose, are of the more complex kind proper to Astronomy, the application is concrete. In the former case Mill speaks of the Geometrical or Abstract Method, in the latter of the Physical or Concrete Deductive Method. 'The Social Science, therefore (which, by a convenient barbarism, has been termed Sociology), is a deductive science; not, indeed, after the model of geometry, but after that of the more complex physical sciences. It infers the law of each effect from the laws of causation on which that effect depends; not, however, from the law merely of one cause, as in the geometrical method, but by considering all the causes which conjunctly influence the effect, and compounding their laws with one another. Its method, in short, is the Concrete Deductive Method—that of which astronomy furnishes the most perfect, natural philosophy a somewhat less perfect example, and the employment of which, with the adaptations and precautions required by the subject, is beginning to regenerate physiology' (Book VI., ch. ix., § 1).

But not only is there a distinction drawn between an abstract and a concrete application of the method; the concrete application itself may be either direct or inverse. Thus, we may start with laws of human nature, deduce conclusions from these through a process of ratiocination, and then verify these by comparing them with observed facts.

This is the Direct Method. But, as Mill goes on to add, 'there is a kind of sociological inquiries to which, from their prodigious complication, the method of direct deduction is altogether inapplicable' (*ibid.*). Resort must then be had to the Inverse Method.

Here we start by direct consideration of the facts of social life and of history, and obtain thence empirical laws or generalizations which we then proceed to verify, not by comparison with further facts, but by deduction from the laws of human nature. It is 'an imperative rule' of the Historical Method 'never to introduce any generalization from history into the social science unless sufficient grounds can be pointed out for it in human nature' (Book VI., ch. x., § 4). Verification here takes the new form of successful deduction, and we have a striking inversion of the ordinary procedure. 'Whereas in the "method of direct deduction" we compare the results of deduction with observed facts, we here begin by provisionally formulating empirical laws gathered from facts of observation, and then verify these laws by deducing them from "the principles of human nature."'*

In the Deductive Method, with its inverse development, we seem to reach what is, perhaps, the concretest form which Scientific Method can take. The march of Mill's exposition from the opening of the third Book onwards may be described as a progressive development from the abstract to the concrete. Beginning with abstract views of the universe, and with the partial methods that correspond to them, he gradually takes in more and more elements of Reality, and shows at each step how the partial methods are in themselves inadequate to cope with these newly recognized aspects of the world. Every step towards a concreter view of the world is shown to necessitate a development of Method, until he has welded all his partial Methods together, and comes out facing a concrete (material) universe with weapons fitted at length to deal with its complexity.†

But there is a sense in which even the Deductive Method is not his final standpoint, for in his paragraphs on 'Hypotheses' Mill gathers up a further and a most important thread into the tissue of logical method. He enriches the conception of Method by an explicit reference to the part played in it by Hypothesis. In Book III., ch. xiv., § 5, he writes: 'The function . . . of hypotheses is one which must be reckoned absolutely indispensable in science. . . . Without such assumptions, science could never have attained its present state. . . . Nearly everything which is now theory was once hypothesis.'

This 'Hypothetical Method,' to use Mill's own expression, is regarded by Mill as a modification of the Deductive Method. It differs from the latter in one important particular. The law, or laws, which are developed through ratiocination are, in the case of the Hypothetical Method, not proved by experimental methods, but assumed. The Hypothetical Method, as Mill conceives it, is

* I believe this to be a quotation, but cannot trace it to its source.

† For a comparison between Mill's 'expository' development of the Theory of Scientific Method and our own exposition of Induction, *vide* pp. 419, 420.

therefore a simplification of the Deductive Method. It consists of two stages only—that of Ratiocination, or the deductive development of Hypothesis, and that of Verification.

This abridgment, however, does not imply any loss of inductive soundness. For, as Mill expressly points out, this abridged method is legitimate 'on one supposition' only—'namely, if the nature of the case be such that the final step, the verification, shall amount to and fulfil the conditions of a complete induction' (Book III., chap. xiv., § 4). The testing through experimental methods, though not here present in the first stage of the inquiry, is in the third stage reinforced and applied with additional vigour. The Method of Difference exercises its verificatory function in its most conclusive form. As Mill expresses it in an earlier chapter of his work, 'The great generalizations which begin as Hypotheses must end by being proved, and are in reality (as will be shown hereafter) proved, by the Four Methods' (Book III., ch. ix., § 6).

The Hypothetical Method, when thus concretely conceived, is distinguishable from the Inductive Method, as we have interpreted it, only in its exclusion of the preliminary stages of Observation and Formulation of Hypothesis. As Mill understands Induction, these are not parts of Induction itself, but operations subsidiary to it.

It is in connexion with the application of the Deductive Method that Mill's *Method of Residues* finds its natural place. But, as formulated by Mill himself, the method at first appears under the head of Induction, and as preliminary to Deduction and Verification. Mill's enunciation runs as follows :

'Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.'* The distinction between two quite different uses of this method we owe to Professor Laurie.† In one application of the method we are acquainted with all the antecedents concerned. Professor Laurie (followed by Dr. Mellone) believes this to be the application which Mill has in mind. But this is surely a misinterpretation, and is in direct opposition to the following statement of the method made by Mill himself :

'Subducting from any given phenomenon all the portions which, by virtue of preceding inductions, can be assigned to *known causes*, the remainder will be the effect of *the antecedents which had been overlooked*, or of which the effect was as yet an unknown quantity.'

The expressions we have 'underlined' seem to show that Mill's Method of Residues is not really different from that which Dr. Mellone characterizes as regulated by 'a distinct rule,' which he thus formulates :

'When any part of a complex phenomenon is still unexplained

* 'A System of Logic,' Book III., ch. viii., § 5.

† 'Methods of Inductive Inquiry,' *Mind*, New Series, vol. ii. (1893), pp. 335, 336.

by the causes which have been assigned, a further cause for this remainder must be sought.'*

Here the antecedents are not all 'known causes.' Some of them (one or more) are causes that have hitherto been overlooked. This is the form of the Method of Residues contemplated by Mill, and Dr. Mellone rightly regards it as the more important of the two.

Suppose the total cause in any given instance to contain the known conditions, A, B, C; and suppose the total effect to be represented by a, β, γ, δ. Singling out the known causes A, B, C, we can deduce from their known modes of operation the effect that they will produce when acting together under the given conditions. Suppose this effect to be represented by a, β, γ; we are then left with a residual phenomenon δ, to which we must assign the unknown cause x.

Example.—The Perturbation of the Planet Uranus.

Before the discovery of Neptune in 1846 this astronomical problem might have been roughly stated thus :

Cause ABC . . . gives $a\beta\gamma\delta$ as effect.
I.e., Tangential momentum of Uranus at a given moment (A)
+ Attraction of the Sun (B)
+ Perturbing influence of known planets (C)
+ Perturbing influence of unknown cause

gives, as the observed effect, the known orbit of Uranus ($a\beta\gamma\delta$). Taking the known causes A, B, C, astronomers were able, by mathematical deduction, to ascertain the effect in the motion of Uranus due to these causes acting in combination. This effect was represented by $a\beta\gamma$, and the investigators were left with the problem of a residual effect δ, which represented the discrepancy between the observed effect $a\beta\gamma\delta$ and the effect $a\beta\gamma$ already deduced from the laws of all the known causes. John Couch Adams in England and Urbain J. J. Leverrier in France (working quite independently of one another) then argued that this effect δ must be due to the perturbing influence of some unknown planet, which we may call x.† In this way an application of the Method of Residues pointed to the fact of the planet Neptune's existence.

From the foregoing it will be seen that the essential function of the Method of Residues is to ensure the *adequacy* of a causal explanation. The ascribed cause must be *qualitatively* adequate to account for the observed effect—*i.e.*, all the requisite conditions must be considered, and the cause established as a *total* cause.

When the Method of Residues is further refined through insistence on *quantitative* adequacy, we reach a conception of Causal Explana-

* 'An Introductory Text-Book of Logic,' first edition, ch. ix., p. 287.

† For a brief authoritative account of the history of the discovery of Neptune, *cf.* Simon Newcomb's 'Astronomy for Everybody,' pp. 232-235.

tion which can be satisfactorily realized only through the application of quantitative methods. It was through precise quantitative investigation of residual errors that James Bradley discovered the nutation of the earth's axis and the aberration of light, and that Leverrier and Adams ascertained not only the existence of Neptune, but also the mass and the orbit of the yet unseen planet. It was also through this same method that Argon, Helium, and other constituents of our atmosphere were recently brought to light.

As characteristic of these more exact modes of scientific procedure, we may note the Elimination of Error in Observation. The following extract from a popular treatise on Astronomy* may serve to illustrate the method : 'Observations made with the transit circle must . . . be corrected for errors in the instrument itself. The astronomer will see to it that his instrument is made and is set up as perfectly as possible. The pivots on which it turns must be exactly on the same level ; they must point exactly east and west, and the axis of the telescope must be exactly at right angles to the lines joining the pivots in all positions of the instrument. These conditions are never absolutely fulfilled. Day by day, therefore, the astronomer has to ascertain just how much his instrument is in error in each of these three matters. Were his instrument absolutely without error to-day, he could not assume that it would remain so. The astronomer finds that his own presence near the instrument is sufficient to disturb it. . . . The great interest attaching to transit circle work is this striving after ever greater and greater precision, with the result of bringing out fresh little discordances, which, at first sight, appear purely accidental, but which, under further scrutiny, show themselves to be subject to some law. Then comes the hunt for this new unknown law. Its discovery follows. It explains much, but when it is allowed for, though the observations now come much closer together, little deviations still remain to form the subject of a fresh inquiry.'

Where errors cannot be individually eliminated, we have to resort to the best argument whereby we can temper our ignorance—the argument from probability. Our reasoning is here based on the reduction of errors to average and probable errors, for which due allowance is made.

III. THE ESSENCE OF INDUCTIVE METHOD AS AN INSTRUMENT OF CAUSAL INQUIRY.

Mill maintains (Book III., ch. viii., § 1) that his methods involve two simple principles :

(1) That of the Method of Agreement ;
(2) That of the Method of Difference.

* I regret that I am unable to state the reference more precisely.

1. The Method of Agreement consists in comparing together different instances in which a phenomenon occurs.

2. The Method of Difference consists in comparing instances in which a phenomenon does occur with instances in other respects similar in which it does not.

Both methods, according to Mill, agree in this—that they are methods of Elimination. 'The Method of Agreement,' he says, 'stands on the ground that whatever can be eliminated is not connected with the phenomenon by any law. The Method of Difference has for its foundation that whatever cannot be eliminated is connected with the phenomenon by a law' (Book III., ch. viii., § 3).

It would appear from the foregoing that Mill regards Inductive Method as fundamentally a weapon of Elimination. Whether we make use of the Method of Agreement or of the Method of Difference, we proceed, according to Mill, by Elimination. But by Elimination Mill means, not a logical process, but a physical exclusion. The term, he says, 'is well suited to express the operation . . . which has been understood since the time of Bacon to be the foundation of experimental inquiry—namely, the successive exclusion of the various circumstances which are found to accompany a phenomenon in a given instance, in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon' (*ibid.*).

In thus reducing his Inductive Methods to Methods of so-called Elimination, Mill here explicitly attaches his view of Induction to that adopted by Francis Bacon.

To Bacon the data of observation presented themselves as complex groups of properties, each property having as its essence some corresponding simple cause. These single causes he called *forms;* and the aim of Induction, in his opinion, is the discovery of these forms. The Method of Induction which Bacon adopted he called the Method of Exclusion. In order to discover the form or cause of a certain phenomenon—for instance, that of heat—we must, according to this method, take three preliminary steps:

1. We must collect all the known cases in which the phenomenon of heat is present. These constitute a *tabula essentiæ et præsentiæ.*

2. We must make a collection of instances, cognate or similar to the former, in which heat is absent. These constitute a *tabula absentiæ in proximo* parallel to the first table.

3. We must compile a *tabula graduum,* in which the varying intensity with which the phenomenon appears may be compared with the varying intensities of other phenomena that accompany it.

We then seek to discover the form of heat by applying the principle that this form can be nothing but that which is always present where heat is found, which is not present where heat is lacking, and which perpetually decreases as heat decreases, and increases as heat increases.

This investigation, we are told, must be conducted by means of progressive exclusion. Thus we argue that the form of heat is not tenuity ; for the form of heat is always present where heat is, and tenuity is not always where heat is. Gold and other metals, though they may be heated, are still of great density throughout. For this reason tenuity must be rejected. Again, the form of heat is not present where heat is lacking ; but tenuity *is* where heat is not, for the air is often cold, and yet remains uncondensed. Thus, for this reason also, tenuity must be rejected. And so it is with many other phenomena that in some cases accompany heat.*

It will be observed that, whether Bacon makes use of the positive or of the negative test-instance, he makes use of it in the interest of logical exclusion. His aim is to include all possible conjectures as to the nature of the form under investigation in one disjunctive judgment, and then to reach the true nature by excluding all conjectures which fail to satisfy the two Canons of Causation—viz. :

1. The cause or form must always be present where the effect is found.

2. The cause or form must never be present where the effect is lacking.

Now, when we compare Bacon's procedure with that of Mill, it becomes evident that the latter is not throughout a Method of Exclusion in the same rigorous and logical sense as is the former. Mill's Elimination is not logical but physical exclusion. In the logical and proper sense of the word, the Method of Agreement only, and not that of Difference, can be called a Method of Elimination. So far as the Method of Agreement is concerned, Mill remains faithful to Bacon's Method of Exclusion ; but the Method of Difference is emphatically not a method of *logical* Exclusion or Elimination. Its essence is not the elimination of the non-cause, but the establishment of the cause. 'The Method of Difference has for its foundation, that whatever cannot be eliminated is connected with the phenomenon by a law.'†

Here the word 'eliminated' is not well chosen, but its meaning is not doubtful. Mill here uses the word 'elimination' consistently (though, as we think, unadvisedly) in the sense of physical, not logical, exclusion. It is quite clear that in the enunciation of the Canon of the Method of Difference the idea of logical Elimination is nowhere implied. 'If an instance,' says Mill, 'in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former ; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon' (Book III., ch. viii.,

* *Vide* 'Novum Organum,' lib. ii., c. xi.-xviii.

† 'A System of Logic,' Book III., ch. viii., § 3.

§ 2). Now, we saw, in the case of the Method of Single Agreement, that the singleness of the *agreement* could be established only through a process of logical elimination; but the singleness of the *difference* between two instances is established, not by any such elimination, but by making a single change in a certain given instance. We do not pass from the positive to the negative test-instance, in experiment, by elimination, except in this sense—that we 'eliminate' a circumstance in the positive instance in order to secure the negative instance. But this, of course, is not a logical process of elimination, but a physical process of subtraction. It would therefore have been better if Mill had avoided the use of the term 'eliminated' in his statement of the foundation of the Method of Difference. He would have expressed his own meaning more unambiguously if he had said: 'What cannot be *absent* without doing away with the phenomenon is causally connected with it.'

The view that Induction is essentially a Method of Elimination or Exclusion has recently received the powerful support of Mr. H. W. B. Joseph. Mr. Joseph is convinced, not only that the Method of Exclusion was Bacon's Method, but that it was also Mill's; and he has uncompromisingly adopted it as his own. 'Inductive conclusions,' he roundly asserts, 'are established disjunctively by the disproof of alternatives.'* And again: 'The inductive proof of a conclusion rests on excluding alternative explanations.'†

It will be useful to consider Mr. Joseph's position more closely. He has formulated four 'grounds' of elimination, each of which 'points to some particular requirement of the causal relation, failure to satisfy which disproves that relation as between two given phenomena.'‡

These grounds of Elimination are enumerated as follows: §

'1. Nothing is the cause of a phenomenon in the absence of which it nevertheless occurs.

'2. Nothing is the cause of a phenomenon in the presence of which it nevertheless fails to occur.

'3. Nothing is the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it.

'4. Nothing is the cause of one phenomenon which is known to be the cause of a different phenomenon.'

The application of these four canons of Elimination enables us to pass to a knowledge of the cause by a successive exclusion of all non-causes, and inductive reasoning takes the form of a disjunctive argument of the *Modus Tollendo Ponens*.

Mr. Joseph takes care to point out that 'the character of the reasoning is unaffected either by the completeness of the elimina-

* 'An Introduction to Logic,' ch. xx., p. 408. † *Ibid.*, p. 415.
‡ *Ibid.*, p. 403. § *Ibid.*, pp. 403, 404.

tion . . . or by the ground of elimination used.'* Whatever fails to satisfy any single one of these grounds cannot be a cause. Again, if I am unable to state my disjunctive basis exhaustively, or to eliminate all alternatives save one, this failure on my part in no way alters the character of my argument. The method of reasoning remains precisely the same whether I am able to conclude that the cause of E is C, or only that the cause of E must be C_1 or C_2 or C_3, or some cause hitherto undetected. 'The getting of a positive conclusion, but not the inductive character of the argument, depends on the completeness of the elimination.'†

In a footnote‡ Mr. Joseph connects these grounds of elimination with Mill's Inductive Canons. 'On these grounds of elimination,' he writes, 'Mill's "Inductive Methods" severally repose. The first is the foundation of his "Method of Agreement," the second of his "Method of Difference," the first and second jointly of his "Joint Method of Agreement and Difference," the third of his "Method of Concomitant Variations," and the fourth of his "Method of Residues."' Mill's statement that the Methods of Agreement and Difference are both Methods of (physical) Elimination is thus developed into the surprising consequence that Inductive Method generally is a Method of Logical Elimination. 'It would be well to recognize that Mill has not formulated four (or five) but one "Method of Experimental Inquiry"—as indeed Bacon might have shown him—of which the essence is that you establish a particular hypothesis about the cause of a phenomenon by showing that, consistently with the nature of the relation of cause and effect, the facts do not permit you to regard it as the effect of anything else (and *mutatis mutandis* if you are inquiring into the effect of anything).'§

Now, it is true that Mill has laid himself open to this interpretation of his position by his statement that not only the Method of Agreement, but the Method of Difference also, is a Method of 'Elimination.' But, as we have already said, that statement does not refer to logical Elimination at all, nor would such an interpretation of it tally with Mill's actual handling of the Method of Difference. Nor is there any hint of the *Methodus Exclusiva* in his formulation and treatment of the Method of Concomitant Variations. Where Mr. Joseph says, 'Nothing is the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it,' Mill says: 'Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.'‖ It is impossible not to be struck at once by the opposition between the negative character of the third ground of Elimination

* 'An Introduction to Logic,' ch. xx., p. 396. † *Ibid.*, p. 417.
‡ *Ibid.*, p. 404. § *Ibid.*, p. 399. ‖ Book III., ch. viii., § 6.

and the positive character of Mill's fifth Canon. Nor is there anything to suggest that the latter reposes on the former, in the sense that the application of the former paves the way for the statement of the latter. If we consider, for instance, Mill's illustration of the experimental corroboration of the first law of motion through the Method of Concomitant Variations (Book III., ch. viii., § 7), we find that the application is perfectly positive and direct. Nor can I find any evidence for supposing that the fourth ground of Elimination is the foundation upon which the Method of Residues rests.

As the gist of our criticism of Mr. Joseph's view consists in the contention that the Method of Difference is not really a Method of logical Elimination, it may be useful to make our meaning clearer by reference to a particular example. Mr. Joseph illustrates the way in which inductive conclusions are established through the disproof of alternatives by reference to the problem as to the power possessed by chameleons and frogs of changing colour according to the colour of their surroundings. 'Granting,' he says, 'in the absence of any other alternative, that it depends on the colour as such, we may ask in what way the differently coloured rays affect the animal. Lord Lister showed that they affected it through the eyes; for a specimen of *Rana temporaria* whose eyes had been removed was no longer affected by any change in the colour of the surroundings in which it was placed; thus the alternative, otherwise not unreasonable, is excluded, that the reaction is somehow determined through the skin, the principle applied being that no circumstance in the presence of which the phenomenon fails to occur is its cause.'* But, we ask, did not the experiment, on Mr. Joseph's own explicit showing, prove the eye-theory rather than disprove the skin-theory? Did not Lord Lister's experiment show that the rays affected the frog through the eyes? And was not the principle applied just this —that the circumstance in the absence of which the phenomenon fails to occur is causally related to that phenomenon?

The pivot upon which Mr. Joseph's argument in favour of the Method of Elimination appears to turn is the conviction that to *disprove* through a ground of elimination is a far easier and surer process than to *prove* through a ground of verification. Referring to Bacon's *Methodus Exclusiva*, he writes: 'We must proceed, then, by *exclusions*. Where a hundred instances will not *prove* an universal connexion, one will *disprove* it.'†

This is, no doubt, true of causal methods in which stress is laid on the *number* of positive instances; but in the application of the Method of Difference the verification is as easy and as convincing as the disproof. The positive verification-method of Mill is in no sense weaker or more unwieldy than the negative elimination-method of Mr. Joseph. Even when the verification of C as causally connected with E does not give the *total* cause on which the phenomenon E

* 'An Introduction to Logic,' ch. xx., p. 409. † *Ibid.*, ch. xviii., p. 365.

depends, still, at the very least, we have ascertained (as Mill puts it in discussing the force of the Method of Agreement) 'one invariable antecedent . . . however many other invariable antecedents . . . may still remain unascertained.'* This result is surely as definite and as valuable for the purposes of causal elaboration as the disproof of the causal connexion in question for those of causal elimination.

But there is, I believe, a deeper reason for the discrepancy between Mr. Joseph's standpoint and our own than any which the preceding argument has yet suggested. It appears to me that there is involved in the issue a conflict between logical and methodological ideals. The strictly logical tendency is to *presuppose* the ideal; the methodological tendency is to aim at progressively realizing it. Where the ideal is presupposed, we tend to frame our rules of procedure on the assumption that it is realized. Thus, in Mill's case, the start is made, not methodologically, from the actual conditions of sense-experience, but from a simplified and idealized conception of such experience. Mill, as his own words and the testimony of his own artificial symbolism assure us, assumes as the basis for the application of his canons a causal situation in which there is neither any plurality of causes nor any intermixture of effects.† On this simplified basis a conception of Induction is developed which can proceed without the aid of Deduction, and, as we might perhaps add, without the aid of Hypothesis, or even of Verification. Unfortunately these presuppositions are not explicitly embodied in the enunciations of the canons, so that the latter wear an air of unreality which haunts the student even when he fails to suspect the abstract formal character of the methods in question. It is only when these formal limitations are abandoned, when the Method of Agreement is shown to be at the mercy of plurality of causes, and forthwith depressed to the level of a mere preliminary or adjunct to the Method of Difference, and the latter, again, shown to be unable to cope with intermixture of effects, and, in its turn, transformed into a preliminary to Deduction, that we approach the true conception of a working Inductive Method. The stages of Observation, Hypothesis, Deductive development and Application of Hypothesis, Progressive Verification—stages which, from the strictly methodological point of view, impose themselves from the outset as the first essentials of inductive procedure, are thus gradually made more and more familiar to the reader, until, in the concrete development of the

* Book III., ch. viii., § 3.

† 'In the preceding exposition of the four methods of observation and experiment, by which we contrive to distinguish among a mass of co-existent phenomena the particular effect due to a given cause, or the particular cause which gave birth to a given effect, it has been necessary to suppose, in the first instance, for the sake of simplification, that this analytical operation is encumbered by no other difficulties than what are essentially inherent in its nature; and to represent to ourselves, therefore, every effect, on the one hand, as connected exclusively with a single cause, and on the other hand as incapable of being mixed and confounded with any other co-existent effect' (Book III., ch. x., § 1).

'Deductive' and 'Hypothetical' Methods, we reach at last a true methodological conception of Inductive Inquiry.

It is an interesting question whether Mr. Joseph has not been developing his Methodology on a similar plan. We would readily admit that the ideal of a reversible or reciprocal causal connexion is common alike to the 'logical' and the 'methodological' points of view; but the presupposing of the ideal as already realized seems to have the effect of diverting the energies of the logician from the task of seeking how progressively to realize it, and concentrating them on methods for the elimination of whatever fails to satisfy certain standard rules or canons. The logical presupposition supplements the methodological goal, though presupposition and goal embody, each in its own way, the same inductive ideal. The difference is one of Means rather than of Ends, though it is a difference which touches very closely on fundamental principles.

The balance between *terminus a quo* and *terminus ad quem*—for both are essential—might, perhaps, be adjusted by a purposive change in the character of the former. For the logical Presupposition we might substitute the methodological Postulate; or, in less technical language, we might place at the forefront of Inductive Inquiry not an ideal assumed to be realized, but a condition which must be satisfied in order to render Inductive Inquiry, not indeed ideal, but *possible*. The methodological Postulate would state the condition on which a *legitimate* scientific explanation depends, and the Unideterminate Ideal—the ideal of a system of reversible causal connexions—would then become the methodological guiding idea. It is in the light of this ideal that Science presses forward to the *terminus ad quem* of all Inductive Inquiry—namely, complete fidelity to fact, in so far as fact is relevant to the demands of the Inductive Postulate.

If we are asked to define the method which we propose to substitute for the Method of Elimination, we can hardly do better than point to Professor Bosanquet's Method of Analysis, to his view of Induction as the progressive 'moulding of Hypothesis' ('Logic,' vol. ii., ch. v., pp. 166, 167) through a process which he elsewhere describes as a purifying by exceptions and a limiting by negations (vol. ii., ch. iv., p. 117). In this process we have central importance attached to the positive, progressive elaboration of causal connexions, with a clear recognition of the part played by negation, and, in particular, by elimination (ch. iv.). The view of Induction adopted in the present work has been largely influenced byProfessor Bosanquet's treatment of the Inductive Problem in this second volume of his 'Logic,' though the inspiration has been tempered by pragmatic leanings for which the author of the 'Logic' might have little sympathy.

In Professor Bosanquet's view, the *primary* and essential business of an inductive investigator is not to attack all rival theories,

supporting the truth of his own theory by showing that no other can explain the facts so well: his main endeavour should be to strengthen his own theory in a positive and constructive spirit. This he must do by showing that all objections advanced against it are ungrounded, or, if not wholly ungrounded, still, by limiting the theory in determinate ways, support rather than refute it. A theory can establish itself only by successfully meeting all relevant objections. These objections will be brought forward as *exceptions* to the truth of the statement which expresses the theory. If the exception proves to be genuine, the theory must be correspondingly modified by the introduction of a limiting condition; if only apparent, it is refuted by a negative instance confirmatory of the theory. A genuine exception modifies the rule by restricting its universality; an apparent exception is but a mistaken interpretation of the facts which the negative instance interprets rightly.

As exemplifying this typical form of inductive procedure, we may cite an investigation discussed by Professor Welton,* who also is confessedly indebted to Professor Bosanquet for his general line of treatment. Professor Welton—from the point of view of the inductive interest—is analysing a part of Charles Darwin's inquiry into the formation of vegetable mould through the action of earth-worms.†

A number of careful observations had gone to support the statement that the 'formation of vegetable mould is due to the action of earth-worms.' The sign by which the fresh production of vegetable mould was tested was the sinking of objects strewed on the ground, and the process through which these objects were gradually buried deeper and deeper in the soil.

But here the investigator was met by an apparent exception to the universal activity of earthworms, which he desired to establish. Large boulders do not sink. Darwin, however, showed that this fact was no genuine exception, but that it furnished confirmatory negative instances. 'If a boulder is of such huge dimensions,' he writes, 'that the earth beneath is kept dry, such earth will not be inhabited by worms, and the boulder will not sink into the ground.'‡ Thus the positive statement, 'Where there are worms there is vegetable mould,' was corroborated and defined by the establishment, through negative instances, of the negative statement, 'Where there are no worms there is no vegetable mould.'

This same example furnishes also a simple instance of the way in which a *genuine* exception necessitates a modification of the original statement of a hypothesis—a restriction of its universality. It might be urged that soil is brought from beneath the surface by moles and other burrowing creatures no less than by worms. The

* 'A Manual of Logic,' vol. ii., Book V., ch. v., pp. 124-127.

† *Vide* 'The Formation of Vegetable Mould through the Action of Worms,' ch. ii., pp. 131-177.

‡ *Ibid.*, p. 152.

fact cannot be denied ; but it can be shown that their contribution to the surface layer of soil is far less than that of worms. The original statement should, then, be modified by the introduction of the word ' mainly,' and should be enunciated thus : ' The formation of vegetable mould is *mainly* due to the action of earthworms.' A series of accurate comparative measurements of the castings of earthworms, on the one hand, and of the soil thrown up by moles, etc., on the other, would complete the causal value of the proposition by giving ' mainly ' its proper fractional values (expressed in average form) for different soils and climates. This illustrates the fact that, before exact results can be obtained, the data upon which the judgment is based must be of a quantitative kind, and that corresponding numerical *limits* must be introduced into our general statement.

In conclusion, we would add that, as we conceive the matter, the part played by Elimination in the complete Method of Analysis is essentially heuristic, as in the case of the Method of Single Agreement. Elimination has a further function to fulfil in connexion with the necessary though secondary work of meeting and refuting rival hypotheses, especially in the devising of crucial instances and experiments which shall approve one hypothesis by disproving its rivals. But even here it seems gratuitous to bring these rival theories into preliminary relation with each other as co-alternatives of a disjunctive proposition. The disjunction has force only in so far as the verification of the theory we are defending rests upon the disproof of the rival theories. But the verification rests *essentially* on the positive value of the theory as a means for systematically explaining the relevant facts, and the disproof of rival theories—a disproof which, if undertaken in an impartial spirit, may be almost as arduous a piece of work as the justification of the theory itself—is an operation subsidiary and supplementary to this.

CHAPTER XLVI.

XIII. (iv.) ILLUSTRATIONS OF THE APPLICATION OF INDUCTIVE METHOD.

As examples of the ways in which Inductive Method is applied, we may take the following :

1. The Problem of Fermentation.*
2. The Case of Algol, the Demon Star.
3. The Rigidity of the Earth.

* This illustration is given by Dr. Mellone as an example of the application of the Method of Single Agreement and the ' Double Method of Difference ' (' An Introductory Text-Book of Logic,' first edition, ch. ix., pp. 272, 273, 282, 283). Our own account of this investigation is mainly an adaptation of Professor W. Dittmar's article on ' Fermentation ' in the ' Encyclopædia Britannica,' ninth edition, vol. ix.

1. *The Problem of Fermentation.*

By the word ' Fermentation ' we designate, as Professor Dittmar tells us, ' a peculiar class of metamorphoses which certain complex organic materials are liable to, and of which the well-known change which grape-juice undergoes when it " ferments " into wine, the souring of wine or milk, and the putrefaction of animal or vegetable matter, may be cited as familiar examples.'*

Chemically, the processes of fermentation admit, as a rule, of fairly simple statement. Thus, the change through which milk turns sour consists chemically in the passing of the milk-sugar in the milk into lactic acid by a rearrangement of atoms symbolized by the chemical equation :

$$\underset{\text{(Milk-sugar)}}{C_{12}H_{22}O_{11}.H_2O} = \underset{\text{(Lactic acid)}}{4C_3H_6O_3}.$$

So, again, in vinous fermentation we find that the formation of alcohol can be chemically explained as the result of the decomposition of the sugar contained in grape-juice, which breaks up into alcohol, carbon dioxide gas, and small quantities of other compound substances. The effervescence which is so noticeable a feature in the fermentation of must is caused by the rapid evolution of the carbon dioxide gas which rises in bubbles to the surface.

There remains, however, the question, What is the ' exciting cause ' of this chemical change ?

Let us, for simplicity's sake, follow the investigation of the alcoholic fermentation of grape-juice only. The facts are these : Grape-juice, clarified by filtration into a perfectly limpid and transparent liquid, may remain unchanged for an indefinite time. But when the smallest quantity of unfiltered juice is introduced into this pure liquid, it becomes turbid, the turbidity being due partly to the evolution of carbon dioxide gas, partly to the presence of a finely divided solid material, some of which rises to the surface and forms a scum. This solid material is known as yeast. These changes include a growing effervescence of the must, which may reach the stage of violent ebullition ; and as the effervescence grows, so does the yeast. A climax is reached, and the effervescence at last dies away. The yeast then settles down as a slimy deposit, above which there is left a clear yellow alcoholic liquid, characterized no longer by its original sweetness, but by a vinous taste and other properties of ' fermented liquors.'

Thus, in all ordinary cases of the phenomenon we are investigating, the formation of alcohol is found to be accompanied by the presence of yeast. Here, then, is an opportunity for the application of the Method of Agreement. It had long been noticed that

* *Loc. cit.*, p. 91.

yeast was apparently an invariable concomitant of vinous fermentation. Accordingly, a *causal hypothesis* was formulated :

First Hypothesis : The substance known as yeast is a causal antecedent of the alcoholic fermentation of grape-juice.

This earliest form of the hypothesis could not easily be developed or applied. While microscopic research was still in its infancy, and the microscope itself hardly yet worthy of its name, such a hypothesis was of necessity comparatively barren. But about the year 1840 it was discovered by Schwann and Cagniard-Latour that the yeast which is found in fermenting grape-juice consists of millions of globules possessing the morphological characteristics of vegetable cells. This, coupled with the fact that these cells multiply as the fermentation proceeds, convinced the investigators of the fact that yeast is a plant. Thus, they were led to put forward an important *modification of the original hypothesis.*

Second Hypothesis : The chemical changes known as the alcoholic fermentation of grape-juice are caused by the physiological activity of the living yeast-cell.

Thus, we see how advancing knowledge, due to a deeper and more precise analysis of the data, results in the *moulding* of hypotheses. Historically, the hypothesis, thus modified, was considerably strengthened by the close *analogies* observed between the processes of putrefaction and alcoholic fermentation, tending to show that in both processes living agency is involved. Moreover, we notice that in its second form the hypothesis is no longer barren. Firmly rooted in the systems of morphological and physiological Science, it can be logically developed into forms which are capable of experimental verification. The recognition of the yeast-cell as a micro-organism possessing the properties of vegetable cells in general enables us to draw upon botanical Science for premisses with which to combine our hypothesis. Such premisses are the following :

(i.) Vegetable cells, including the yeast-plant, cannot live at very high or at very low temperatures.

(ii.) They are immediately killed by being treated with certain substances known as 'antiseptics'—*e.g.*, corrosive sublimate, sulphuric acid, carbolic acid.

(iii.) Micro-organisms cannot (so far as we know) arise 'spontaneously.' Thus, a liquid which by prolonged boiling or other means has been freed from all living micro-organisms will remain so if kept from all contact with free atmospheric air and otherwise uninfected.

Combining these statements in turn with our modified hypothesis, we obtain, as 'developed' forms of that same hypothesis, the state-

ments that, in the process which we are investigating, the exciting cause of the chemical change (and therefore in this instance the chemical change itself) is (i.) something that cannot exist at extremely high or extremely low temperatures; (ii.) something that cannot exist in the immediate presence of certain antiseptics; (iii.) something that, if not antecedently present, cannot arise spontaneously in any substance kept under aseptic conditions.

These, and other developed forms of the hypothesis, were duly applied, and tested by means of a long series of experiments, of which these are some of the results:

(i.) Grape-juice ferments within a certain range of temperature only. It does not ferment at any temperature much higher than 60° C. If the fermenting liquid is boiled, the fermentation invariably ceases; nor does it recommence on cooling unless after the liquid has been left for some time in contact with atmospheric air.

(ii.) The fermentation may always be arrested by treating the liquid with sulphuric acid, bisulphide of carbon, carbolic acid, or any other of a number of antiseptics. By an experiment in antiseptic treatment Schwann verified that part of the modified hypothesis which assigned a vegetable as distinguished from an animal organism as the exciting cause in alcoholic fermentation, and at the same time experimentally discovered the limit of the analogy between this kind of fermentation, on the one hand, and putrefaction on the other. 'He found that white arsenic and corrosive sublimate, being poisonous to both plants and animals, stop both putrefaction and fermentation, while extract of nux vomica, being destructive of animal but not of vegetable life, prevents putrefaction, but does not interfere with vinous fermentation.'*

(iii.) 'Perfectly pure grape-juice does not ferment, unless the process has been started by at least temporary contact with ordinary air. This cardinal fact was observed by Gay-Lussac in a now classical series of experiments. He caused clean grapes to ascend through the mercury of a large barometer into the Torricellian vacuum, where he crushed them by means of the mercurial column. The juice thus produced and preserved remained unchanged, but the addition to it of ever so small an air-bell (as a rule) induced fermentation, which, when once started, was always found to take care of itself.'†

Such experiments (and others of which we have still to speak) were the means of a prolonged and varied application of the Method of Difference—an application in which it is noticeable that the positive is in many cases prior to the negative instance. Instead of *introducing* the suspected causal antecedent, and so obtaining the positive instance from the negative, the investigators more often *subtracted* the suggested agent, and so obtained the negative instance from the positive—though, in the case of the air-bubble

* 'Encyclopædia Britannica,' *loc. cit.*, p. 95. † *Ibid.*, p. 94.

admitted to the Torricellian vacuum, and the atmospheric air left in contact with the boiled and cooling must, the positive instance was re-obtained from the negative.

The obvious danger in each case was that of *subtracting too much.* For instance, when air was excluded from the germless liquid, was it not possible that the micro-organisms in atmospheric air were *not* the only relevant and important circumstance thereby subtracted? Might not the exciting cause of the fermentation be the air itself, or some constituent of it? This point was decided by means of a *crucial instance* devised by Schwann. He boiled grape-juice, thus subtracting all organic life, and then admitted air to it, but only through a red-hot glass tube, which, while allowing the passage of the air, must destroy any living germs that it might contain. The grape-juice so treated did not ferment, and thus, by a striking application of the Method of Single Difference, it was proved that air, *qua* air, is not responsible for the fermentation of grape-juice.

Meanwhile the rapid advance of Science was making possible a fuller analysis of the data and a further development of the hypothesis. Rooted now in the system of chemical as well as in that of biological Science, that hypothesis could be combined with such premisses as—

(iv.) Living vegetable cells cannot, as such, constitute a soluble substance; while advancing knowledge of micro-organisms and their movements had furnished the premiss—

(v.) A certain thickness of cotton-wool is an impassable barrier to micro-organisms [which become enmeshed in it, and cannot slip through].

Combined in turn with each of these premisses, the hypothesis develops into the statements that the causal antecedent for which we are seeking is (iv.) not soluble, and (v.) something of which the passage may be prevented by the interposition of cotton-wool.

From the statement that the required agent is not soluble Helmholz deduced the still further developed hypothesis that it is unable to pass through the wall of a bladder. This form he was able to verify experimentally by suspending a sealed bladder containing grape-juice in a quantity of fermenting must. The liquid contained in the bladder remained unchanged.

Many years after this Hoffmann 'took a test-tube full of sugar-water, and by a plug of cotton-wool inserted within it divided the liquid into two parts. To the upper part he added yeast, which, of course, induced fermentation there; but the change did not propagate itself through the cotton-wool to the lower portion.'* Thus, again, a developed form of the hypothesis was conclusively verified.

All these investigations, taken together, pointed irresistibly to

* 'Encyclopædia Britannica,' *loc. cit.*, p. 95.

the truth of the Second Hypothesis—*i.e.*, the original hypothesis as modified by Schwann and Cagniard-Latour. That the chemical changes known as the alcoholic fermentation of grape-juice are caused by the physiological activity of the living yeast-cell could no longer be doubted. It had been triumphantly established as a causal law. The ideal of 'reversibility,' indeed, had not been, and possibly will never be, reached in this case. In view of the facts (1) that a number of fermentations have been proved to be independent of physiological activity, and (2) that even alcoholic fermentation of minute quantities of sugar seems in some cases to have occurred without the intervention of micro-organisms, we cannot yet venture to assert that aseptic fermentation of minute quantities of grape-juice has never taken place. But that the ordinary process of vinous fermentation is in all cases due to yeast-plants has long been an established fact.

But the Second Hypothesis could not be regarded as final. In several points it is lacking in scientific precision. What is meant by the word 'caused'? Is the yeast-cell to be regarded as an immediate or only as a remote causal antecedent? Is the presence of the *living* cell in all cases necessary, or is it only a separable product of the living cell that is indispensable? What precisely is meant by the use of the word 'physiological'? Does it imply that the process of fermentation is in itself anything but a purely chemical reaction? And, finally, what is meant by 'the yeast-cell'? Is it one species or many? If there are different species of the yeast-plant, are there corresponding differences in the fermentations which they severally induce?

We have not space in which to follow out in detail the investigations to which these questions have given rise. With regard to the interesting botanical researches into the nature of the yeast-plant, we can only say that they have resulted in the establishment of the fact that vinous fermentation results from an activity (probably to be regarded as pathological) of the genus of fungus known as Saccharomyces. This genus includes a considerable number of species, of which the most important is *S. cerevisiæ*. Each plant of this species is a single cell of globular form measuring about $\frac{1}{100}$ millimetre in diameter.

From the logical point of view, the most interesting question suggested is that of remote or immediate causation. This problem gave rise to two *rival hypotheses*—that is, the hypothesis was, by different investigators, moulded in two (apparently) opposite ways; its progressive modification advanced along divergent lines.

> *Third Hypothesis: The changes known as the alcoholic fermentation of grape-juice are of a purely chemical character. The living organism is a remote causal antecedent.*

We may call this the Hypothesis of Liebig.

Fourth Hypothesis: The changes known as the alcoholic fermentation of grape-juice represent a physiological process. They take place within the organism of the yeast-plant, and are immediately caused thereby in this sense—that the products of the fermentation are to be regarded as actual products of the metabolism of the cell.

This is the Hypothesis of Pasteur.

The question between these rival hypotheses cannot yet be said to have been finally decided. In forms modified by recent research they still confront one another. But for a time Pasteur's theory was completely triumphant, and until the year 1897 that of Liebig was regarded as finally discredited. Both hypotheses were duly developed, applied, and tested. The history of the (temporary) establishment of Pasteur's theory is logically the more interesting.

Pasteur's work was at first of a purely revisionary and critical kind. 'He did the whole of the work of Schwann and the rest of his predecessors over again, modifying and perfecting the experimental methods, so as to silence any objection or doubt that might possibly be raised, repeating and multiplying his experiments until every proposition was firmly established.'*

Pasteur's positive work consisted essentially in *generalizing* the theory of the connexion of alcoholic fermentation with organic agency. He supported his hypothesis not as an isolated thesis, but as part of a larger theoretical system. To express the matter in a way less adequate but more logically precise, he supported his hypothesis by strong *analogical arguments* which his extended researches had made possible.

It is well known that alcoholic or 'vinous fermentation is only one of a number of fermentative changes to which sugar is liable. The same† substance—sugar—which, when placed under certain conditions, breaks up into alcohol and carbonic acid, under certain other sets of conditions ferments into lactic acid, and through lactic into butyric acid, or into gum plus mannite. . . . What Pasteur showed is that each of these changes is the exclusive function of a certain species (or at least genus) of organism. What the yeast-plant is for vinous a certain other organism is for lactic, a third for mannitic, a fourth for butyric fermentation. No two of these species, even if they belong to the same genus, will ever pass into each other. Pasteur arrived at this great generalization by means of his invention of an ingenious method for cultivating pure growths of the several species, so that each of them could be examined separately for its chemical functions.'‡ More recently this method has been superseded by that of Hansen, a Danish chemist and

* 'Encyclopædia Britannica,' *loc. cit.*, p. 95.
† This is not quite accurate. Different sugars are variously constituted.
‡ 'Encyclopædia Britannica,' *loc. cit.*, p. 95.

botanist, who showed that Pasteur's so-called pure cultures did not really deserve the name, and that, in order to obtain a culture that is certainly and strictly pure, it is necessary to begin with a single cell; but Pasteur must still be regarded as having done inestimable service to Science in the investigation of yeast-fermentation, however imperfect some of his methods may have been. His researches were accepted by the scientific world as having conclusively established the theory that yeast-fermentation is a 'vital phenomenon' taking place within the living cell.

But the Chemical as opposed to the Physiological Hypothesis had not yet been finally disproved. In 1897 an important discovery was made by Buchner. By means which Professor Ray Lankester has truly characterized as 'heroic mechanical methods,' he succeeded in obtaining from brewing-yeast what he regards as an unorganized substance, which certainly is able, quite apart from the living yeast-cell, to induce alcoholic fermentation. By prolonged grinding with quartz sand, and by then subjecting the mass of disintegrated cells to high pressure, he at length produced the liquid 'zymase' which he regards as an enzyme that by a purely chemical process effects alcoholic fermentation. This substance appears to be soluble—at least, the liquid may be made to pass through a filter of porous porcelain without losing its property of exciting fermentation. Thus, the Chemical Hypothesis of Liebig, so long discredited, may be said to have been revived, though in a modified form, in the Enzyme Hypothesis of Buchner. The Physiological Hypothesis of Pasteur can no longer stand as he formulated it. At present it is struggling to save its life by submitting to an important modification. The fermentation (says the physiological faction) induced by Buchner's wrongly termed enzyme is due not to a ferment separable from the substance of the cell, but to minute particles of *living protoplasm*, which are not, indeed, soluble, but are sufficiently miscible with water to be able to pass through the porcelain filter.

Thus, the Chemical and the Physiological hypotheses both (in modified forms) still survive. Will some investigator of the future devise at last some crucial instance which will lead to the final triumph of one, the final defeat of the other? Or will some reconciliation between them be brought about by a deeper analysis of the meaning of that difference between chemical and physiological process of which we speak so readily, but which yet is not without that suspicion of vagueness which may be the indication of the need of preciser definition? Is it possible that the two divergent forms of the hypothesis may be so modified as to converge again, and finally to coincide? As Organic Chemistry advances, may it prove that our distinction between physiological and 'purely chemical' process was, after all, in this case a distinction without a difference?

2. *The Case of Algol, the Demon Star.**

Algol is a variable star in the constellation Perseus. It goes through a regular cycle of changes that are visible to the naked eye, and have therefore been known for centuries. Algol is the brightest of those variable stars which, instead of varying continuously, remain constantly at their maximum brightness during the greater part of their period, and then temporarily lose a part of their light, soon to regain their usual brilliancy. Stars which vary in this peculiar way are relatively very few, only twelve being as yet known to us.

The problem of causally accounting for the variability of Algol is of long standing, and some of the earlier hypotheses were crude and fanciful. The star was at one time believed to be the eye of a demon. The development and application of this attractive hypothesis may safely be left to the ingenuity of the reader, who will find no difficulty in combining it with such premisses concerning the blinking of demons as Mythology or Folk-lore may supply. We need hardly say that, considered from the Astronomical standpoint, this prescientific hypothesis is unfortunately barren.

From the later stages of this long-continued inquiry we select a few salient points, without aiming at strictly chronological order.

First Hypothesis: The variability of Algol is due to a series of explosions.

This hypothesis did not prove itself fruitful in any precise or satisfactory sense. Too vague to take firm root in any scientific system of Astronomy, it put forth only feeble shoots of indeterminate probability. Yet, slight as its vitality was, the hypothesis developed far enough to be quite sufficiently disproved. Into whatever specific forms it ramified, the feebly swaying branches all pointed away from the facts. Explosive action is sudden, seldom regular, likely to result in a maximum brilliancy of short duration and long intervals of diminished light. The variation of Algol is steady and regularly recurring, and consists in the occurrence of a relatively short interval of diminished brilliancy between much longer times of maximum brightness. Even the idea of a venerable demon indulging every three days in one solemn and deliberate wink was in some respects less incredible than the Explosion Hypothesis. Such a suggestion had not strength enough even to bear modification. It was simply discarded.

The next hypothesis that we select was based on an ingenious argument from Analogy.

* Our account of this investigation is partly based upon that given by Sir Robert Ball in the volume entitled 'In the High Heavens,' pp. 179-190.

Second Hypothesis: The case of Algol is analogous to that of our sun. The periodic increase and decrease of sun-spots constitutes our sun a variable star. The periodic variation of Algol is due to similar causes.

Here was a hypothesis that was not wanting in vitality. It could be developed in various ways, but all of them pointed to its own disproof. If we were as far from our Sun as we are from Algol, even were the sun-spots much larger and more frequent than they are, the variation in brightness would be altogether imperceptible. And when the two periods had been precisely ascertained, the analogy entirely broke down. That of the sun-spots is a period of eleven years, that of Algol's variation less than three days; and 610 days is the very longest period that we know as that of any variable star. We need not follow out in detail the process which led to the rejection of this attractive but untenable conjecture.

Third Hypothesis: The variation of Algol is due to a rotatory movement of the star, of which one aspect is brighter than another.

Here we stand on much firmer ground. The laws of rotatory motion are well established, and furnish undoubted premisses with which to combine our hypothesis so as to cause it to develop in more than one direction. Moreover, we are now considering a stage of the investigation at which the original problem has been more precisely stated. The telescope and the camera have both been brought to bear upon the problem, and fuller data have thus been acquired for scientific analysis. The results of observation as they stand at the present time are as follows:

For about two days, ten hours, the star remains constant in brilliancy. Then its brightness declines so rapidly that in a few hours it has lost three-fifths of its brightness. In this stage of lowest brilliancy it remains about twenty minutes, and then begins to grow brighter again, so that in a few hours more Algol is as brilliant as ever.

	Days.	Hours.	Minutes.	Seconds.
Time of maximum brilliancy ..	2	10 (approximately)	—	—
Time of decline and rise of brilliancy	—	10	48	52
Time of whole period	2	20	48	52 (approximately)

This period is liable to fluctuations of a few seconds, but otherwise it is wonderfully uniform.

Now, when the rotation-hypothesis is developed into forms which can be precisely applied, and compared with these data, verification is found to be impossible. No form of the hypothesis can be obtained which does not require the time of maximum brightness

to be much more nearly equal to that of diminished lustre than is actually the case. If the hypothesis is not to be entirely discarded, it must at least submit to a radical reconstruction.

Fourth Hypothesis: The apparent variation of Algol is due to its being periodically eclipsed by a darker companion revolving round it.

This hypothesis was not chronologically the latest, but it was the last to be so developed that a conclusive verification could be attempted. Even our present telescopes enable us to see Algol only as a pin-point of light, and the star is at an immeasurable distance from us. When first suggested, and for long afterwards, the hypothesis was comparatively barren. Even the modern telescopic camera, with its films so much more sensitive than the human eye, could alone give no clue for its profitable development. It could, indeed, be combined with premisses furnished by the Law of Gravitation and the Laws of Motion, and so developed into a wide range of possible results, some of which agreed with the observed data; but such a test as this, though it sufficed to disprove the rotation-hypothesis, did not afford for the eclipse-hypothesis an adequate verification. To disprove an erroneous hypothesis is often easy; to establish a true one is usually very difficult.

But at length the spectroscope gave the required clue. It was observed that during that half of Algol's period which follows the first half of the time of minimum brilliancy the dark lines in the spectrum of the star shifted slightly towards the violet end, and that during the other half of the period they shifted towards the red end of the spectrum.

Now let us again consider our Fourth Hypothesis, to see whether it can in any way be developed, so that this observed fact may be used for its verification.

For brevity's sake, we formulated our hypothesis inaccurately. What the Law of Gravitation requires it to mean is not that Algol stands still and his dark companion revolves round him, but that the Algol system consists of two bodies (of which one is luminous) revolving about their common centre of gravity. Now, a little consideration will show that if that be really the case, then the luminous body will always be approaching us for some time after the eclipse, and retreating from us for an equal time before it, these two times, together with the time of the eclipse itself, constituting the time of the whole period. As the luminous body moves not in a straight line, but in an elliptical orbit, the velocity of the luminous body in the line of sight will, of course, not be uniform. Now, it can be deductively shown that, assuming the truth of the now well-established Undulatory Theory of Light, the dark lines in the spectrum of a source of light which is moving towards the observer will shift slightly towards the violet end; and that if the

source of light be moving away from the observer, they will shift slightly towards the red end of the spectrum.

Thus we see that, by combining our Fourth Hypothesis with premisses furnished by the Law of Gravitation and the Laws of Motion, it has been possible to develop it into a form in which it can be applied by means of deductions from an accepted theory of physical Science. Thus the eclipse-hypothesis was at length established by a convincing verification. Further accurate observation of the photographic spectrum of Algol showed the exact length of the times during which the dark lines moved towards the violet and the red end respectively, and it was shown that during just one-half of its period Algol must be moving towards us with a maximum velocity of twenty-six miles per second in the line of sight, and that during the other half it must be receding from us with the same maximum speed of twenty-six miles per second. These spectroscopic observations confirmed the hypothesis not only in regard to the motion of Algol, but also in the statement that the companion-star must be dark relatively to Algol, since, were both stars bright, *two* sets of dark lines would have appeared instead of only one. Moreover, from these observations, combined with premisses furnished by the Law of Gravitation and the Laws of Motion, it was found possible to deduce a form of motion for Algol which, while perfectly agreeing with the undeveloped hypothesis, could now be more precisely stated. Of the dark companion and its movements the spectrum analysis could, of course, say nothing.

The previously observed facts as to the variability of Algol could now be interpreted in the light of the established hypothesis, and of the results of deduction, themselves based upon the analysis of the spectral phenomena. *Each detail* of the original observations can now be interpreted in terms of the newly established theory. Thus :

(*a*) The period of Algol's variation *means* the period of Algol's revolution in its orbit. Therefore, the time of this period is two days, twenty hours, forty-eight minutes, fifty-two seconds (approximately).

(*b*) The periodic fall and rise of brilliancy in Algol *means* the regularly recurring eclipse of Algol by its dark companion. The brilliancy begins to diminish as Algol begins to pass behind the dark body; it begins to increase as the dark body begins to pass from the disc of Algol. The whole time of eclipse is, therefore, about eleven hours.

(*c*) The time of minimum brilliancy *means* the time during which the *whole* of the dark body is eclipsing Algol. Therefore this time of the transit of the dark body over Algol is twenty minutes.

(*d*) Algol's losing three-fifths of its brightness during this time of greatest eclipse *means* that the dark companion is then covering three-fifths of its disc.

All these are not *deductions* in the true sense of the word, but *interpretations* of one set of symbols in terms of another.

We come now to a stage in the investigation which we may call that of *quantitative deduction.* Given (*a*), (*b*), (*c*), (*d*) (as above) in conjunction with what we know about the velocity of Algol in its nearly circular orbit, we are able to deduce the following results :

(1) The magnitude of Algol's orbit about the common centre of gravity.
(2) The distance between the two globes.
(3) The masses of the two globes, and also their dimensions, if we are allowed to assume what we have reason for believing is more or less true—that the two bodies are of the same density.
(4) The nature of the dark body's orbit, and its velocity in that orbit, provided that we are allowed to assume that this dark body, as well as Algol itself, obeys the Law of Gravitation—*i.e.*, a law of attraction varying directly with the mass of two attracting bodies, and inversely as the square of their distance.

The following are some of the results thus obtained :

Algol is twice as large as our sun, having a diameter of more than a million miles. Its weight is only one-half that of our sun, its average density being less than that of water, a little greater than that of cork.

Algol's dark companion is of the same size as our sun, and has one-quarter of our sun's mass. Its orbit, like that of Algol, is nearly circular, and its velocity in that orbit almost uniform—about fifty miles per second.

The distance between the two globes is about three million miles.

So ends for the present the story of Algol, the Demon Star. The results may be regarded as perfectly trustworthy, provided we have been right in assuming—

(i.) The truth of the Law of Gravitation as applicable to Algol and its companion.
(ii.) The truth of the Laws of Motion.
(iii.) The truth of the Undulatory Theory of Light.
(iv.) The equal densities of the two stars.

Of these, (i.), (ii.), and (iii.) are fundamental, and about as certain as any fact of knowledge whatsoever, especially (i.) and (ii.). The

only element of real uncertainty lies in (iv.). But even if the densities should be found not to be equal, the only consequence would be a comparatively small modification of the *numerical* results. Substantially our account of the Algol system would remain the same.

If the story of Algol is to close with a logical moral, we would point out that the great lesson it teaches is that of the importance of Analysis. But for the minute and careful analysis of the spectroscopic data the causal hypothesis could never have been established or the quantitative deductions made. We must not, indeed, confuse logical with physical analysis. The so-called spectrum analysis—the spreading out of the light of Algol into its elements—was not a logical but a physical process. The logical analysis came afterwards. It was the careful distinction between those spectroscopic phenomena which concerned the qualitative properties of the light and those which pointed to movements of the source of light itself. It is noticeable that this analysis was exercised upon the results not of direct, but of what we may call indirect observation. When the direct observation of Algol's luminous changes had been carried as far as the case admitted, it gave place to the observation of the phenomena of Algol's spectrum. Thus, it often happens in scientific investigation that the analysis of some obscure phenomenon enables us sooner or later to discover some hitherto uninterpreted indications which, though not obviously or directly bearing upon the problem before us, yet enable us to apply to its elucidation some part of our previously acquired knowledge. Analysis is that essential process in scientific investigation whereby it is possible to bring old knowledge to bear effectively on that which is yet unknown.

3. *The Rigidity of the Earth.**

The high temperature of the interior of the Earth—*i.e.*, of the whole bulk of our planet with the exception of a relatively very thin crust—is a well-established fact. From early times such phenomena as those of hot springs and the lava-streams of molten rock ejected by volcanoes had attracted attention and called for explanation. More recently, the fact that the temperature in the lower parts of deep mines is higher than that at the earth's surface was recognized as pointing in the same direction, and experimental investigations have now established the fact that, from a point about 100 feet beneath the surface, the temperature of the Earth increases at the rate of 1° F. for every 66 feet of descent—*i.e.*, 80° for the first mile.

The causal investigation of such phenomena as pointed to the

* Our account of this investigation is based mainly upon Sir Robert Ball's discussions in 'The Earth's Beginning,' ch. ix., and 'In the High Heavens,' ch. iii.

existence of extremely high temperatures beneath the Earth's crust, when viewed from the standpoint suggested by the initial stages of the Nebular Theory, gained immensely in interest and importance, and necessitated the formulation of some definite hypothesis as to the conditions upon which these phenomena depend. The earlier forms in which the hypothesis took shape were founded on a consideration of the high-temperature conditions of the Earth's interior, while the high-pressure conditions, which are quite as important, were comparatively ignored. If we may be allowed to use the word 'fluid' in that popularly accepted sense in which it is opposed to 'rigid,' we may briefly formulate the initial hypothesis thus :

First Hypothesis : The interior of the Earth is fluid.

We have not space in which to follow out the various processes whereby this hypothesis was developed, applied, and disproved. Speaking quite briefly, and aiming at logical rather than chronological arrangement, we may say that this early form of the hypothesis was developed in two main directions :

(i.) Combined with premisses furnished by the Laws of Motion, it was developed into consequences which could be compared with facts to which Sir Robert Ball somewhat vaguely refers as 'certain astronomical phenomena connected with the way in which the earth turns round on its axis.'

(ii.) Combined with the Law of Gravitation and certain well-known laws of Hydrostatics, it was developed into consequences which could be compared with established facts regarding the ebb and flow of the tides.

It was found that in both these developed forms the hypothesis broke down under the test of attempted verification.

Meanwhile, Astronomers had been implicitly building many of their theories and investigations upon an hypothesis directly opposed to that which we have been considering. It is true that absolute rigidity was not supposed to have any actual existence, but it was in many investigations taken for granted that the rigidity of the Earth is so great that without practical risk of error the Earth may be considered as behaving like an absolutely rigid body. We must now consider the fortunes of this form of hypothesis.

Second Hypothesis : The Earth may be taken as behaving like an absolutely rigid body.

Passing over many interesting developments of this hypothesis, and the partial verifications which in many cases justified its use

as a simplification resulting in no practical error, we must give some account of one development which is of very special interest, since it ultimately led to the disproof of the hypothesis itself. Moreover, it is specially interesting as being an *unintentional* development in this sense—that it was not carried out with any intention of testing or verifying the hypothesis of which the truth was implicitly assumed. The development took the form of an investigation respecting a supposed movement of the North Pole of the Earth relative to the Earth's surface.

Most of us can identify the Pole Star in the heavens. Observation shows that its apparent position remains practically the same all through the night, while the other stars appear to revolve round it. As a matter of fact, it does every twenty-four hours apparently describe a small circle about another point in the heavens. *This* point, which is entirely stationary relatively to the diurnal apparent motion of the stars, is called the Celestial Pole.

Let the reader imagine that his eye is placed at the centre of the Earth, and that a long, slender tube passes from that centre to the surface. If this tube be so placed that, when looking through it from the centre of the Earth, the eye is directed exactly to the celestial pole, then that spot at which the end of the tube passes out through the surface of the Earth is the North Pole.

We have now to consider whether this imaginary tube or axis, about which the Earth daily rotates, always cuts the surface of the Earth (north of the Equator) at exactly the same point of that surface. We must remember that it is only an *imaginary* axis, or we shall tend to think that this must necessarily be the case. If I make an orange spin round a steel knitting-needle that pierces its centre, that needle, however it moves, will always cut the surface of the orange at the same point of the peel. But there is no such axis as this to exert a physical compulsion on the Earth; and the question arises: 'Assuming that the Celestial and North Poles remain in fixed positions, does the Earth move at all in such a way as to make it necessary to identify in succession different points of its surface with the North Pole?' Or, to put it differently, is there *a movement of the pole* over the surface of the Earth?

Now, the great mathematician Euler, assuming the absolute rigidity of the Earth, and combining that assumption with well-established laws of Astronomical Science, was able to show that a rotatory movement of the North Pole is physically possible, and that, if it takes place, the period must be completed in ten months.

This statement about a ten months' period, having been deduced from the assumed absolute rigidity of the Earth, we may regard as a developed form of our Second Hypothesis. That Euler did not call it so, or regard it as a means of testing

that hypothesis, need not disturb us. We are regarding the investigation from the logical, and not at all from the historical point of view.

Euler's calculation remained unchallenged for more than a century. At length it occurred to a certain Mr. Chandler that Euler's deductive result should be tested by comparison with the results of observations made during the past century as to the movement of the North Pole. A careful scrutiny of these recorded observations enabled him to discover that the rotatory movement does indeed take place, that the pole describes a circle not more than a dozen yards in radius, but that the period is completed, not in ten months, but in fourteen.

Now, Chandler was not consciously testing the absolute rigidity assumption any more than Euler had been consciously developing it, and the true import of the discrepancy between the results of deduction and the results of observation was not at first perceived by anyone. Chandler's observational results were received with incredulity by the theorists, who maintained that Euler's reasoning was faultless. The outcome was a veritable dead-lock between theory and observation.

At length Simon Newcomb reasoned thus within himself : Euler, in making his calculations, assumed that the Earth is a perfectly rigid body. His assigning of a ten months' period for the rotatory movement may depend entirely on that assumption. It is possible that, if the Earth is not taken as being perfectly rigid, there may still be deducible a rotatory movement of the pole, but that this movement may be shown to have a period of fourteen months instead of ten. These ideas were worked out, and theory and observation, after many hitches, were finally reconciled. Thus Newcomb showed the true meaning of the discrepancy discovered by Chandler. He showed that it amounted to a disproof of the absolute-rigidity assumption, to the establishment of the theory that the Earth is far from being ideally rigid. More recent researches have confirmed this conclusion, and have shown that the measure of the Earth's plasticity is exactly given by the observations which Chandler adduced.

We see, then, how, by a long and intermittent process, carried out at different periods by different investigators, and by means of a method of which the application was to a great extent unconscious, the assumption which we have called the Second Hypothesis was developed, applied, and in the end disproved.

This Second Hypothesis concerned the behaviour of the Earth considered as a whole. Its disproof and the establishment of the plasticity of the Earth as measured by Chandler's results bear only indirectly on the original question as to the conditions of the interior of the Earth as distinguished from its outer crust. We

have now to consider the recent investigations by which the rigidity of the Earth's interior *relative to that of its outer crust* has been accurately ascertained. We have to consider the development, verification, precise formulation, and final establishment of the theory which, for the sake of simplicity and clearness, we may call the Third Hypothesis, it being, of course, understood that our numbers have no precise chronological significance.

Third Hypothesis: The interior of the Earth is at least as rigid as its outer crust.

In order to understand the possible developments of this hypothesis, we must make sure that our conception of rigidity is sufficiently precise. The question to be decided is not exactly the question whether the matter of which the interior of the Earth consists is solid rather than liquid. Not only is that matter intensely hot; it is also subjected to a pressure of many thousands of tons on the square inch. We have no experience of matter under such conditions. Whether it is solid or liquid we cannot tell. It is quite possible that, as Sir Robert Ball suggests, the words 'liquid' and 'solid' may be equally inapplicable to it in the senses in which we understand them. As 'many, if not all, solids may be made to flow like liquids if only adequate pressure be applied,'* so it may be that (for instance) water, if subjected to the enormous pressure under which the materials of the bulk of the Earth exist, would in respect of rigidity behave like cast-iron. Our hypothesis, then, does not assert that the materials of which the interior of the earth consists are solid as opposed to liquid. It asserts only their comparative *rigidity*. Now, the rigidity of any substance is measured by the amount of force required to make its particles change their positions relatively to one another. The greater the force required to bring about a certain given amount of such change, the more rigid is the body in question.

What, then, do we know about rigid bodies so defined? What statements about them does Physical Science supply which may serve as premisses with which to combine our hypothesis with a view to its fruitful development? There are two such statements which, representing well-established laws, enable the hypothesis to develop in two important directions. These statements of fact are the following:

(i.) When a body receives a blow, the disturbance, propagated in undulatory tremors through the substance of the body, travels at greater speed the greater is the rigidity of the body struck.

(ii.) When such tremors are propagated through a body, the greater the body's rigidity, the less will be the consequent displacement of its particles.

* 'The Earth's Beginning,' ch. ix., p. 161.

These well-known facts of Physical Science enable the hypothesis to be developed into two forms capable of being experimentally tested :

(i.) Tremors propagated through the interior of the Earth travel with at least as great a speed as those propagated through the outer crust.

(ii.) Tremors propagated through the interior of the Earth cause no more, and possibly less, displacement of the particles of the substances traversed than do tremors propagated through the Earth's outer crust.

The development of the hypothesis in these two directions was no difficult matter. But what of the experimental verification which was necessary for its establishment ? What fabled giant of the wildest fairy-tale could make or wield a sledge-hammer heavy enough to give such a blow to the Earth as might result in the propagation of tremors throughout the globe ? The developed forms of the hypothesis might have waited long for their experimental testing had not the Earth itself come to the rescue and supplied (as, indeed, it had supplied during countless ages) a series of phenomena which were now for the first time recognized as *natural experiments* of the very kind that the hypothesis needed for its convincing verification.

We have spoken above of the enormous amount of heat that exists in the interior of the Earth. Now this heat is very slowly indeed, but still unceasingly, passing away from the Earth. It is continually rising by conduction to the surface, and thence is lost by radiation into space. Professor J. D. Everett has estimated that, were the whole globe covered with a shell of ice one-fifth of an inch in thickness, that shell could be entirely melted by the amount of heat which thus escapes annually from the Earth.

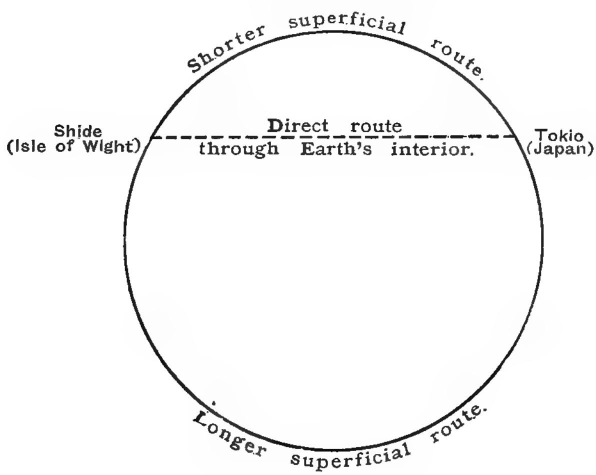
As a consequence of this continual loss of heat, the Earth contracts. Further, the Earth-crust has to accommodate itself to this perpetual shrinkage, and this adjustment causes violent shocks, which reveal themselves at the Earth's surface in the form of earthquakes. Suppose an adjustment of this kind to take place among the rocks at a depth of ten miles, where the pressure is thirty-five tons on the square inch. Under such pressure as this even a slight adjustment must produce an exceedingly violent shock, of which the effect is propagated in the form of undulatory tremors throughout the globe. If the shock is sufficiently intense, the surface of the Earth above the centre of disturbance will shake and rend as the vibrations reach it. The wave-commotion spreads in all directions, decreasing in violence as the distance from the centre of disturbance increases, until it is no longer directly perceptible. Yet thousands of miles away from the disturbed area, though *we* may not feel the tremor, delicate instruments can. A *seismometer*, set up at any

point on the Earth's surface, will be sensitive to earthquake tremors which have become far too faint to be directly perceived by our senses, and faithfully records on its revolving paper drum the particulars of all the earthquakes that take place even in the most distant countries. These seismograms vary in character. Those which represent Earth-tremors originating in any specific area have a family resemblance.

Thus, Professor Milne, in his laboratory at Shide, in the Isle of Wight, has accurate news of every earthquake a very short time after its occurrence. He looks at the seismogram, and observes the nature of the tracing. If it is of a certain kind—σ_1—he is able to say: 'This is the tracing proper to the Japanese group of earthquakes. Therefore an earthquake has been taking place in Japan during the last half-hour, and the tracing tells me the magnitude of the shock.' If the seismogram is of another kind—σ_2—he will say: 'This is the tracing proper to the West Indian earthquakes; I see that an exceptionally violent earthquake has just now taken place in that region.'

Japan is the scene of very frequent earthquakes; about a thousand take place there every year. Let us suppose that a vigorous earthquake has occurred in the neighbourhood of Tokio. The earth-tremors are propagated thence over the surface and through the interior of the globe in all directions. Speaking roughly, and putting the matter in the simplest way possible, we may say that they reach the Isle of Wight by three main routes:

(1) The direct route through the interior of the Earth;
(2) The shorter superficial route through the Earth's crust;
(3) The longer superficial route through the Earth's crust.



The tremors propagated along these different routes reach Shide at different times :

(1) About a quarter of an hour after an earthquake-shock has been felt in Japan the pencil of the seismometer at Shide begins to record the tremors arriving by the direct route through the interior of the Earth.
(2) Three-quarters of an hour after this the pencil makes another record of precisely the same form, but on a much larger scale. This represents the tremors arriving by the shorter superficial route.
(3) About half an hour later still a precisely similar record is made on a larger scale than the first, but on a smaller scale than the second.

We are chiefly interested in two deductions that have been made from these observations :

(i.) By comparing times of arrival and distances travelled, it has been shown that *the tremors propagated through the interior of the Earth travel at greater speed than do those propagated by way of the Earth's crust.* Indeed, it has been shown that the velocity varies with the square root of the depth beneath the surface. When the tremors are traversing the Earth's centre the velocity of propagation is more than ten miles per second ; near the surface it is not two miles per second.

(ii.) Measurements of the amplitudes of the tremors, as represented by their respective seismograms, show that the tremors which have travelled by the direct route have much smaller amplitudes than those which have travelled by either of the superficial routes. Hence is deduced the fact that *the particles of which the materials in the Earth's interior are composed are less displaced by any given earthquake-shock than are those of the Earth's crust.*

Thus the two developed forms of our hypothesis have been experimentally tested and established ; and so precise are the results obtained that they not only verify the hypothesis itself, but enable us to state it in a far preciser form than that in which it was first proposed. It is now an established fact of knowledge that the materials of which the interior of the Earth are composed are more rigid than solid steel as we know it at ordinary temperatures and under atmospheric pressure at the Earth's surface. Thus we have reached the solution of the problem of the Earth's rigidity.

The investigation of this problem, when studied from the methodological standpoint, offers several points of interest and instruction.

In the first place, the long acceptance and the ultimate fate of the assumption which we have called the Second Hypothesis teaches

us the important lesson that, when deductive theory on the one hand, and the results of observation on the other, come to a deadlock, we must fall back upon such questions as these : Are the assumptions adopted by Theory for simplicity's sake really trustworthy ? May not these simplifications have ignored some element which, if taken into account, might bring about a reconciliation between the results of Deduction and the data of Observation ? Instead of discrediting the results of Observation, may not the discrepancy be interpreted as pointing to the disproof of some hypothetical assumption ?

A second important feature of the investigation is the fact that, in the testing of the Second and the Third Hypotheses respectively, the problem was approached from two different and independent standpoints. We have here two separate groups of observed facts : (1) Observations respecting the movements of the pole ; (2) observations respecting earth-tremors. From each of these groups are deduced certain conclusions respecting the rigidity of the Earth. The first set of observations tends to emphasize its plasticity, the second set its rigidity. In such a case it sometimes happens that the conclusions drawn from the one point of view simply serve to give greater definiteness to those drawn from the other standpoint. In the present case it is possible that the amount of rigidity required by the seismic observations may be found to be incompatible with the amount of plasticity required by the observations regarding the movements of the pole. If this is so, the whole argument will have to be revised on both sides until the two groups of observations can be shown to be concordant. If such complete harmony can be reached, it will involve the assigning of a narrowly restricted *degree of rigidity*. The interior of the Earth must be shown to be just rigid enough to please Mr. Milne, and just plastic enough to content Mr. Chandler. The hope of an accurate solution of the problem is thus greatly increased by the fact that the investigation has been carried on from two points of view which are apparently antagonistic in their requirements. If solutions of a problem offered from two opposite sides of the question can be shown to be coincident, the probability that the common result is true is very great indeed. Generally speaking, there is no more convincing vindication of the truth of a questioned fact than the proof that various independent lines of evidence all converge in maintaining it.

A third point which this investigation illustrates is the intricate complexity with which phenomena cohere in Nature. Whether we consider the ocean tides, or the rotatory movements of the Earth and of its poles, or its internal tremors, the question of the Earth's rigidity is seen to be equally involved. The methodologically important conclusion that we draw from such instances as this is that, whenever we reason from any group of observed facts, we

are drawing conclusions to which the remaining facts of Nature are not by any means indifferent. Our conclusions are always liable to be called to account by reasonings based on other groups of facts, perhaps apparently remote from those on which we reasoned, but really together with them belonging to one and the same harmonious system.

XIV.

THE INDUCTIVE POSTULATE.

CHAPTER XLVII.

THE INDUCTIVE POSTULATE: THE POSTULATE OF CAUSAL EXPLANATION.

The Meaning of the Term 'Postulate.'

In Scientific Theory we have four important terms which are too often confused with one another, though they are by no means synonymous. These are the terms 'presupposition,' 'working idea,' 'hypothesis,' and 'postulate.'

The *presuppositions* of an abstract science are the established results of the simpler sciences, results which it accepts uncritically, or takes for granted. Thus, Geometry presupposes the fundamental results of the Science of Number, Physics presupposes an already elaborated science of quantity in general, Chemistry presupposes the results both of Mathematics and of Physics—*i.e.*, it assumes, as already established, the properties of Quantity and Motion, dealing mainly with the atomic and molecular composition of bodies. Biology presupposes the results of Chemistry, Physics, and Mathematics—the properties of Quantity, Motion, and molecular composition—as already established, dealing directly with organic relations only.

The *working ideas* of a Science are certain fundamental assumptions which, in the attempt to elucidate its own subject-matter, it finds itself called upon to make. They differ from presuppositions in this—that they concern the subject-matter of the science itself, and not that of other more elementary sciences; and they differ from hypotheses in this—that they represent the relatively permanent, whilst hypothesis represents the more fluctuating, element in the inquiry. As in military strategy, so in scientific research, it is necessary to have a basis of operations which remains fixed, however the plan of the inquiry be modified. A science of Comparative Biology may posit *Evolution* as its fundamental working idea, while holding itself free to change or modify its hypotheses as to the nature of the evolutionary process—Aristotelian, Lamarckian, Darwinian, and the more specific hypotheses that depend on these. Such an inquiry consists in the constant remodelling of a system of hypotheses so as to adjust it to the facts, the working

idea, in close relation to which the hypotheses are framed, remaining steady throughout the inquiry. If any discrepancy occurs between calculated results and facts, the hypothesis must first be modified or changed so as to fit the facts, and only in last resort must the Idea be brought in question.

Postulate and Working Idea.

The term 'postulate' may conveniently be used in one or the other of two main senses. It may stand either for an *a priori* necessity of the reason, or for a 'methodological guiding principle.' The Principle of Logical Consistency would be an *a priori* postulate —that is, a postulate which, if denied, would leave the Reason irrational. An *a priori* postulate is the Reason's demand to have its own intrinsic nature respected as an indispensable precondition to its functioning at all. Such a postulate cannot be denied without self-contradiction. For what is to guarantee the rational character of the denial if the requirement of logical consistency be not respected? What is to hinder the denial from being explained away as a corroborative affirmation in disguise?

Under the second of the two headings we have the Inductive Postulate of Determinism. This deterministic postulate is not *a priori* in the sense in which this term has just been defined. It is not a law of Thought. The Reason and this fundamental stipulation of Inductive Method do not stand and fall together. The *Method*, however, of which the postulate is the principle, *does* stand or fall with the postulate, and it is on this account that the postulate is called 'methodological.' A methodological postulate lies at the root of rational inquiry in this sense—that it defines the *type of explanation* that such inquiry must reach after, and the *type of method* appropriate to such explanation. 'Inductive Principle,' 'Inductive Postulate,' 'Inductive Method,' 'Inductive Ideal of Explanation,' 'Inductive Conception of Fact,' are but different expressions of one and the same dominating determinant: *the scientific point of view, the point of view of the external observer.*

We propose, then, to use the term 'postulate' in the sense of *methodological* postulate. Hence we shall be free to discuss the limits of the Inductive Postulate, for these will not necessarily coincide with the limits of the human understanding.

In a certain sense, a methodological postulate may be described as a working idea. It is a working idea of the Reason, all Reason's methods being working methods to be appraised by their explanatory power. But it is not a working idea of the scientific point of view, for its limitations coincide with the scientific horizon. It is constitutive of the scientific outlook, and not tentatively regulative of it. *Relative to the scientific point of view,* the Inductive Postulate is an *a priori* postulate, a vital requirement which cannot be severed

from the method which it informs without leaving that method incapable of development and condemned to barrenness.

The deterministic postulate should be distinguished from the *uni*deterministic ideal of Natural Science. If the former defines the *legitimacy* of a hypothetical explanation, the latter defines the essential condition on which its *conclusiveness* depends. An explanation may be said to be conclusive when accompanied by the demonstration that no other is possible. Thus, the proof that a given causal relation is necessarily reciprocal—a proof, in other words, which eliminates the possibility of a plurality of causes—would satisfy the unideterministic ideal of Scientific Explanation.

The Inductive Postulate and the Inductive Principle.

The Inductive Principle of Fidelity to Relevant Fact can be fully understood only in the light of the Inductive Postulate. For it is only through the latter that we can clearly perceive the meaning of the word 'relevant.' Fact is relevant to inductive inquiry only in so far as it is conceived as the expression of Natural Law—only in so far, that is, as it is studied under the limitations of the Inductive Postulate. The Inductive Postulate simply specifies the enunciation of the Inductive Principle; an inquiry carried out under the general inspiration of the latter has its direction specifically determined by the meaning put upon the former.

Postulate and Necessary Truth.

Dr. Whewell defined a necessary truth as a proposition the negation of which is not only false, but inconceivable;* and Herbert Spencer's definition is practically the same as Whewell's.

Mill denies that this is an adequate account. He objects, and rightly, to measuring the possibility of things by our human capacity of conceiving them. He explains that 'when we have often seen and thought of two things together, and have never in any one instance either seen or thought of them separately, there is by the primary law of association an increasing difficulty, which may in the end become insuperable, of conceiving the two things apart.' Thus, even eminent persons—*e.g.*, Comte, when he urged (at the very time when the principles of Spectroscopy were being discovered) that it was inconceivable that we should ever discover what the stars were made of—have seemed unable to conceive what was afterwards found to be not only quite conceivable, but quite true.

Mill has done good service by clearly distinguishing between the *inconceivable* and the *unbelievable*;† but a further distinction is

* *Vide* J. S. Mill, 'A System of Logic,' Book II., ch. v., § 6.
† *Ibid.*, Book II., ch. vii., § 3.

29

imperatively called for—that between the *inconceivable* and the *unimaginable*. *These* two words Mill seems to regard as identical in meaning. For our part, we hold that the only genuine inconceivable is the strictly irrational, the self-contradictory, the meaningless. Where it is still conceivable that greater knowledge may shed light upon a mystery, that mystery cannot logically be termed 'inconceivable.' We may feel very sceptical about it, but that is another matter.

Mill's own view of necessary truth is far from satisfactory. He holds that what we call necessary truths are experimental truths, generalizations from observation. 'The proposition, Two straight lines cannot enclose a space . . . is an induction from the evidence of our senses.'* He argues that it is unreasonable to attribute to these truths an origin different from that of all the rest of our knowledge of Nature when their existence is perfectly accounted for by supposing their origin to be the same. Speaking still of the axiom, 'Two straight lines cannot enclose a space,' he writes: 'Experimental proof crowds in upon us in such endless profusion, and without one instance in which there can be even a suspicion of an exception to the rule, that we should soon have stronger ground for believing the axiom, even as an experimental truth, than we have for almost any of the general truths which we confessedly learn from the evidence of our senses.'

We shall presently discuss the weakness of this position of Mill's —*i.e.*, of the attempt to treat a necessary truth (*e.g.*, an axiom of Geometry or the Law of Causation) as a mere hypothesis that is more than usually well grounded. Meanwhile we have already committed ourselves to the conviction that the *demands* or postulates which the Reason is obliged to make as a condition of its effective exercise are necessary truths, and that these are of two kinds:

(*a*) *A priori* postulates—the absolutely necessary truths (*e.g.*, the Law of Non-Contradiction).
(*b*) *Methodological* postulates—truths necessary, not, indeed, for the reason generally, but for the reason as self-limited to some particular universe or aspect of Reality.

The Inductive Postulate as the Postulate of a Mechanical or Deterministic Explanation of Nature.

The Inductive Postulate—the Postulate of Causal Explanation—may be enunciated as follows: *Fact is inductively explained only in so far as it has been determinately brought under Causal Law.* This may be more expressively named the Deterministic Postulate.

* *Vide* J. S. Mill, 'A System of Logic,' Book II., ch. v., § 4.

The demand that the explanation of Nature shall be given in terms of Causal Law is far from being a mere abstract formula, of interest to logicians only. It represents the hardest-won victory of Science, and is a demand over which the keenest controversies are waged even at the present day. It represents the victory of the mechanical view of Nature over the magical. Indeed, the true, living value of the deterministic conception of Explanation is seen only in contrast with the magical, anthropomorphic conception which it supplanted.

To the savage all things appear to have psychical life, and he interprets the movements of inanimate objects as though they had their source in psychological motives. He thinks of plants and animals as quasi-persons, and seeks to determine their behaviour by prayers, sacrifices, etc. This is his anthropomorphism.

Again, the savage sees a sympathetic connexion between objects either because they are like one another (resemblance), or because they have previously been connected together (association). Thus the cut hair of a man, his shadow, image, or picture are conceived of as so closely related to him that it is possible by injuring them to injure the man himself. If a waxen image of a man be set in the sun to melt, the man himself will waste away (*cf.* D. G. Rossetti's 'Sister Helen'). Hence the custom of burying hair or nail-parings, the dislike of being sketched or photographed, and the care with which savages often keep their names, or the names of their gods, a profound secret.

Here are other instances : A savage will wear a ring of iron in order that it may impart its quality of hardness to his body, or when bargaining for a cow, or asking a woman for wife, he will chew a piece of wood to soften the heart of the person he is dealing with. So, again, having discovered in the lion the quality of courage, or in the deer that of swiftness, he eats the former that he may become bold, and the latter that he may run well. So he will eat an enemy to acquire his boldness, or a kinsman to prevent his virtues from going out of the family.

These instances might be indefinitely multiplied. Whether anthropomorphic or magical, the explanations are all equally non-mechanical. The postulate of Induction is the protest of Science against Anthropomorphism and Magic. It requires that natural effects shall follow from natural conditions, and *vice versa* that natural conditions shall give rise to natural effects. It is now recognized as the specific regulative principle of all the natural sciences—a principle, that is, which determines the general method or direction of inquiry, however hypotheses may be modified or working ideas displaced.

Towards the middle of the nineteenth century, certain puzzling irregularities were observed in the movements of the planet Uranus. There were two conceivable ways of hypothetically explaining

these irregularities. The orbit of Uranus must have been modified either mechanically, by the influence of matter in motion elsewhere, or else by the immediate operation of some volitional agency of more than human power. This second alternative was not in itself inconceivable. It was inadmissible only as a scientific explanation. It was an alternative that Astronomy could not possibly have admitted without admitting that it had transgressed its own limitations—*i.e.*, without ceasing to be Astronomy. Supposing, now, that some incalculable demon had really been responsible for the perturbations, could Astronomy, we ask, ever have found it out? By no means. It would still be puzzling its mighty intellect for a mechanical solution, and meanwhile blaming its telescopes, or the irreflective nature of the surface of the disturbing body, or its extraordinary density that resulted in its being too small for visibility, etc.; and so it would go puzzling on for ever, readjusting its hypotheses and working conceptions—even, perhaps, that of gravitation itself—in order to render the phenomenon mechanically intelligible. It would, in fact, simply repeat over again, in its improved modern way, those processes of adjusting epicycles and excentrics which were forced by the same respect for regulative Ideas upon the bewildered observers of the Middle Ages. The postulate or Supreme Idea is a principle for working with, and not for discussing.

The same great regulative principle—the principle of Determinism—is sometimes assumed even in Psychology, and by the most modern writers. 'Psychology, like every other science,' writes Höffding, 'must be deterministic—*i.e.*, it must start from the assumption that the causal law holds good even in the life of the will, just as this law is assumed to be valid for the remaining conscious life and for material nature. If there are limits to this assumption, they will coincide with the limits to Psychology.'* Professor James speaks in a precisely similar manner: 'Psychology, as a would-be Science, must, like every other Science, *postulate* complete determinism in its facts, and abstract consequently from the effects of free-will even if such a force exists.'†

The identification of the Inductive with the *Deterministic* Postulate may appear to some to be arbitrary, and to involve a gratuitous restriction of the Inductive Method. This, however, is not the case. There is inherent in the very attitude of Science towards its facts a restriction which compels the identification in question. An intrinsic requirement of scientific method is that the theorist shall approach his facts from a standpoint external to the facts themselves, and this *external* attitude is responsible for the form which Inductive Method necessarily takes. There are two main ways in

* 'Outlines of Psychology,' English translation, by Mary Lowndes, p. 345.

† 'Text-book of Psychology,' p. 456. For a criticism of these views, see 'Personal Idealism, Philosophical Essays,' edited by Henry Sturt, pp. 166 *et seq.*

which we can study facts. We may study them in relation either to a scientific or to a philosophic interest. In the former case, we are concerned solely with the relations in which the facts stand to each other. In the second case we are concerned with their relation to us, who know them and observe them, with their function as factors in a concrete spiritual experience. The dominant question here is : 'How do the facts we are studying express spiritual purpose ?' Such teleological investigation, however, is possible only when the inner standpoint of personal experience is adopted. When, by the nature of the case, the object is approached from a standpoint external to it, it is only the external, sense-perceivable behaviour of the object that admits of being studied. Thus, in this case the only question which we can legitimately ask about our object is : 'How does this object embody natural law ?' Now, the Sciences of Nature are obliged to approach their object from the outside. 'Whatever life or mind may constitute the inner being of so-called inanimate Nature, the scientist cannot share it in such a way as to make any knowledge of its procedure as a purposeful agent a basis for his investigations.'* But just in so far as we fail to regard an object *from the inner point of view* of the end or purpose that object may be tending to realize, a procedure of tentative explanation becomes imperative. Laws of behaviour must be hypothetically superinduced upon the object, and be left entirely dependent on verification for their objective acceptance.

If the question should be asked, 'Why do we need to make postulates at all ?' the reply is simply this : that the postulate is needed to define what we mean by a legitimate explanation. It is of no use to begin an inquiry without a test which shall enable us to decide whether an alleged explanation is legitimate or not—whether, in fact, it can be accepted as a *possible* explanation. The postulate of scientific method is a test of this kind. If a suggested explanation violates the postulate of mechanical connexion, Inductive Science will have none of it. The true interests—indeed, the necessities—of Inductive Science demand or postulate that no explanation of a magical or otherwise indeterministic character be entertained for a moment, even as a possible explanation.

In a famous chapter of his 'Logic' (Book III., ch. xxi.) Mill has endeavoured to present the Inductive Postulate, or, as he calls it, 'the Ground of Induction,' as a generalization from experience, and as standing on precisely the same inductive footing as a well-established hypothesis or law of Nature. He enunciates this Law of Causation, or Ground of Induction, as follows :

'Every event, or the beginning of every phenomenon, must have some cause, some antecedent, on the existence of which it is invariably and unconditionally consequent.'†

* From 'A Philosophical Introduction to Ethics,' p. 54.

† 'A System of Logic,' Book III., ch. xxi., § 1.

This law, he maintains, can be inductively proved, and that by a process of Simple Enumeration. In certain cases, he says, we may have the completest proof based on simple enumeration of instances—namely, in those cases in which our survey over instances is so extensive as to leave us convinced that, had there been any instance contrary to the law, we should have met with it.

In order to be sure of the truth of an Induction by Simple Enumeration, we must, according to Mill, be able to affirm two things: (1) that 'we have never known an instance to the contrary'; and (2) 'that if there were in Nature any instances to the contrary, we should have known of them.'* In most cases of this kind of Induction the first affirmation has to be made without the second. But the peculiarity of Induction by Simple Enumeration, argues Mill ingeniously, is that it 'is delusive and insufficient exactly in proportion as the subject-matter of the observation is special and limited in extent. As the sphere widens, this unscientific method becomes less and less liable to mislead; and the most universal class of truths—the law of causation, for instance, and the principles of number and of geometry—are duly and satisfactorily proved by that method alone, nor are they susceptible of any other proof.'†

Mill sustains this point by the following argument: An Induction by Simple Enumeration can be affirmed as true only 'within certain limits of time, place, and circumstance,' the reason for not extending its application beyond those limits being 'that the fact of its holding true within them may be a consequence of collocations, which cannot be concluded to exist in one place because they exist in another; or may be dependent on the accidental absence of counteracting agencies, which any variation of time or the smallest change of circumstances may possibly bring into play.'

Now, argues Mill, 'if we suppose . . . the subject-matter of any generalization to be so widely diffused that there is no time, no place, and no combination of circumstances, but must afford an example either of its truth or of its falsity, and if it be never found otherwise than true, its truth cannot be contingent on any collocations, unless such as exist at all times and places'—*i.e.*, it must hold good for all collocations;‡ 'nor can it be frustrated by any counteracting agencies, unless by such as never actually occur. It

* 'A System of Logic,' Book III., ch. iii., § 2.

† *Ibid.*, ch. xxi., § 3.

‡ By 'collocation' (an expression borrowed from Dr. Chalmers) Mill means the coexistence of causes or causal tendencies in certain relative positions and relations. For example, a lady is troubled, in cycling, by a constant noisy rattle, and succeeds in tracing this result to an interference between gear-case and loose chain. The gear-case is seen to be slightly displaced, but this of itself could not have caused the rattle provided the chain had been sufficiently taut, nor would the mere looseness of the chain have caused it if the gear-case had been in order. The fact of the two circumstances being present together—their '*collocation*,' as Mill would put it—is indispensable for the production of the effect.

is, therefore, an empirical law coextensive with all human experience, at which point the distinction between empirical laws and laws of Nature vanishes.' In other words, the original limitations characteristic of an enumerative induction (in Mill's sense of the term) are then entirely cancelled. The law established by such an induction, says Mill, 'takes its place among the most firmly established as well as largest truths accessible to science.'

Criticism of Mill's Attempted Proof of the Law of Causation.

Our criticism of this justly celebrated proof may conveniently be arranged under three heads :

(i.) *Even if the Law of Causation were a generalization from experience,* it could not be established by the simple enumeration of instances.*

Mill's proof supposes that the uniform working of the Law of Causation is much more obvious than is, in fact, the case. He does, indeed, clearly distinguish the Uniformity of Nature (the 'universality of the Law of Causation '†) from any obvious uniformity in the succession of 'physical facts';‡ and we must also remember that Mill regards the Law of Causation as a generalization not from individual instances of causal sequence, but as obtained 'by generalization from many laws of inferior generality,'§ as 'obtained by induction from particular laws of causation.'‖ But the process by which these particular laws are in the first instance established is 'the loose and uncertain mode of induction *per enumerationem simplicem*'; nor are we given to understand that the higher generalization is reached by means of any other less superficial method. Now, this method is essentially unanalytic, and therefore powerless to elicit from the apparent confusion of natural phenomena their underlying regularity and order. As Mill truly says, 'The course of Nature . . . is not only uniform; it is also infinitely various.'¶ But if this is so, then investigation by means of Simple Enumeration alone would as often oppose the hypothesis of Causal Law as it would confirm it. As Dr. Sigwart has shown, all we can attain to by this method is a number of empirical rules accompanied by a large and equally imposing number of exceptions. The process leads naturally to the conviction 'that law and disorder bear sway in wild alternation.'** To this rudimentary form of observation 'the universe becomes

* The term 'experience' is here naturally, indeed inevitably, used in the restricted sense of 'experience as relevant to a pre-scientific or scientific interest.'

† 'A System of Logic,' Book III., ch. xxi., § 1.

‡ *Cf. Ibid.*, § 2, second footnote.

§ *Ibid.*, § 2.

‖ *Ibid.*, § 3.

¶ *Ibid.*, ch. iii., § 2.

** 'Logik,' vol. ii., ch. v., § 93, 12. English translation, pp. 306, 307.

divided into one sphere, in which we feel at home, and are accustomed to expect results with certainty, and another composed of phenomena which are changing, variable, and fortuitous.'*

(ii.) *The Law of Causation, as the Ground of Induction, cannot be a mere generalization from experience ; it must have a methodological significance.*

Let us, then, suppose that this proof by Enumeration is abandoned, and that the attempt is made to prove the Law of Causation by a scientific induction. We will suppose that the surface-view of fact, inseparable from the Method of Simple Enumeration, is given up, and that a thorough-going analysis takes its place. We will further suppose that, as the result of such analysis, Nature has never failed to reveal the uniformity which the hypothesis requires for its verification. There are, indeed, indications that some such procedure as this is what Mill really has in mind, and that his Enumeration process is not so unanalytic as its name implies : for he says that it is as phenomena 'become better known to us' that they are found to obey the law of uniformity of succession; that it is 'after due examination,' and when we know a phenomenon 'sufficiently well,' that we are able to perceive its obedience to causal law.† In any case, it might be thought that, if the suppositions suggested above were granted, Mill's main contention that the Law of Causation is a generalization from experience would be justified, though we might be dissatisfied with his method of supporting his thesis. Let us consider this point.

We must admit that, if Mill were prepared to accept the Law of Causation as a simple hypothesis of precisely the same standing as the hypothesis of gravitation or any other well-grounded hypothesis, and did not attempt to erect the law into a Ground of Induction on which 'the validity of all the Inductive Methods depends,'‡ the proof might be accepted as amounting to a very satisfactory verification. And, indeed, Mill does not regard the Uniformity of Nature as anything more than the most general and the most extensively verified of causal laws, as witness the following famous passage :

'The uniformity in the succession of events, otherwise called the Law of Causation, must be received not as a law of the universe, but of that portion of it only which is within the range of our means of sure observation, with a reasonable degree of extension to adjacent cases.'§

But if the Law of Causation is itself an induction, it cannot be at the same time the Criterion of Induction. The Ground of Induction, as Mill understands it, cannot be the Standard of Induc-

* 'Logik,' vol. ii., ch. v., § 93, 12. English translation, p. 306.
† 'A System of Logic,' Book III., ch. xxi., § 4. ‡ *Ibid.*, § 1.
§ *Ibid.*, § 4.

tion. It cannot tell us what we are to understand by an inductive explanation, and so enable us to distinguish between an explanation that is inductively legitimate and one that is inductively illegitimate. In so far, then, as the *methodological* significance of a ground of Induction depends on its ability to supply a criterion for the legitimate application of Inductive Method, the Law of Causation, as interpreted by Mill, cannot be said to possess any methodological significance.

The question then remains : What significance other than this methodological significance can a ground of Induction possess ? Or, to state the question in the narrower form which alone is relevant to the present criticism : What significance other than this methodological significance can *Mill's* Ground of Induction lay claim to ? The function of the Ground of Induction, according to Mill, is to serve as the ultimate major premiss for every specific induction, 'not contributing at all to prove it, but being a necessary condition of its being proved' (Book III., ch. iii., § 1). Thus, the statement, 'The course of Nature is uniform'—Mill's favourite expression for the Ground of Induction—is a necessary condition for proving, for instance, that heat causes evaporation. For if we had no warrant for assuming the uniformity of Nature, we should not feel safe in concluding from our observations on heat that it tends to cause—*i.e.*, uniformly tends to cause—the evaporation of liquids. Now, our sole guarantee, according to Mill, for assuming that Nature is uniform is that *Experience shows* with convincing consistency that 'it is a law that there is a law for everything' (Book III., ch. v., § 1). The word 'Experience,' however, is ambiguously vague. So far as the term has any relevancy to Mill's argument, it should mean 'Experience as relevant to the inductive interest.' It is, then, 'Experience as relevant to the inductive interest' which establishes on a firm inductive basis the uniformity of Nature. Hence, since the inductive interest itself defines the very experience which establishes this uniformity, we are driven to ask whether, apart from that interest, the uniformity could ever be established. The query, so formulated, leads us at once to the crux of the indictment. We shall see that this inevitable reference to the inductive interest requires that our confidence in the uniformity of Nature as the Ground of Induction shall itself rest on the postulate that Nature is uniform. It is only as depending on this methodological demand that the Ground of Induction can guarantee specific inductions in the sense indicated by Mill.

(iii.) *Mill's proof of the Law of Causation involves, either directly or indirectly, a Petitio Principii.*

That the Law of Causation, the 'ultimate major premise' (as Mill calls it) of every scientific induction, should itself have been obtained by 'Induction' does not, from Mill's point of view,

involve any *petitio principii ;* for he maintains that the major premiss is never ' the proof of the conclusion, but is itself proved, along with the conclusion, from the same evidence.'* But we hold that the fallacy of *petitio principii* vitiates the very process by which, according to Mill, the ' ultimate major premise ' has been obtained. A successful proof of the Law of Causation, even within the limits of a restricted range of experience, must necessarily be based on the assumption that there is more uniformity in Nature than at first meets the eye. As we have already seen, the facts do not thrust the idea of Causal Law upon the impressionable investigator. Apparent exceptions to the reign of law will inevitably be met with. But if the scientist is not assuming that a fact must exemplify law, there is no reason why he should not accept these exceptions as final. Why should he suspect that the exceptions are but disguised exemplifications of law ?

The simple truth is that we are logically justified in treating apparent exceptions to order as merely disguised instances of it only on condition that we deliberately make it a postulate of the search after knowledge that we shall *look for* order even where order is not palpably manifested in the facts. The mind must make its own demand for Causal Uniformity, or it will never find it realized. So far as Mill finds order beneath the surface of natural phenomena, he does so by implicitly assuming the fundamental postulate of all inductive inquiry ; he assumes that the intelligibility of fact for Science depends on its being conceived in the light of a law.

Limits of the Inductive or Deterministic Postulate.†

Provided that the Postulate of Determinism be treated simply as a methodological guiding principle—as limiting the sphere in which Inductive Method is applicable, and as having no relevance to facts that refuse to be explained on the ground of it—this principle needs no other limitation than that imposed upon it by the facts themselves.

From our present point of view facts may be roughly classed under three heads :

(1) Inorganic.
(2) Organic.
(3) Self-conscious.

In reference to this rough division of facts, we may say that the postulate works well within the first two realms of fact (though the

* ' A System of Logic,' Book III., ch. xxi., § 4.

† In what follows I am much indebted to Dr. Sigwart's section on ' Explanation by the Nature of Substances ' in the fifth chapter of his already classical Methodology (' Logik,' vol. ii., ch. v., § 100. English translation by Helen Dendy, pp. 460-480).

nature of the determinism has to be understood differently in the two cases), whereas with what is central and essential in the third group of facts it is entirely inadequate to deal.

Whether Determinism is an effective postulate or not, and how, when adequate and fruitful, it is exactly to be interpreted, depends on the ultimate nature of the substance whose movements or activities it is concerned with interpreting. We require Ideas or working concepts as to the nature of the substances between which causal interaction takes place, as well as Ideas concerning the nature of that causal interaction itself.

Thus, in dealing with inorganic phenomena, Science has adopted the concepts of the molecule, the atom, and the electron. In attempting to deal with organic phenomena on the basis of the same suppositions as to the nature of substance, it has found itself unable to give complete explanation of the facts, and it has been obliged to adopt the further concepts of *individuality* and *development*. Finally, in attempting to explain mental activity, the concept of *freedom*, which has been found indispensable for giving any meaning to human action, has sprung up in direct antagonism to the deterministic postulate.

Let us consider these points more closely :

1. The concept of the *atom*. The atom (or the electron, which seems likely to take its place as representing the ultimate nature of substance for physical Science) is conceived as an indivisible, invariable force-centre, inherently possessing certain fundamental force-attributes; and upon this view of the ultimate nature of (material) substance is based 'the conception of a mechanism of the universe which attempts to represent all perceptible events as the motion of invariable atoms according to invariable laws.'*

The mechanism of the heavens, of which this mechanism of the universe is the imaginative extension, furnishes the best example of what such a conception involves. It is needless to say that the extended conception of a thorough-going mechanical (atomistic) explanation of the universe has not the same justification as that of the mechanical explanation of the planetary system. That could be the case only were this explanation to interpret the whole course of the universe as intelligibly as the motions of the planets and their satellites are interpreted by the laws of gravitation and inertia. In the one case the Given is completely explained on the ground of the mechanical hypothesis; in the other it is far indeed from admitting of any such complete explanation.

Thus, the attempt to explain the facts of development by the interactions of atoms that mutually attract or repel each other is far from being satisfactory. We need here other concepts or categories than those of 'atom' and 'inherent force.' The thorough-

* Dr. Christoph Sigwart, 'Logik.' vol. ii., ch. v., § 100, 11. English translation by Helen Dendy, p. 469.

going atomist tells us that each individuality is only a collective aggregate of atoms which interact by virtue of their inherent forces, and that the development of these individualities, their disposition to pass through successive stages, is already pre-established in the original configuration of the atoms relative to one another. It need hardly be pointed out that upon this view 'life' and 'mind' are by-products, the world's course being already mechanically predetermined. Thus, consistent atomists regard Consciousness as epiphenomenal, as a mere spectator of its own predetermined changes.

2. Truer conceptions of development and of individuality are obviously needed when we come to deal with organic life. Let us first consider the meaning of 'development,' as this meaning has itself gradually developed. In the meaning of the term 'organic process' or 'development' we can, as Dr. Sigwart says, distinguish several stages of growth.

First, we have the original meaning of the word, a mere unfolding, as in the opening of a rolled-up scroll or the expansion of a bud into the full-blown flower. This conception is then enlarged, so as to take in at the same time the idea of growth, a growth not only in volume, but also in differentiation. This meaning also is illustrated by the development of a flower-bud. Not only do its parts unfold, but they also change in size and shape, and their tissues become continually less and less homogeneous. A still fuller meaning is gained when all the particular stages of the process are explicitly referred to one developing individual, and an antithesis is drawn between the beginning *from* which and the end *towards* which the subject develops. The end of the development is then conceived as revealing what the beginning contained, as the oak reveals the true nature of the acorn. Finally, when the concept of development is made to extend 'beyond particular individuals to the whole range of the organic universe,' it has reached its deepest meaning; but in establishing this we find a difficulty which lies in our inability to fix definitely upon 'the subject to which this universal development is to be ascribed.'

Now, from the point of view of our logical inquiry, the main thing to note is that, though we may still profitably adhere to our deterministic postulate in the investigation of organic as in that of inorganic forms, we have to introduce into the former investigation a new conception of causal explanation. The earlier stages of the growth of an organism cannot be said to account for the later stages in the development in the same sense as that in which the original distribution of atoms in space can be said to account for the later distribution of these atoms. For the change we call 'development' is always *qualitative* as well as quantitative, and atomistic explanation practically ignores all qualitative changes.

Thus, in the case of organic development, we cannot account for

any one phase by merely pointing out the phases antecedent to it. All we can truly say is that the phenomena of the beginning of a life are imperfect manifestations of a principle which is more completely manifested in the later stages. Hence, the study of *antecedents* gives us the *least* distinct clue to this inner principle. The forest-tree, with its deep-striking, ramifying root, its massively towering trunk, and its far-spreading system of branches, more truly expresses the nature of its species than did the embryonic infant-plant contained in the seed from which it sprang. The strongly developed root-system that is able to withstand the mighty strain and leverage of the storm-wind was but feebly represented by the minute and unbranched radicle ; the giant branches upholding their dense cloud of luxuriant foliage are more explicitly significant than the tiny and delicate plumule ; the fully differentiated tissues of leaf and twig, of pith and bast and woody fibre, were but dimly foreshadowed in the soft, rudimentary features of the small white folded embryo. Most significant of all is the mature tree in its flowering and fruiting seasons, for an organism is never so truly or so explicitly itself as in that process of reproduction which is the culminating-point of its development. In the physical, no less than in the moral, world does the saying hold good : 'By their fruits ye shall know them.'

3. That the end explains the beginning in a profounder and completer sense than that in which the beginning explains the end is a principle that applies to all developing life as such, whether the life, like that of a tree, is unconscious of its own development, or is conscious, or at least partially conscious, of it, as in the case of the life of Mind. This type of explanation, which is known as teleological, is indeed characteristic of all our attempts to understand conscious experience. Just as a finished essay explains a writer's idea far more truly than do the first rough, incoherent jottings—as these jottings, though they do explain something, though they tell us how the thoughts struggled into being, and give us the early history of the idea, yet do not give us its true meaning—so Man is explained, indeed, both by his past and by his future, but more truly by his future—by his destiny—than by his past. The teleological explanation reaches deeper than the genetic.

The attempt, when dealing with mental development, to explain the subsequent completely by means of its antecedents inevitably issues in fallacy. Thus, even if we were to grant that the religious sentiment was at first no other than a belief in ghosts, we could not reasonably go on to argue that, since the religious sentiment of to-day is but a development of the religious sentiment of primitive man, it must therefore still be essentially a mere transfigured belief in ghosts. We might just as profitably argue that, since the first

efforts of Science produced nothing but fanciful conjecture, therefore Science must be at bottom a mere collection of fancies.

But it is when we come to deal with the self-conscious activities of moral beings that we most clearly realize the limits of all deterministic—*i.e.*, of all inductive—explanation. For any attempt to explain morality on the basis of determinism completely stultifies that which it seeks to explain. Where there is no freedom, there is no responsibility, no duty, no ideal to be striven after—for why should we strive against the inevitable ?—so that morality is a question no longer of character or even of conduct, but merely of customary behaviour ; and ethics a science no longer of what ought to be, but of what has been and must be.

With the conception of Freedom, and still more so with the further concepts of Immortality and God—concepts needed, we believe, for explaining the deepest rooted of our moral difficulties—we leave the deterministic postulate far behind. The category of mechanical causation must be transcended. What categories of explanation there may be which can transcend it—this is a question which, when systematically conceived, forms the main problem of a Philosophical Logic.

INDEX, VERBAL AND ANALYTIC.

THE END

BILLING AND SONS, LTD., PRINTERS, GUILDFORD